ANALYTICAL THEORY OF THE LIBRATION OF THE MOON

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Abstract. This paper presents a new theory of the libration of the Moon, completely analytical with respect to the harmonic coefficients of the lunar gravity field. This field is represented through its third degree harmonics for the torque due to the Earth (second degree for the torque due to the Sun).

The orbital motion of the Moon is described by the ELP 2000 solution (Chapront-Touzé, 1980) of the main problem of lunar theory.

The physical libration variables are obtained as Poisson series and comparisons with the results of Eckhardt (Eckhardt, 1981) and Migus (Migus, 1980) are presented.

1. Introduction

Since 1970, when the precision of the observations on the Moon's motion jumped by a factor 10^3 with the LRRR data, much work has been done in order to produce a theory with an equivalent level of accuracy.

The numerical integration approach is well known and presents essentially two advantages: facility of implementation and high precision of results; we are concerned here with an analytical theory. This theory may be separated in two parts: the orbital motion of the Moon and its rotational motion about its center of mass, the second one taking the first one as data and being strongly dependent on it. This fact is probably the major reason explaining the difference between the states of art in the two parts: when the orbital motion theory is in the process of completion (Chapront-Touzé, 1982), an important work on the libration remains to be done. Up to recently the only theories taking into account the planetary perturbations (Migus, 1977; Eckhardt, 1981) were still based on the ILE (Eckert *et al.*, 1954). Eckhardt (1982) has just now published new tables based on Chapront's solution (Chapront and Chapront-Touzé, 1982).

Nevertheless, very precise theories have been built these last years concerning the effect of the Earth and the Sun on the rotational motion of the Moon (Migus, 1980; Eckhardt, 1981; Moons, 1982). The results of one of them are presented here in details. The method used being already explained in previous papers (Henrard and Moons, 1978; Moons, 1981, 1982), we will be satisfied with a brief summary of the theory.

2. Presentation of the Method

2.1 Phase space and Hamiltonian of the problem

To describe the rotational motion of the Moon, we adopt two direct reference frames centered at the center of mass of the Moon:

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- $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ which is an inertial system of reference, $(\mathbf{e}_1, \mathbf{e}_2)$ being the ecliptic 2000 plane with $\mathbf{e}_1$ towards the perigee of the Sun;
- $(\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3)$ which is the system of the Moon's principal axis of interia coresponding to the moment of inertia matrix

$$\begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix} \text{with } A \leqslant B \leqslant C.$$

As shown in Figure 1, the position of the second frame with respect to the first one is described with the Andoyer's canonical variables

$$\begin{aligned} &\mu_1 \qquad M_1 = \|\mathbf{L}\| \cos I, \\ &\mu_2 \qquad M_2 = \|\mathbf{L}\| \\ &\mu_3 \qquad M_3 = \|\mathbf{L}\| \cos b, \end{aligned}$$

where $\mathbf{L}$ denotes the angular momentum of the Moon.

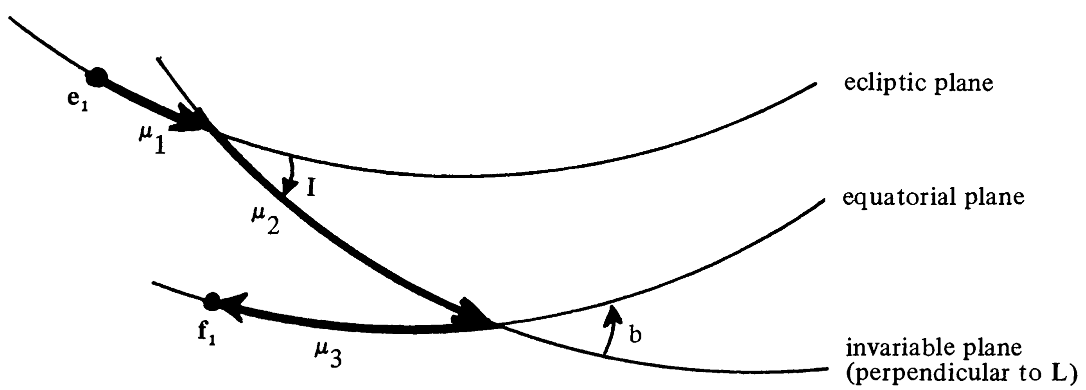

Fig. 1.

In fact, in order to control the virtual singularities of the Andoyer's elements, we use the modified Andoyer's variables

$$\begin{aligned} \lambda_1 &= \mu_1 + \mu_2 + \mu_3, & \Lambda_1 &= M_2, \\ \lambda_2 &= -\mu_3, & \Lambda_2 &= M_2 - M_3, \\ \lambda_3 &= -\mu_1, & \Lambda_3 &= M_2 - M_1; \end{aligned}$$

the singularities of which are of polar-coordinates type.

With these variables, Cassini's laws (Tisserand, 1898) are expressed by

$$\lambda_1 = \lambda + \pi, \qquad \Lambda_1 = nC, \qquad \Lambda_2 = 0, \qquad \lambda_3 = -h, \qquad \Lambda_3 = ct,$$

where λ denotes the mean longitude of the Moon with respect to the Earth, h is the longitude of the ascending node of the Moon's orbit and n the mean motion in longitude of the Moon.

Keeping in mind Cassini's laws, we use the following canonical transformation:

$$x_1 = \lambda_1 - \lambda - \pi, \qquad y_1 = \frac{\Lambda_1}{nC} - \nu,$$

$$x_2 = \sqrt{\frac{2\Lambda_2}{nC}} \sin \lambda_2, \qquad y_2 = \sqrt{\frac{2\Lambda_2}{nC}} \cos \lambda_2,$$

$$x_3 = \sqrt{\frac{2\Lambda_3}{nC}} \sin (\lambda_3 + h), \qquad y_3 = \sqrt{\frac{2\Lambda_3}{nC}} \cos (\lambda_3 + h) - 2\mu;$$

in order to obtain dimensionless cartesian-like coordinates (x_i, y_i) expressing deviations with respect to a mean equilibrium position. The constant ν, close to 1, comes from the fact that the value of Λ_1 is close to nC and the constant μ, close to 0.013, is related to the value of $\sin I/2$.

The Hamiltonian for the libration of the Moon describes the rotational motion of the Moon around its center of mass in the gravity field of Earth and Sun. It is expressed by an expansion in (x_i, y_i) and the constants μ and ν are determined in such a way that the quadratic part of the Hamiltonian is as close as possible to three harmonic oscillators:

$$H = \sum_{i=1}^{3} (A_i x_i^2 + B_i y_i^2) + F(x_1, x_2, x_3, y_1, y_2, y_3, t).$$

Considering the smallness of the quantities (x_i, y_i), we truncate the expansion at the fourth order in these variables.

2.2. Application of a perturbation method

Let us first introduce action-angle canonical variables by way of the relations

$$x_1 = \alpha_1 \sqrt{2P} \sin p \qquad y_1 = \frac{1}{\alpha_1} \sqrt{2P} \cos p,$$

$$x_2 = \alpha_2 \sqrt{2Q} \sin q, \qquad y_2 = \frac{1}{\alpha_2} \sqrt{2Q} \cos q,$$

$$x_3 = \alpha_3 \sqrt{2R} \sin r, \qquad y_3 = \frac{1}{\alpha_3} \sqrt{2R} \cos r.$$

The constants α_i are determined so as to equate $A_i \alpha_i^2$ with B_i/α_i^2 and the Hamiltonian becomes

$$H = n_p P + n_q Q + n_r R + F(p, q, r, P, Q, R, t),$$

with the following frequencies for the harmonic oscillators:

$$n_p = 0.025\,954\,386\,109 * n,$$

$$n_q = 0.000\,993\,516\,224 * n,$$

$$n_r = -0.003\,098\,977\,293 * n.$$

We can separate the Hamiltonian in two parts: i.e.,

$$H = H_0 + H_1$$

with H_0 depending only on the momenta

$$H_0 = n_p P + n_q Q + n_r R + n_t T.$$

The artificial momentum T, conjugate to the variable t, is introduced here to obtain a conservative Hamiltonian.

After the separation, we can apply the Lie transform method in order to obtain an expression of the Hamiltonian depending only on new momenta (which will be then constants). The generator of the transformation is then used to express any function of the osculating elements in terms of the new variables which are linear functions of the time. In particular, we can transform the functions determining the position of the moving reference frame with respect to the fixed one and, in this way, describe the libration.

In order to compare our results with others, we choose the variables p_1, p_2, the first two direction cosines of the pole of the ecliptic relative to the axes of inertia and τ, the libration in longitude (Eckhardt, 1965).

3. Presentation of the Results

3.1. Basic Assumptions and Data

We expand the figure of the Moon (assumed to be rigid) in spherical harmonics and consider the Earth and Sun as point masses. We take into account the influence of the Earth and Sun on the second harmonics and the influence of the Earth only on the third ones. The solution ELP 2000 (Chapront-Touzé, 1980) of the main problem of lunar theory supplies us with the position of the Earth. For the Sun, we adopt an elliptical orbit around the Earth–Moon center of mass.

The constants used are:

$$\kappa = M/E = 0.012\,300\,02,$$

$$\kappa' = (E+M)/S = 0.000\,003\,040,$$

$$C/(MR^2) = 0.394,$$

$$e' = 0.016\,708,$$

$$R/a = 0.004\,517\,2,$$

$$a/a' = 0.002\,571\,881\,3,$$

$$n' = 0.074\,800\,657\,5 * n,$$

$$n_g = 0.012\,473\,653\,1 * n,$$

$$n_h = -0.004\,021\,752\,14 * n,$$

where M is the mass of the Moon, E the mass of the Earth, S the mass of the Sun, R the equatorial radius of the Moon, e' the eccentricity of the apparent orbit of the Sun, a the mean Earth–Moon distance defined by $n^2a^3 = G(E+M)$ with G the universal gravitational constant, a' the mean Earth–Sun distance defined by $n'^2a'^3 = G(S+E+M)$ with n' the mean motion in longitude of the Sun, n_g the mean motion of the Moon's perigee, and n_h the mean motion of the node of the Moon's orbit.

The difference between these last three values and those of our 1982 paper causes some differences on the results we present here and previous results (Moons, 1982).

The choice of the values of R/a and a/a' implies the value of 1.002 726 for the λ constant of Jeffreys (Jeffreys, 1961).

The value of the mean motion of the Moon n is, in our theory, a scale factor and does not need to be specified at this stage.

3.2. Evaluation of the Series of the Forced Libration

The series p_1, p_2, τ we present in Table I are completely analytical with respect to the lunar gravitational field model parameters. They consist in a set of terms in the form

$$\text{COEFF} * \text{FACTOR} * {}^{\sin}_{\cos}(j_1 l + j_2 l' + j_3 F + j_4 D),$$

where COEFF is a numerical amplitude in arc seconds, $j_1, \ldots, j_4$ are integer coefficients, l is the mean anomaly of the Moon, l' the mean anomaly of the Sun, $F = \lambda - h$, D is the difference between Moon's and Sun's mean longitudes, and FACTOR an expression depending on MR^2/C and the harmonic coefficients C_{nm}, S_{nm}.

More precisely,

$$\text{FACTOR} = \prod_{i=1}^{9} (E_i)^{k_i},$$

where $k_1, \ldots, k_9$ are integer exponents

$$E_1 = \frac{\delta - 10.3 * 10^{-4}}{12 * 10^{-6}}, \quad \text{with} \quad \delta = \frac{2C - A - B}{C} = -\frac{2MR^2}{C} C_{20},$$

$$E_2 = \frac{\gamma - 2.3 * 10^{-4}}{6 * 10^{-6}}, \quad \text{with} \quad \gamma = \frac{B - A}{C} = \frac{4MR^2}{C} C_{22},$$

$$E_3 = \frac{-\eta * C_{30}}{15 * 10^{-6}}, \quad \text{with} \quad \eta = 0.394 \frac{MR^2}{C};$$

$$E_4 = \frac{\eta * C_{31}}{35 * 10^{-6}},$$

$$E_5 = \frac{\eta * S_{31}}{10^{-5}},$$

TABLE I

Series of the forced libration

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
0	0	0	0												
					-68.7462	0	0	0	0	0	1	0	0	0	P1
					-20.6096	0	0	1	0	0	0	0	0	0	P1
					0.6612	1	0	0	0	0	1	0	0	0	P1
					0.3300	0	1	0	0	0	1	0	0	0	P1
					0.1998	1	0	1	0	0	0	0	0	0	P1
					-0.1789	0	0	0	1	0	1	0	0	0	P1
					0.0999	0	1	1	0	0	0	0	0	0	P1
					0.0815	0	0	0	0	0	0	1	0	1	P1
					-0.0542	0	0	1	1	0	0	0	0	0	P1
					0.0417	0	0	0	0	0	1	0	1	0	P1
					0.0135	0	0	0	0	1	0	1	0	0	P1
					0.0127	0	0	1	0	0	0	0	1	0	P1
					-0.0063	2	0	0	0	0	1	0	0	0	P1
					-0.0062	1	1	0	0	0	1	0	0	0	P1
					0.0034	1	0	0	1	0	1	0	0	0	P1
					-0.0025	0	1	0	0	0	0	1	0	1	P1
					-0.0019	2	0	1	0	0	0	0	0	0	P1
					-0.0019	1	1	1	0	0	0	0	0	0	P1
					0.0017	0	1	0	1	0	1	0	0	0	P1
					-0.0016	0	2	0	0	0	1	0	0	0	P1
					0.0010	1	0	1	1	0	0	0	0	0	P1
					-0.0008	1	0	0	0	0	1	0	1	0	P1
					-0.0008	1	0	0	0	0	0	1	0	1	P1
					-0.0006	0	0	0	0	0	0	1	1	1	P1
					0.0005	0	0	0	0	0	1	0	0	2	P1
					0.0005	0	1	1	1	0	0	0	0	0	P1
0	0	0	1												
				0.0006		0	0	0	0	0	0	1	0	0	P1
0	0	0	2												
					0.0011	0	0	0	0	0	1	0	0	0	P1
				0.0008		0	0	0	0	0	0	1	0	0	P1
0	0	1	-4												
				-0.0025		0	0	0	0	0	0	0	0	0	P1
0	0	1	-3												
				0.0006		0	0	0	0	0	0	0	0	0	P1
0	0	1	-2												
					-0.0035	0	0	0	0	0	0	0	0	1	P1
				2.9086		0	0	0	0	0	0	0	0	0	P1
				0.0277		1	0	0	0	0	0	0	0	0	P1
				-0.0081		0	0	0	1	0	0	0	0	0	P1
				0.0047		0	1	0	0	0	0	0	0	0	P1
				-0.0011		0	0	0	0	0	0	0	1	0	P1
0	0	1	-1												
				0.1159		0	0	0	0	0	0	0	0	0	P1
				0.0020		1	0	0	0	0	0	0	0	0	P1
				-0.0007		0	0	0	1	0	0	0	0	0	P1
				-0.0007		0	1	0	0	0	0	0	0	0	P1
0	0	1	0												
					8.1765	0	0	0	0	0	0	0	0	1	P1
					1.3595	0	0	0	0	1	0	0	0	0	P1
					-0.1634	0	1	0	0	0	0	0	0	1	P1
					0.1009	1	0	0	0	0	0	0	0	1	P1
					-0.0458	0	0	0	0	0	0	0	1	1	P1
					-0.0298	0	0	0	1	0	0	0	0	1	P1
					-0.0273	0	1	0	0	1	0	0	0	0	P1
					0.0165	1	0	0	0	1	0	0	0	0	P1
					-0.0076	0	0	0	0	1	0	0	1	0	P1
					-0.0049	0	0	0	1	1	0	0	0	0	P1
					0.0043	0	2	0	0	0	0	0	0	1	P1
					-0.0024	1	1	0	0	0	0	0	0	1	P1
					0.0023	0	1	0	0	0	0	0	1	1	P1
					-0.0018	0	0	0	0	0	1	1	0	0	P1
					0.0007	0	2	0	0	1	0	0	0	0	P1
					-0.0006	1	0	0	0	0	0	0	1	1	P1
					-0.0005	0	0	1	0	0	0	1	0	0	P1
					0.0005	0	1	0	1	0	0	0	0	1	P1
				5566.1599		0	0	0	0	0	0	0	0	0	P1
				68.7840		1	0	0	0	0	0	0	0	0	P1
				34.0458		0	1	0	0	0	0	0	0	0	P1
				-25.1022		0	0	0	1	0	0	0	0	0	P1
				5.7743		0	0	0	0	0	0	0	1	0	P1
				0.1922		2	0	0	0	0	0	0	0	0	P1
				0.1906		1	1	0	0	0	0	0	0	0	P1
				-0.1386		1	0	0	1	0	0	0	0	0	P1
				-0.0689		0	1	0	1	0	0	0	0	0	P1
				0.0473		0	2	0	0	0	0	0	0	0	P1
				0.0322		1	0	0	0	0	0	0	1	0	P1

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
				0.0250		0	0	0	2	0	0	0	0	0	P1
				0.0160		0	1	0	0	0	0	0	1	0	P1
				-0.0117		0	0	0	1	0	0	0	1	0	P1
				0.0095		0	0	0	0	1	0	0	0	1	P1
				0.0055		0	0	0	0	0	2	0	0	0	P1
				0.0053		0	0	0	0	0	0	0	0	2	P1
				0.0047		0	0	1	0	0	1	0	0	0	P1
				0.0014		0	0	0	0	2	0	0	0	0	P1
				0.0014		0	0	0	0	0	0	0	2	0	P1
				0.0009		0	0	2	0	0	0	0	0	0	P1
				0.0008		2	1	0	0	0	0	0	0	0	P1
				-0.0006		1	1	0	1	0	0	0	0	0	P1
				-0.0006		2	0	0	1	0	0	0	0	0	P1
				0.0005		3	0	0	0	0	0	0	0	0	P1
0	0	1	1	-0.0090		0	0	0	0	0	0	0	0	0	P1
0	0	1	2	0.1190		0	0	0	0	0	0	0	0	0	P1
				0.0014		1	0	0	0	0	0	0	0	0	P1
				0.0007		0	1	0	0	0	0	0	0	0	P1
				-0.0006		0	0	0	1	0	0	0	0	0	P1
0	0	1	4	0.0009		0	0	0	0	0	0	0	0	0	P1
0	0	2	0		0.0007	0	0	0	0	0	1	0	0	0	P1
0	0	3	-2	-0.0129		0	0	0	0	0	0	0	0	0	P1
0	0	3	0	0.0126		0	0	0	0	0	0	0	0	0	P1
0	1	-3	2	0.0006		0	0	0	0	0	0	0	0	0	P1
0	1	-1	-2	-0.0083		0	0	0	0	0	0	0	0	0	P1
0	1	-1	0		-0.0017	0	0	0	0	0	0	0	0	1	P1
					-0.0007	0	0	0	0	1	0	0	0	0	P1
				1.0319		0	0	0	0	0	0	0	0	0	P1
				0.0487		0	1	0	0	0	0	0	0	0	P1
				0.0119		0	0	0	0	0	0	0	1	0	P1
				0.0116		1	0	0	0	0	0	0	0	0	P1
				-0.0055		0	0	0	1	0	0	0	0	0	P1
				0.0005		1	1	0	0	0	0	0	0	0	P1
0	1	-1	1	0.3059		0	0	0	0	0	0	0	0	0	P1
				0.0056		1	0	0	0	0	0	0	0	0	P1
				-0.0020		0	1	0	0	0	0	0	0	0	P1
				-0.0018		0	0	0	1	0	0	0	0	0	P1
				-0.0008		0	0	0	0	0	0	0	1	0	P1
0	1	-1	2	0.1181		0	0	0	0	0	0	0	0	0	P1
				0.0011		1	0	0	0	0	0	0	0	0	P1
				0.0005		0	1	0	0	0	0	0	0	0	P1
0	1	0	0		-0.0021	0	0	0	0	0	1	0	0	0	P1
					-0.0006	0	0	1	0	0	0	0	0	0	P1
0	1	0	1		-0.0054	0	0	0	0	0	1	0	0	0	P1
					-0.0015	0	0	1	0	0	0	0	0	0	P1
				-0.0065		0	0	0	0	0	0	1	0	0	P1
0	1	1	-2	0.0804		0	0	0	0	0	0	0	0	0	P1
				0.0008		1	0	0	0	0	0	0	0	0	P1
0	1	1	0		0.0017	0	0	0	0	0	0	0	0	1	P1
				1.2520		0	0	0	0	0	0	0	0	0	P1
				0.0392		0	1	0	0	0	0	0	0	0	P1
				0.0164		1	0	0	0	0	0	0	0	0	P1
				0.0093		0	0	0	0	0	0	0	1	0	P1
				-0.0069		0	0	0	1	0	0	0	0	0	P1
0	1	1	1	0.0013		0	0	0	0	0	0	0	0	0	P1
0	1	1	2	-0.0013		0	0	0	0	0	0	0	0	0	P1
0	1	3	-2	-0.0006		0	0	0	0	0	0	0	0	0	P1
0	2	-1	0	0.0013		0	0	0	0	0	0	0	0	0	P1
0	2	-1	2	0.0237		0	0	0	0	0	0	0	0	0	P1
0	2	1	-2	0.0020		0	0	0	0	0	0	0	0	0	P1
0	2	1	0	0.0034		0	0	0	0	0	0	0	0	0	P1
1	-2	-1	0	0.0013		0	0	0	0	0	0	0	0	0	P1
1	-2	1	-2	-0.0458		0	0	0	0	0	0	0	0	0	P1
				-0.0009		1	0	0	0	0	0	0	0	0	P1
1	-1	-1	-2	0.0023		0	0	0	0	0	0	0	0	0	P1

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
1	-1	-1	-1												
				-0.0134		0	0	0	0	0	0	0	0	0	P1
1	-1	-1	0												
				0.1274		0	0	0	0	0	0	0	0	0	P1
				0.0021		1	0	0	0	0	0	0	0	0	P1
				-0.0008		0	0	0	1	0	0	0	0	0	P1
				-0.0007		0	1	0	0	0	0	0	0	0	P1
1	-1	-1	2												
				0.0016		0	0	0	0	0	0	0	0	0	P1
1	-1	0	0												
					-0.0007	0	0	0	0	0	1	0	0	0	P1
1	-1	1	-2												
				0.0382		0	0	0	0	0	0	0	0	0	P1
				0.0006		1	0	0	0	0	0	0	0	0	P1
1	-1	1	-1												
				-0.0364		0	0	0	0	0	0	0	0	0	P1
1	-1	1	0												
				0.0113		0	0	0	0	0	0	0	0	0	P1
1	-1	1	2												
				0.0010		0	0	0	0	0	0	0	0	0	P1
1	0	-3	0												
				0.0068		0	0	0	0	0	0	0	0	0	P1
1	0	-3	2												
				-0.0068		0	0	0	0	0	0	0	0	0	P1
1	0	-2	0												
					-0.0858	0	0	0	0	0	1	0	0	0	P1
					0.0080	0	0	1	0	0	0	0	0	0	P1
					0.0024	0	1	0	0	0	1	0	0	0	P1
					-0.0014	1	0	0	0	0	1	0	0	0	P1
					0.0007	0	0	0	0	0	1	0	1	0	P1
				-0.0124		0	0	0	0	0	0	1	0	0	P1
				0.0005		0	1	0	0	0	0	1	0	0	P1
1	0	-1	-4												
				-0.0027		0	0	0	0	0	0	0	0	0	P1
1	0	-1	-2												
				-0.3499		0	0	0	0	0	0	0	0	0	P1
				-0.0040		1	0	0	0	0	0	0	0	0	P1
				0.0018		0	0	0	1	0	0	0	0	0	P1
				-0.0015		0	1	0	0	0	0	0	0	0	P1
1	0	-1	-1												
				-0.0469		0	0	0	0	0	0	0	0	0	P1
				-0.0018		0	1	0	0	0	0	0	0	0	P1
				-0.0006		1	0	0	0	0	0	0	0	0	P1
1	0	-1	0												
					-0.1709	0	0	0	0	0	0	0	0	1	P1
					-0.0320	0	0	0	0	1	0	0	0	0	P1
					0.0019	0	1	0	0	0	0	0	0	1	P1
					-0.0011	1	0	0	0	0	0	0	0	1	P1
					0.0008	0	0	0	0	0	0	0	1	1	P1
					0.0008	0	0	0	1	0	0	0	0	1	P1
				124.1783		0	0	0	0	0	0	0	0	0	P1
				2.1898		1	0	0	0	0	0	0	0	0	P1
				-0.7697		0	1	0	0	0	0	0	0	0	P1
				-0.7425		0	0	0	1	0	0	0	0	0	P1
				-0.2985		0	0	0	0	0	0	0	1	0	P1
				0.0060		2	0	0	0	0	0	0	0	0	P1
				-0.0040		1	0	0	1	0	0	0	0	0	P1
				-0.0008		0	2	0	0	0	0	0	0	0	P1
				0.0008		0	0	0	2	0	0	0	0	0	P1
				-0.0007		0	0	0	0	0	0	0	0	2	P1
				0.0006		1	1	0	0	0	0	0	0	0	P1
1	0	-1	1												
				0.0029		0	0	0	0	0	0	0	0	0	P1
1	0	-1	2												
				0.0358		0	0	0	0	0	0	0	0	0	P1
1	0	0	-2												
					-0.0063	0	0	0	0	0	1	0	0	0	P1
					-0.0026	0	0	1	0	0	0	0	0	0	P1
				0.0119		0	0	0	0	0	0	1	0	0	P1
1	0	0	0												
					-0.7197	0	0	0	0	0	1	0	0	0	P1
					-0.1825	0	0	1	0	0	0	0	0	0	P1
					0.0030	0	1	0	0	0	1	0	0	0	P1
					-0.0015	0	0	0	1	0	1	0	0	0	P1
					0.0012	0	0	0	0	0	1	0	1	0	P1
					-0.0008	1	0	0	0	0	1	0	0	0	P1
					-0.0006	0	0	1	1	0	0	0	0	0	P1
				-0.8313		0	0	0	0	0	0	1	0	0	P1
				-0.0059		0	0	0	0	0	1	0	0	1	P1
				-0.0015		0	0	0	0	1	1	0	0	0	P1
				0.0010		0	1	0	0	0	0	1	0	0	P1
1	0	1	-4												
				-0.0082		0	0	0	0	0	0	0	0	0	P1
1	0	1	-3												
				-0.0009		0	0	0	0	0	0	0	0	0	P1
1	0	1	-2												
					-0.0029	0	0	0	0	0	0	0	0	1	P1
				-2.6718		0	0	0	0	0	0	0	0	0	P1
				-0.0460		1	0	0	0	0	0	0	0	0	P1
				0.0164		0	1	0	0	0	0	0	0	0	P1

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
				0.0162		0	0	0	1	0	0	0	0	0	P1
				0.0060		0	0	0	0	0	0	0	1	0	P1
1	0	1	-1												
				-0.0524		0	0	0	0	0	0	0	0	0	P1
				-0.0015		0	1	0	0	0	0	0	0	0	P1
				-0.0007		1	0	0	0	0	0	0	0	0	P1
1	0	1	0												
					0.0008	0	0	0	0	0	0	0	0	1	P1
					0.0007	0	0	0	0	1	0	0	0	0	P1
				1.5768		0	0	0	0	0	0	0	0	0	P1
				0.0180		1	0	0	0	0	0	0	0	0	P1
				-0.0080		0	0	0	1	0	0	0	0	0	P1
				0.0074		0	1	0	0	0	0	0	0	0	P1
				0.0015		0	0	0	0	0	0	0	1	0	P1
1	0	1	1												
				-0.0008		0	0	0	0	0	0	0	0	0	P1
1	0	1	2												
				0.0131		0	0	0	0	0	0	0	0	0	P1
1	0	2	-2												
					0.0005	0	0	0	0	0	1	0	0	0	P1
1	0	3	-2												
				-0.0011		0	0	0	0	0	0	0	0	0	P1
1	0	3	0												
				0.0011		0	0	0	0	0	0	0	0	0	P1
1	1	-1	-2												
				-0.0166		0	0	0	0	0	0	0	0	0	P1
1	1	-1	0												
				0.1160		0	0	0	0	0	0	0	0	0	P1
				0.0020		1	0	0	0	0	0	0	0	0	P1
				-0.0008		0	1	0	0	0	0	0	0	0	P1
				-0.0008		0	0	0	1	0	0	0	0	0	P1
1	1	-1	1												
				0.0014		0	0	0	0	0	0	0	0	0	P1
1	1	-1	2												
				-0.0009		0	0	0	0	0	0	0	0	0	P1
1	1	0	0												
					-0.0014	0	0	0	0	0	1	0	0	0	P1
				-0.0008		0	0	0	0	0	0	1	0	0	P1
1	1	1	-4												
				-0.0007		0	0	0	0	0	0	0	0	0	P1
1	1	1	-2												
				-0.0826		0	0	0	0	0	0	0	0	0	P1
				-0.0014		1	0	0	0	0	0	0	0	0	P1
				0.0005		0	1	0	0	0	0	0	0	0	P1
				0.0005		0	0	0	1	0	0	0	0	0	P1

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
1	1	1	0												
				-0.0084		0	0	0	0	0	0	0	0	0	P1
1	2	-1	-2												
				-0.0006		0	0	0	0	0	0	0	0	0	P1
1	2	1	-2												
				-0.0023		0	0	0	0	0	0	0	0	0	P1
2	-2	-1	-2												
				0.0055		0	0	0	0	0	0	0	0	0	P1
2	-2	1	-2												
				0.0066		0	0	0	0	0	0	0	0	0	P1
2	-1	-1	-2												
				0.0132		0	0	0	0	0	0	0	0	0	P1
				0.0005		0	1	0	0	0	0	0	0	0	P1
2	-1	-1	-1												
				-0.0010		0	0	0	0	0	0	0	0	0	P1
2	-1	-1	0												
				0.0011		0	0	0	0	0	0	0	0	0	P1
2	-1	1	-2												
				0.0112		0	0	0	0	0	0	0	0	0	P1
2	-1	1	0												
				0.0010		0	0	0	0	0	0	0	0	0	P1
2	0	-3	0												
					0.0117	0	0	0	0	0	0	0	0	1	P1
					-0.0040	0	1	0	0	0	0	0	0	1	P1
					0.0014	0	2	0	0	0	0	0	0	1	P1
					-0.0011	0	0	0	0	0	0	0	1	1	P1
					-0.0009	0	0	0	0	1	0	0	0	0	P1
					0.0007	0	1	0	0	0	0	0	1	1	P1
				0.2092		0	0	0	0	0	0	0	0	0	P1
				-0.0544		0	1	0	0	0	0	0	0	0	P1
				0.0188		0	2	0	0	0	0	0	0	0	P1
				-0.0065		0	3	0	0	0	0	0	0	0	P1
				0.0062		0	1	0	0	0	0	0	1	0	P1
				-0.0039		0	0	0	0	0	0	0	1	0	P1
				-0.0038		0	2	0	0	0	0	0	1	0	P1
				0.0018		1	0	0	0	0	0	0	0	0	P1
				-0.0007		0	1	0	0	0	0	0	2	0	P1
				-0.0006		0	3	0	0	0	0	0	1	0	P1
				-0.0006		0	1	0	1	0	0	0	0	0	P1
2	0	-1	-4												
				-0.0033		0	0	0	0	0	0	0	0	0	P1
2	0	-1	-2												
					0.0006	0	0	0	0	0	0	0	0	1	P1
				0.2319		0	0	0	0	0	0	0	0	0	P1
				0.0041		0	1	0	0	0	0	0	0	0	P1

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
				0.0032		1	0	0	0	0	0	0	0	0	P1
				-0.0014		0	0	0	1	0	0	0	0	0	P1
				0.0008		0	0	0	0	0	0	0	1	0	P1
2	0	-1	-1												
				-0.0014		0	0	0	0	0	0	0	0	0	P1
2	0	-1	0												
					0.0217	0	0	0	0	0	0	0	0	1	P1
					-0.0046	0	1	0	0	0	0	0	0	1	P1
					0.0016	0	2	0	0	0	0	0	0	1	P1
					-0.0013	0	0	0	0	0	0	0	1	1	P1
					0.0008	0	1	0	0	0	0	0	1	1	P1
					0.0006	0	0	0	0	1	0	0	0	0	P1
					-0.0005	0	3	0	0	0	0	0	0	1	P1
				0.3610		0	0	0	0	0	0	0	0	0	P1
				-0.0297		0	1	0	0	0	0	0	0	0	P1
				0.0210		0	2	0	0	0	0	0	0	0	P1
				0.0123		0	0	0	0	0	0	0	1	0	P1
				-0.0072		0	3	0	0	0	0	0	0	0	P1
				-0.0072		1	0	0	0	0	0	0	0	0	P1
				0.0070		0	1	0	0	0	0	0	1	0	P1
				-0.0060		0	0	0	1	0	0	0	0	0	P1
				-0.0042		0	2	0	0	0	0	0	1	0	P1
				-0.0007		0	1	0	0	0	0	0	2	0	P1
				-0.0007		0	1	0	1	0	0	0	0	0	P1
				-0.0007		0	3	0	0	0	0	0	1	0	P1
2	0	-1	2												
				0.0014		0	0	0	0	0	0	0	0	0	P1
2	0	0	-2												
				-0.0024		0	0	0	0	0	0	1	0	0	P1
2	0	0	0												
					0.0010	0	0	0	0	0	1	0	0	0	P1
				0.0006		0	0	0	0	0	0	1	0	0	P1
2	0	1	-4												
				0.0014		0	0	0	0	0	0	0	0	0	P1
2	0	1	-3												
				0.0006		0	0	0	0	0	0	0	0	0	P1
2	0	1	-2												
					0.0008	0	0	0	0	0	0	0	0	1	P1
				0.1523		0	0	0	0	0	0	0	0	0	P1
				0.0027		1	0	0	0	0	0	0	0	0	P1
				0.0025		0	1	0	0	0	0	0	0	0	P1
				-0.0012		0	0	0	1	0	0	0	0	0	P1
2	0	1	0												
				0.0730		0	0	0	0	0	0	0	0	0	P1
				0.0008		1	0	0	0	0	0	0	0	0	P1

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
2	0	1	2												
				0.0013		0	0	0	0	0	0	0	0	0	P1
2	1	-1	-2												
				0.0042		0	0	0	0	0	0	0	0	0	P1
2	1	-1	0												
				0.0026		0	0	0	0	0	0	0	0	0	P1
2	1	1	-2												
				0.0020		0	0	0	0	0	0	0	0	0	P1
2	1	1	0												
				-0.0008		0	0	0	0	0	0	0	0	0	P1
3	0	-3	0												
				-0.0017		0	0	0	0	0	0	0	0	0	P1
3	0	-1	-2												
				-0.0039		0	0	0	0	0	0	0	0	0	P1
3	0	-1	0												
				-0.0010		0	0	0	0	0	0	0	0	0	P1
3	0	1	-4												
				-0.0013		0	0	0	0	0	0	0	0	0	P1
3	0	1	-2												
				-0.0020		0	0	0	0	0	0	0	0	0	P1
3	0	1	0												
				0.0042		0	0	0	0	0	0	0	0	0	P1
0	0	0	0												
					0.3043	0	0	0	0	0	0	1	0	0	P2
					0.1621	0	0	0	0	0	1	0	0	1	P2
					0.0480	0	0	1	0	0	0	0	0	1	P2
					0.0274	0	0	0	0	1	1	0	0	0	P2
					0.0109	1	0	0	0	0	0	1	0	0	P2
					0.0081	0	0	1	0	1	0	0	0	0	P2
					0.0076	0	1	0	0	0	0	1	0	0	P2
					-0.0056	0	0	0	1	0	0	1	0	0	P2
					-0.0029	0	1	0	0	0	1	0	0	1	P2
					-0.0023	1	0	0	0	0	1	0	0	1	P2
					-0.0012	0	0	0	0	0	1	0	1	1	P2
					-0.0009	0	1	1	0	0	0	0	0	1	P2
					-0.0007	1	0	1	0	0	0	0	0	1	P2
					0.0007	0	0	0	0	0	0	1	1	0	P2
0	0	0	1												
					0.0006	0	0	0	0	0	0	1	0	0	P2
0	0	1	-4												
					0.0006	0	0	0	0	0	0	0	0	0	P2
0	0	1	-2												
					-3.1971	0	0	0	0	0	0	0	0	0	P2
					-0.0289	1	0	0	0	0	0	0	0	0	P2
					0.0080	0	0	0	1	0	0	0	0	0	P2

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
					-0.0064	0	1	0	0	0	0	0	0	0	P2
					0.0008	0	0	0	0	0	0	0	1	0	P2
				-0.0030		0	0	0	0	0	0	0	0	1	P2
0	0	1	-1												
					0.0939	0	0	0	0	0	0	0	0	0	P2
					0.0011	1	0	0	0	0	0	0	0	0	P2
					-0.0007	0	0	0	1	0	0	0	0	0	P2
0	0	1	0												
					5544.0162	0	0	0	0	0	0	0	0	0	P2
					68.5112	1	0	0	0	0	0	0	0	0	P2
					33.9085	0	1	0	0	0	0	0	0	0	P2
					-25.0026	0	0	0	1	0	0	0	0	0	P2
					5.7508	0	0	0	0	0	0	0	1	0	P2
					0.1914	2	0	0	0	0	0	0	0	0	P2
					0.1898	1	1	0	0	0	0	0	0	0	P2
					-0.1380	1	0	0	1	0	0	0	0	0	P2
					-0.0686	0	1	0	1	0	0	0	0	0	P2
					0.0471	0	2	0	0	0	0	0	0	0	P2
					0.0321	1	0	0	0	0	0	0	1	0	P2
					0.0249	0	0	0	2	0	0	0	0	0	P2
					0.0159	0	1	0	0	0	0	0	1	0	P2
					-0.0116	0	0	0	1	0	0	0	1	0	P2
					0.0095	0	0	0	0	1	0	0	0	1	P2
					0.0058	0	0	0	0	0	2	0	0	0	P2
					0.0053	0	0	0	0	0	0	0	0	2	P2
					0.0049	0	0	1	0	0	1	0	0	0	P2
					0.0014	0	0	0	0	2	0	0	0	0	P2
					0.0014	0	0	0	0	0	0	0	2	0	P2
					0.0009	0	0	2	0	0	0	0	0	0	P2
					0.0008	2	1	0	0	0	0	0	0	0	P2
					-0.0006	2	0	0	1	0	0	0	0	0	P2
					-0.0006	1	1	0	1	0	0	0	0	0	P2
					0.0005	3	0	0	0	0	0	0	0	0	P2
				-8.2093		0	0	0	0	0	0	0	0	1	P2
				-1.3651		0	0	0	0	1	0	0	0	0	P2
				0.1640		0	1	0	0	0	0	0	0	1	P2
				-0.1013		1	0	0	0	0	0	0	0	1	P2
				0.0460		0	0	0	0	0	0	0	1	1	P2
				0.0300		0	0	0	1	0	0	0	0	1	P2
				0.0274		0	1	0	0	1	0	0	0	0	P2
				-0.0165		1	0	0	0	1	0	0	0	0	P2
				0.0077		0	0	0	0	1	0	0	1	0	P2
				0.0049		0	0	0	1	1	0	0	0	0	P2
				-0.0044		0	2	0	0	0	0	0	0	1	P2
				0.0024		1	1	0	0	0	0	0	0	1	P2
				-0.0023		0	1	0	0	0	0	0	1	1	P2
				0.0018		0	0	0	0	0	1	1	0	0	P2
				-0.0007		0	2	0	0	1	0	0	0	0	P2
				0.0006		1	0	0	0	0	0	0	1	1	P2
				0.0005		0	0	1	0	0	0	1	0	0	P2
				-0.0005		0	1	0	1	0	0	0	0	1	P2
0	0	1	1												
					-0.0018	0	0	0	0	0	0	0	0	0	P2
0	0	1	2												
					0.0080	0	0	0	0	0	0	0	0	0	P2
0	0	2	-2												
				-0.0005		0	0	0	0	0	1	0	0	0	P2
0	0	3	-2												
					-0.0063	0	0	0	0	0	0	0	0	0	P2
0	0	3	0												
					0.0108	0	0	0	0	0	0	0	0	0	P2
0	1	-3	2												
					-0.0007	0	0	0	0	0	0	0	0	0	P2
0	1	-1	-2												
					0.0005	0	0	0	0	0	0	0	0	0	P2
0	1	-1	0												
					-1.0700	0	0	0	0	0	0	0	0	0	P2
					-0.0487	0	1	0	0	0	0	0	0	0	P2
					-0.0123	1	0	0	0	0	0	0	0	0	P2
					-0.0118	0	0	0	0	0	0	0	1	0	P2
					0.0058	0	0	0	1	0	0	0	0	0	P2
					-0.0005	1	1	0	0	0	0	0	0	0	P2
				-0.0017		0	0	0	0	0	0	0	0	1	P2
				-0.0008		0	0	0	0	1	0	0	0	0	P2
0	1	-1	1												
					-0.0620	0	0	0	0	0	0	0	0	0	P2
					0.0022	1	0	0	0	0	0	0	0	0	P2
					-0.0018	0	1	0	0	0	0	0	0	0	P2
					-0.0009	0	0	0	0	0	0	0	1	0	P2
0	1	-1	2												
					0.1268	0	0	0	0	0	0	0	0	0	P2
					0.0012	1	0	0	0	0	0	0	0	0	P2
					0.0005	0	1	0	0	0	0	0	0	0	P2
0	1	0	0												
					0.0068	0	0	0	0	0	0	1	0	0	P2
				-0.0008		0	0	0	0	0	1	0	0	0	P2
0	1	0	1												
					-0.0065	0	0	0	0	0	0	1	0	0	P2
				0.0054		0	0	0	0	0	1	0	0	0	P2
				0.0015		0	0	1	0	0	0	0	0	0	P2

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
0	1	1	-2												
					-0.0861	0	0	0	0	0	0	0	0	0	P2
					-0.0007	1	0	0	0	0	0	0	0	0	P2
0	1	1	-1												
					0.0006	0	0	0	0	0	0	0	0	0	P2
0	1	1	0												
					1.2918	0	0	0	0	0	0	0	0	0	P2
					0.0393	0	1	0	0	0	0	0	0	0	P2
					0.0171	1	0	0	0	0	0	0	0	0	P2
					0.0092	0	0	0	0	0	0	0	1	0	P2
					-0.0072	0	0	0	1	0	0	0	0	0	P2
				-0.0017		0	0	0	0	0	0	0	0	1	P2
0	2	-1	0												
					-0.0017	0	0	0	0	0	0	0	0	0	P2
0	2	-1	2												
					0.0238	0	0	0	0	0	0	0	0	0	P2
0	2	1	-2												
					-0.0018	0	0	0	0	0	0	0	0	0	P2
0	2	1	0												
					0.0037	0	0	0	0	0	0	0	0	0	P2
1	-2	-1	0												
					-0.0013	0	0	0	0	0	0	0	0	0	P2
1	-2	1	-2												
					-0.0560	0	0	0	0	0	0	0	0	0	P2
					-0.0011	1	0	0	0	0	0	0	0	0	P2
1	-1	-1	-1												
					0.0135	0	0	0	0	0	0	0	0	0	P2
1	-1	-1	0												
					-0.1369	0	0	0	0	0	0	0	0	0	P2
					-0.0016	1	0	0	0	0	0	0	0	0	P2
					0.0010	0	0	0	1	0	0	0	0	0	P2
					-0.0008	0	1	0	0	0	0	0	0	0	P2
1	-1	-1	2												
					0.0011	0	0	0	0	0	0	0	0	0	P2
1	-1	0	0												
				0.0009		0	0	0	0	0	1	0	0	0	P2
1	-1	1	-2												
					0.0291	0	0	0	0	0	0	0	0	0	P2
1	-1	1	-1												
					-0.0363	0	0	0	0	0	0	0	0	0	P2
1	0	-3	0												
					-0.0018	0	0	0	0	0	0	0	0	0	P2
1	0	-3	2												
					0.0029	0	0	0	0	0	0	0	0	0	P2
1	0	-2	0												
					0.0125	0	0	0	0	0	0	1	0	0	P2
					-0.0005	0	1	0	0	0	0	1	0	0	P2
				-0.0863		0	0	0	0	0	1	0	0	0	P2
				0.0080		0	0	1	0	0	0	0	0	0	P2
				0.0024		0	1	0	0	0	1	0	0	0	P2
				-0.0014		1	0	0	0	0	1	0	0	0	P2
				0.0007		0	0	0	0	0	1	0	1	0	P2
1	0	-1	-2												
					0.0787	0	0	0	0	0	0	0	0	0	P2
				0.0006		0	0	0	0	0	0	0	0	1	P2
1	0	-1	-1												
					0.0471	0	0	0	0	0	0	0	0	0	P2
					0.0018	0	1	0	0	0	0	0	0	0	P2
					0.0006	1	0	0	0	0	0	0	0	0	P2
1	0	-1	0												
					-75.7656	0	0	0	0	0	0	0	0	0	P2
					-0.6278	0	1	0	0	0	0	0	0	0	P2
					0.5843	0	0	0	1	0	0	0	0	0	P2
					-0.5729	1	0	0	0	0	0	0	0	0	P2
					-0.2222	0	0	0	0	0	0	0	1	0	P2
					-0.0025	1	1	0	0	0	0	0	0	0	P2
					0.0019	2	0	0	0	0	0	0	0	0	P2
					0.0013	0	1	0	1	0	0	0	0	0	P2
					-0.0012	1	0	0	0	0	0	0	1	0	P2
					-0.0009	0	2	0	0	0	0	0	0	0	P2
					-0.0006	0	1	0	0	0	0	0	1	0	P2
				0.1338		0	0	0	0	1	0	0	0	0	P2
				-0.0625		0	0	0	0	0	0	0	0	1	P2
				0.0063		0	1	0	0	0	0	0	0	1	P2
				-0.0028		1	0	0	0	0	0	0	0	1	P2
				0.0016		0	0	0	1	0	0	0	0	1	P2
				0.0012		0	1	0	0	1	0	0	0	0	P2
				0.0009		0	0	0	0	0	0	0	1	1	P2
1	0	-1	1												
					0.0018	0	0	0	0	0	0	0	0	0	P2
1	0	-1	2												
					0.0192	0	0	0	0	0	0	0	0	0	P2
1	0	0	-2												
					-0.0136	0	0	0	0	0	0	1	0	0	P2
				-0.0088		0	0	0	0	0	1	0	0	0	P2
				-0.0034		0	0	1	0	0	0	0	0	0	P2
1	0	0	0												
					-0.8382	0	0	0	0	0	0	1	0	0	P2
					-0.0059	0	0	0	0	0	1	0	0	1	P2
					-0.0015	0	0	0	0	1	1	0	0	0	P2

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
					0.0010	0	1	0	0	0	0	1	0	0	P2
				0.7293		0	0	0	0	0	1	0	0	0	P2
				0.1852		0	0	1	0	0	0	0	0	0	P2
				-0.0030		0	1	0	0	0	1	0	0	0	P2
				0.0015		0	0	0	1	0	1	0	0	0	P2
				-0.0012		0	0	0	0	0	1	0	1	0	P2
				0.0008		1	0	0	0	0	1	0	0	0	P2
				0.0006		0	0	1	1	0	0	0	0	0	P2
1	0	1	-4												
					0.0049	0	0	0	0	0	0	0	0	0	P2
1	0	1	-3												
					0.0005	0	0	0	0	0	0	0	0	0	P2
1	0	1	-2												
					-1.6156	0	0	0	0	0	0	0	0	0	P2
					-0.0211	1	0	0	0	0	0	0	0	0	P2
					0.0129	0	0	0	1	0	0	0	0	0	P2
					-0.0042	0	1	0	0	0	0	0	0	0	P2
					-0.0013	0	0	0	0	0	0	0	1	0	P2
				-0.0025		0	0	0	0	1	0	0	0	0	P2
				0.0019		0	0	0	0	0	0	0	0	1	P2
1	0	1	-1												
					-0.0542	0	0	0	0	0	0	0	0	0	P2
					-0.0015	0	1	0	0	0	0	0	0	0	P2
					-0.0007	1	0	0	0	0	0	0	0	0	P2
1	0	1	0												
					0.2996	0	0	0	0	0	0	0	0	0	P2
					0.0027	0	1	0	0	0	0	0	0	0	P2
					-0.0011	0	0	0	1	0	0	0	0	0	P2
					0.0009	1	0	0	0	0	0	0	0	0	P2
					0.0008	0	0	0	0	0	0	0	1	0	P2
				-0.0026		0	0	0	0	0	0	0	0	1	P2
				-0.0007		0	0	0	0	1	0	0	0	0	P2
1	0	1	2												
					-0.0022	0	0	0	0	0	0	0	0	0	P2
1	0	3	0												
					0.0009	0	0	0	0	0	0	0	0	0	P2
1	1	-1	-2												
					0.0036	0	0	0	0	0	0	0	0	0	P2
1	1	-1	0												
					-0.1614	0	0	0	0	0	0	0	0	0	P2
					-0.0020	1	0	0	0	0	0	0	0	0	P2
					-0.0013	0	1	0	0	0	0	0	0	0	P2
					0.0013	0	0	0	1	0	0	0	0	0	P2
					-0.0005	0	0	0	0	0	0	0	1	0	P2
1	1	-1	1												
					0.0015	0	0	0	0	0	0	0	0	0	P2
1	1	0	0												
					-0.0007	0	0	0	0	0	0	1	0	0	P2
				0.0013		0	0	0	0	0	1	0	0	0	P2
1	1	1	-2												
					-0.0530	0	0	0	0	0	0	0	0	0	P2
					-0.0007	1	0	0	0	0	0	0	0	0	P2
1	2	-1	0												
					-0.0006	0	0	0	0	0	0	0	0	0	P2
1	2	1	-2												
					-0.0015	0	0	0	0	0	0	0	0	0	P2
2	-2	-1	-2												
					-0.0055	0	0	0	0	0	0	0	0	0	P2
2	-2	1	-2												
					0.0066	0	0	0	0	0	0	0	0	0	P2
2	-1	-1	-2												
					-0.0134	0	0	0	0	0	0	0	0	0	P2
					-0.0005	0	1	0	0	0	0	0	0	0	P2
2	-1	-1	-1												
					0.0005	0	0	0	0	0	0	0	0	0	P2
2	-1	-1	0												
					0.0030	0	0	0	0	0	0	0	0	0	P2
2	-1	1	-2												
					0.0115	0	0	0	0	0	0	0	0	0	P2
2	0	-3	0												
					-0.2102	0	0	0	0	0	0	0	0	0	P2
					0.0545	0	1	0	0	0	0	0	0	0	P2
					-0.0189	0	2	0	0	0	0	0	0	0	P2
					0.0065	0	3	0	0	0	0	0	0	0	P2
					-0.0063	0	1	0	0	0	0	0	1	0	P2
					0.0039	0	0	0	0	0	0	0	1	0	P2
					0.0038	0	2	0	0	0	0	0	1	0	P2
					-0.0018	1	0	0	0	0	0	0	0	0	P2
					0.0007	0	1	0	0	0	0	0	2	0	P2
					0.0006	0	3	0	0	0	0	0	1	0	P2
					0.0006	0	1	0	1	0	0	0	0	0	P2
				0.0117		0	0	0	0	0	0	0	0	1	P2
				-0.0040		0	1	0	0	0	0	0	0	1	P2
				0.0014		0	2	0	0	0	0	0	0	1	P2
				-0.0011		0	0	0	0	0	0	0	1	1	P2
				-0.0009		0	0	0	0	1	0	0	0	0	P2
				0.0007		0	1	0	0	0	0	0	1	1	P2
2	0	-2	0												
					-0.0006	0	0	0	0	0	0	1	0	0	P2
				0.0008		0	0	0	0	0	1	0	0	0	P2

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
2	0	-1	-2		-0.2191	0	0	0	0	0	0	0	0	0	P2
					-0.0042	0	1	0	0	0	0	0	0	0	P2
					-0.0030	1	0	0	0	0	0	0	0	0	P2
					0.0014	0	0	0	1	0	0	0	0	0	P2
					-0.0008	0	0	0	0	0	0	0	1	0	P2
				0.0006		0	0	0	0	0	0	0	0	1	P2
2	0	-1	-1		0.0033	0	0	0	0	0	0	0	0	0	P2
2	0	-1	0		0.4645	0	0	0	0	0	0	0	0	0	P2
					-0.0294	0	1	0	0	0	0	0	0	0	P2
					0.0211	0	2	0	0	0	0	0	0	0	P2
					0.0124	0	0	0	0	0	0	0	1	0	P2
					-0.0072	0	3	0	0	0	0	0	0	0	P2
					0.0070	0	1	0	0	0	0	0	1	0	P2
					-0.0069	0	0	0	1	0	0	0	0	0	P2
					-0.0058	1	0	0	0	0	0	0	0	0	P2
					-0.0042	0	2	0	0	0	0	0	1	0	P2
					-0.0007	0	1	0	0	0	0	0	2	0	P2
					-0.0007	0	1	0	1	0	0	0	0	0	P2
					-0.0007	0	3	0	0	0	0	0	1	0	P2
				-0.0216		0	0	0	0	0	0	0	0	1	P2
				0.0046		0	1	0	0	0	0	0	0	1	P2
				-0.0016		0	2	0	0	0	0	0	0	1	P2
				0.0013		0	0	0	0	0	0	0	1	1	P2
				-0.0008		0	1	0	0	0	0	0	1	1	P2
				-0.0008		0	0	0	0	1	0	0	0	0	P2
				0.0005		0	3	0	0	0	0	0	0	1	P2
2	0	0	-2		0.0011	0	0	0	0	0	0	1	0	0	P2
				-0.0032		0	0	0	0	0	1	0	0	0	P2
				0.0010		0	0	1	0	0	0	0	0	0	P2
2	0	1	-4		0.0069	0	0	0	0	0	0	0	0	0	P2
2	0	1	-3		0.0005	0	0	0	0	0	0	0	0	0	P2
2	0	1	-2		0.1922	0	0	0	0	0	0	0	0	0	P2
					0.0033	1	0	0	0	0	0	0	0	0	P2
					0.0024	0	1	0	0	0	0	0	0	0	P2
					-0.0014	0	0	0	1	0	0	0	0	0	P2
				-0.0007		0	0	0	0	0	0	0	0	1	P2
2	0	1	0		-0.0090	0	0	0	0	0	0	0	0	0	P2

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
2	1	-1	-2		-0.0035	0	0	0	0	0	0	0	0	0	P2
2	1	-1	0		0.0015	0	0	0	0	0	0	0	0	0	P2
2	1	1	-4		0.0007	0	0	0	0	0	0	0	0	0	P2
2	1	1	-2		0.0035	0	0	0	0	0	0	0	0	0	P2
3	-1	-1	-2		-0.0005	0	0	0	0	0	0	0	0	0	P2
3	0	-3	0		0.0039	0	0	0	0	0	0	0	0	0	P2
					-0.0007	0	1	0	0	0	0	0	0	0	P2
3	0	-1	-2		-0.0049	0	0	0	0	0	0	0	0	0	P2
3	0	-1	0		0.0013	0	0	0	0	0	0	0	0	0	P2
3	0	1	-4		-0.0022	0	0	0	0	0	0	0	0	0	P2
3	0	1	-2		0.0012	0	0	0	0	0	0	0	0	0	P2
3	0	1	0		-0.0012	0	0	0	0	0	0	0	0	0	P2
0	0	0	0		304.2334	0	0	0	0	0	0	0	0	1	TAU
					50.6036	0	0	0	0	1	0	0	0	0	TAU
					-7.9397	0	1	0	0	0	0	0	0	1	TAU
					-2.0198	0	0	0	0	0	0	0	1	1	TAU
					-1.3246	0	1	0	0	1	0	0	0	0	TAU
					-0.3366	0	0	0	0	1	0	0	1	0	TAU
					0.2634	0	0	0	1	0	0	0	0	1	TAU
					0.2071	0	2	0	0	0	0	0	0	1	TAU
					0.1054	0	1	0	0	0	0	0	1	1	TAU
					-0.0674	0	0	0	0	0	1	1	0	0	TAU
					0.0467	0	0	0	1	1	0	0	0	0	TAU
					0.0345	0	2	0	0	1	0	0	0	0	TAU
					-0.0202	0	0	1	0	0	0	1	0	0	TAU
					0.0176	0	1	0	0	1	0	0	1	0	TAU
					-0.0137	0	1	0	1	0	0	0	0	1	TAU
					0.0134	0	0	0	0	0	0	0	2	1	TAU
					-0.0090	1	0	0	0	1	0	0	0	0	TAU
					-0.0059	1	0	0	0	0	0	0	0	1	TAU
					-0.0054	0	3	0	0	0	0	0	0	1	TAU
					-0.0041	0	2	0	0	0	0	0	1	1	TAU
					-0.0035	0	0	0	1	0	0	0	1	1	TAU

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
					-0.0025	0	0	0	0	0	0	0	0	3	TAU
					-0.0024	0	1	0	1	1	0	0	0	0	TAU
					0.0022	0	0	0	0	1	0	0	2	0	TAU
					0.0021	0	1	0	0	0	1	1	0	0	TAU
					-0.0010	0	1	0	0	0	0	0	2	1	TAU
					-0.0009	0	3	0	0	1	0	0	0	0	TAU
					-0.0008	0	0	0	0	1	0	0	0	2	TAU
					-0.0007	0	2	0	0	1	0	0	1	0	TAU
					0.0007	1	0	0	0	0	1	1	0	0	TAU
					0.0006	0	1	1	0	0	0	1	0	0	TAU
					-0.0006	0	0	0	1	1	0	0	1	0	TAU
					0.0005	0	2	0	1	0	0	0	0	1	TAU
0	0	0	1												
				0.1006		0	0	0	0	0	0	0	0	0	TAU
				0.0026		0	1	0	0	0	0	0	0	0	TAU
				0.0007		0	0	0	0	0	0	0	1	0	TAU
0	0	0	2												
					-0.0006	0	0	0	0	0	0	0	0	1	TAU
				-0.4896		0	0	0	0	0	0	0	0	0	TAU
				-0.0127		0	1	0	0	0	0	0	0	0	TAU
				-0.0043		0	0	0	0	0	0	0	1	0	TAU
				0.0006		0	0	0	1	0	0	0	0	0	TAU
0	0	0	3												
				0.0007		0	0	0	0	0	0	0	0	0	TAU
0	0	0	4												
				-0.0038		0	0	0	0	0	0	0	0	0	TAU
0	0	1	-1												
					-0.0038	0	0	0	0	0	1	0	0	0	TAU
				0.0009		0	0	0	0	0	0	1	0	0	TAU
0	0	1	0												
					0.9243	0	0	0	0	0	1	0	0	0	TAU
					0.2771	0	0	1	0	0	0	0	0	0	TAU
					0.0025	1	0	0	0	0	1	0	0	0	TAU
					-0.0018	0	0	0	1	0	1	0	0	0	TAU
					0.0012	0	1	0	0	0	1	0	0	0	TAU
					-0.0011	0	0	0	0	0	0	1	0	1	TAU
					0.0008	1	0	1	0	0	0	0	0	0	TAU
					-0.0005	0	0	1	1	0	0	0	0	0	TAU
				-0.0119		0	0	0	0	0	0	1	0	0	TAU
				0.0007		0	0	0	0	0	1	0	0	1	TAU
0	0	2	-2												
					0.0018	0	0	0	0	1	0	0	0	0	TAU
				1.6560		0	0	0	0	0	0	0	0	0	TAU
				0.0445		0	1	0	0	0	0	0	0	0	TAU
				0.0133		0	0	0	0	0	0	0	1	0	TAU

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
				-0.0021		1	0	0	0	0	0	0	0	0	TAU
				-0.0016		0	0	0	1	0	0	0	0	0	TAU
0	0	2	-1												
				-0.0009		0	0	0	0	0	0	0	0	0	TAU
0	0	2	0												
				-0.0267		0	0	0	0	0	0	0	0	0	TAU
				-0.0030		1	0	0	0	0	0	0	0	0	TAU
				0.0017		0	1	0	0	0	0	0	0	0	TAU
				0.0010		0	0	0	1	0	0	0	0	0	TAU
				0.0006		0	0	0	0	0	0	0	1	0	TAU
0	1	-2	0												
				0.0005		0	0	0	0	0	0	0	0	0	TAU
0	1	-2	1												
				-0.0017		0	0	0	0	0	0	0	0	0	TAU
0	1	-2	2												
				0.1426		0	0	0	0	0	0	0	0	0	TAU
				0.0040		0	1	0	0	0	0	0	0	0	TAU
				0.0011		0	0	0	0	0	0	0	1	0	TAU
0	1	-1	1												
					-0.0048	0	0	0	0	0	1	0	0	0	TAU
				-0.0011		0	0	0	0	0	0	1	0	0	TAU
0	1	0	-4												
				0.0005		0	0	0	0	0	0	0	0	0	TAU
0	1	0	-2												
				0.0377		0	0	0	0	0	0	0	0	0	TAU
				0.0010		0	1	0	0	0	0	0	0	0	TAU
0	1	0	-1												
				-0.0009		0	0	0	0	0	0	0	0	0	TAU
0	1	0	0												
					0.0085	0	0	0	0	0	0	0	0	1	TAU
				91.3399		0	0	0	0	0	0	0	0	0	TAU
				2.7073		0	1	0	0	0	0	0	0	0	TAU
				0.6891		0	0	0	0	0	0	0	1	0	TAU
				-0.0881		0	0	0	1	0	0	0	0	0	TAU
				0.0096		0	2	0	0	0	0	0	0	0	TAU
				0.0049		0	1	0	0	0	0	0	1	0	TAU
				-0.0028		1	0	0	0	0	0	0	0	0	TAU
				0.0016		0	0	0	0	0	0	0	0	2	TAU
				0.0006		0	0	0	0	0	0	0	2	0	TAU
				-0.0006		0	1	0	1	0	0	0	0	0	TAU
0	1	0	1												
				-0.0148		0	0	0	0	0	0	0	0	0	TAU
0	1	0	2												
				0.0073		0	0	0	0	0	0	0	0	0	TAU
0	1	2	-2												

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
				0.0318		0	0	0	0	0	0	0	0	0	TAU
				0.0008		0	1	0	0	0	0	0	0	0	TAU
0	2	-2	2	-0.0250		0	0	0	0	0	0	0	0	0	TAU
0	2	0	-2	0.0020		0	0	0	0	0	0	0	0	0	TAU
0	2	0	0	0.2272		0	0	0	0	0	0	0	0	0	TAU
				0.0061		0	1	0	0	0	0	0	0	0	TAU
				0.0016		0	0	0	0	0	0	0	1	0	TAU
0	2	2	-2	0.0006		0	0	0	0	0	0	0	0	0	TAU
0	3	0	0	0.0014		0	0	0	0	0	0	0	0	0	TAU
1	-2	0	-2	-0.0010		0	0	0	0	0	0	0	0	0	TAU
1	-2	0	0	-0.0034		0	0	0	0	0	0	0	0	0	TAU
1	-2	1	-2		0.0007	0	0	0	0	0	1	0	0	0	TAU
1	-1	-1	0		0.0042	0	0	0	0	0	1	0	0	0	TAU
				0.0017		0	0	0	0	0	0	1	0	0	TAU
1	-1	0	-2	-0.0312		0	0	0	0	0	0	0	0	0	TAU
				-0.0008		0	1	0	0	0	0	0	0	0	TAU
1	-1	0	-1		-0.0051	0	0	0	0	0	0	0	0	1	TAU
					0.0012	0	0	0	0	1	0	0	0	0	TAU
				-1.1516		0	0	0	0	0	0	0	0	0	TAU
				0.0037		0	1	0	0	0	0	0	0	0	TAU
				0.0013		0	0	0	0	0	0	0	1	0	TAU
1	-1	0	0	-0.1648		0	0	0	0	0	0	0	0	0	TAU
				-0.0044		0	1	0	0	0	0	0	0	0	TAU
				-0.0012		0	0	0	0	0	0	0	1	0	TAU
1	-1	0	2	-0.0051		0	0	0	0	0	0	0	0	0	TAU
1	-1	1	-2		-0.0016	0	0	0	0	0	1	0	0	0	TAU
1	0	-2	-2	0.0006		0	0	0	0	0	0	0	0	0	TAU
1	0	-2	0		0.0012	0	0	0	0	1	0	0	0	0	TAU
					0.0011	0	0	0	0	0	0	0	0	1	TAU

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
				-0.4226		0	0	0	0	0	0	0	0	0	TAU
				-0.0152		1	0	0	0	0	0	0	0	0	TAU
				0.0046		0	1	0	0	0	0	0	0	0	TAU
				0.0029		0	0	0	1	0	0	0	0	0	TAU
				0.0022		0	0	0	0	0	0	0	1	0	TAU
1	0	-2	1	0.0062		0	0	0	0	0	0	0	0	0	TAU
1	0	-2	2	0.0054		0	0	0	0	0	0	0	0	0	TAU
1	0	-1	0		-7.1148	0	0	0	0	0	1	0	0	0	TAU
					0.4531	0	0	1	0	0	0	0	0	0	TAU
					0.2298	0	1	0	0	0	1	0	0	0	TAU
					0.0635	0	0	0	0	0	1	0	1	0	TAU
					-0.0200	1	0	0	0	0	1	0	0	0	TAU
					-0.0153	0	1	1	0	0	0	0	0	0	TAU
					0.0084	0	0	0	0	0	0	1	0	1	TAU
					-0.0079	0	2	0	0	0	1	0	0	0	TAU
					-0.0065	0	0	0	1	0	1	0	0	0	TAU
					-0.0042	0	1	0	0	0	1	0	1	0	TAU
					-0.0026	0	0	1	0	0	0	0	1	0	TAU
					-0.0017	0	0	1	1	0	0	0	0	0	TAU
					0.0014	0	0	0	0	1	0	1	0	0	TAU
					0.0011	1	0	1	0	0	0	0	0	0	TAU
					0.0006	1	1	0	0	0	1	0	0	0	TAU
					-0.0005	0	0	0	0	0	1	0	2	0	TAU
				-1.6069		0	0	0	0	0	0	1	0	0	TAU
				0.0499		0	1	0	0	0	0	1	0	0	TAU
				-0.0134		0	0	0	0	0	1	0	0	1	TAU
				0.0128		0	0	0	0	0	0	1	1	0	TAU
				-0.0033		0	0	0	0	1	1	0	0	0	TAU
				-0.0020		1	0	0	0	0	0	1	0	0	TAU
				-0.0017		0	2	0	0	0	0	1	0	0	TAU
				-0.0009		0	0	0	1	0	0	1	0	0	TAU
				-0.0009		0	1	0	0	0	0	1	1	0	TAU
				0.0007		0	1	0	0	0	1	0	0	1	TAU
1	0	0	-4	0.0134		0	0	0	0	0	0	0	0	0	TAU
1	0	0	-3	-0.0015		0	0	0	0	0	0	0	0	0	TAU
1	0	0	-2		-0.0023	0	0	0	0	0	0	0	0	1	TAU
					-0.0006	0	0	0	0	1	0	0	0	0	TAU
				4.1554		0	0	0	0	0	0	0	0	0	TAU
				0.1077		0	1	0	0	0	0	0	0	0	TAU

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
				0.0272		0	0	0	0	0	0	0	1	0	TAU
				-0.0038		0	0	0	1	0	0	0	0	0	TAU
				0.0007		1	0	0	0	0	0	0	0	0	TAU
1	0	0	-1												
					-0.0083	0	0	0	0	0	0	0	0	1	TAU
					0.0016	0	0	0	0	1	0	0	0	0	TAU
				-3.4880		0	0	0	0	0	0	0	0	0	TAU
				-0.1073		0	1	0	0	0	0	0	0	0	TAU
				-0.0277		0	0	0	0	0	0	0	1	0	TAU
				0.0035		0	0	0	1	0	0	0	0	0	TAU
				-0.0005		0	2	0	0	0	0	0	0	0	TAU
1	0	0	0												
					-0.0118	0	0	0	0	0	0	0	0	1	TAU
					-0.0029	0	0	0	0	1	0	0	0	0	TAU
				-16.8912		0	0	0	0	0	0	0	0	0	TAU
				-0.4128		0	1	0	0	0	0	0	0	0	TAU
				-0.1045		0	0	0	0	0	0	0	1	0	TAU
				-0.0349		1	0	0	0	0	0	0	0	0	TAU
				0.0281		0	0	0	1	0	0	0	0	0	TAU
1	0	0	1												
				0.0051		0	0	0	0	0	0	0	0	0	TAU
1	0	0	2												
				-0.0631		0	0	0	0	0	0	0	0	0	TAU
				-0.0016		0	1	0	0	0	0	0	0	0	TAU
				-0.0005		0	0	0	0	0	0	0	1	0	TAU
1	0	0	4												
				-0.0066		0	0	0	0	0	0	0	0	0	TAU
1	0	1	-2												
					0.0411	0	0	0	0	0	1	0	0	0	TAU
					-0.0029	0	0	1	0	0	0	0	0	0	TAU
				-0.0071		0	0	0	0	0	0	1	0	0	TAU
1	0	1	0												
					-0.0009	0	0	0	0	0	1	0	0	0	TAU
				-0.0007		0	0	0	0	0	0	1	0	0	TAU
1	0	2	-4												
				0.0007		0	0	0	0	0	0	0	0	0	TAU
1	0	2	-2												
				0.0223		0	0	0	0	0	0	0	0	0	TAU
1	0	2	0												
				-0.0040		0	0	0	0	0	0	0	0	0	TAU
1	1	-2	0												
				0.0007		0	0	0	0	0	0	0	0	0	TAU
1	1	-2	1												
				0.0025		0	0	0	0	0	0	0	0	0	TAU
1	1	-1	0												
					-0.0067	0	0	0	0	0	1	0	0	0	TAU
				-0.0033		0	0	0	0	0	0	1	0	0	TAU
1	1	0	-4												
				0.0016		0	0	0	0	0	0	0	0	0	TAU
1	1	0	-2												
				0.2324		0	0	0	0	0	0	0	0	0	TAU
				0.0061		0	1	0	0	0	0	0	0	0	TAU
				0.0016		0	0	0	0	0	0	0	1	0	TAU
1	1	0	-1												
				-0.0024		0	0	0	0	0	0	0	0	0	TAU
1	1	0	0												
				0.1029		0	0	0	0	0	0	0	0	0	TAU
				0.0028		0	1	0	0	0	0	0	0	0	TAU
				0.0008		0	0	0	0	0	0	0	1	0	TAU
1	1	0	1												
				-0.0007		0	0	0	0	0	0	0	0	0	TAU
1	1	0	2												
				0.0012		0	0	0	0	0	0	0	0	0	TAU
1	1	1	-2												
					0.0008	0	0	0	0	0	1	0	0	0	TAU
1	1	2	-2												
				0.0007		0	0	0	0	0	0	0	0	0	TAU
1	2	0	-2												
				0.0104		0	0	0	0	0	0	0	0	0	TAU
1	2	0	0												
				0.0007		0	0	0	0	0	0	0	0	0	TAU
2	-2	0	-2												
					0.0008	0	0	0	0	0	0	0	0	1	TAU
				0.4058		0	0	0	0	0	0	0	0	0	TAU
				-0.0079		0	1	0	0	0	0	0	0	0	TAU
				-0.0022		0	0	0	0	0	0	0	1	0	TAU
2	-1	-2	0												
				-0.0033		0	0	0	0	0	0	0	0	0	TAU
2	-1	0	-2												
					0.0011	0	0	0	0	0	0	0	0	1	TAU
				0.9583		0	0	0	0	0	0	0	0	0	TAU
				0.0325		0	1	0	0	0	0	0	0	0	TAU
				0.0113		0	0	0	0	0	0	0	1	0	TAU
				-0.0006		0	0	0	1	0	0	0	0	0	TAU
2	-1	0	0												
				-0.0077		0	0	0	0	0	0	0	0	0	TAU
2	0	-2	-2												
				-0.0009		0	0	0	0	0	0	0	0	0	TAU
2	0	-2	0												
					0.9436	0	0	0	0	0	0	0	0	1	TAU

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
					-0.3229	0	1	0	0	0	0	0	0	1	TAU
					0.1110	0	2	0	0	0	0	0	0	1	TAU
					-0.0887	0	0	0	0	0	0	0	1	1	TAU
					-0.0700	0	0	0	0	1	0	0	0	0	TAU
					0.0588	0	1	0	0	0	0	0	1	1	TAU
					-0.0382	0	3	0	0	0	0	0	0	1	TAU
					-0.0299	0	2	0	0	0	0	0	1	1	TAU
					0.0242	0	1	0	0	1	0	0	0	0	TAU
					0.0090	0	0	0	1	0	0	0	0	1	TAU
					-0.0083	0	2	0	0	1	0	0	0	0	TAU
					-0.0078	0	1	0	0	0	0	0	2	1	TAU
					0.0077	0	0	0	0	0	0	0	2	1	TAU
					-0.0067	0	1	0	1	0	0	0	0	1	TAU
					0.0051	0	0	0	0	1	0	0	1	0	TAU
					0.0040	1	0	0	0	0	0	0	0	1	TAU
					-0.0039	0	1	0	0	1	0	0	1	0	TAU
					0.0036	0	2	0	1	0	0	0	0	1	TAU
					0.0027	0	3	0	0	1	0	0	0	0	TAU
					0.0021	0	2	0	0	1	0	0	1	0	TAU
					0.0017	0	1	0	1	0	0	0	1	1	TAU
					-0.0017	0	0	0	0	0	1	1	0	0	TAU
					-0.0017	0	0	0	1	0	0	0	1	1	TAU
					-0.0015	1	1	0	0	0	0	0	0	1	TAU
					0.0014	0	3	0	0	0	0	0	1	1	TAU
					0.0012	0	4	0	0	0	0	0	0	1	TAU
					-0.0008	0	0	0	1	1	0	0	0	0	TAU
					0.0006	0	2	0	0	0	0	0	2	1	TAU
					-0.0006	0	0	0	0	0	0	0	3	1	TAU
					0.0006	0	1	0	0	0	1	1	0	0	TAU
					0.0005	0	1	0	1	1	0	0	0	0	TAU
					0.0005	0	1	0	0	1	0	0	2	0	TAU
					-0.0005	0	0	0	0	1	0	0	2	0	TAU
				15.2543		0	0	0	0	0	0	0	0	0	TAU
				-4.3412		0	1	0	0	0	0	0	0	0	TAU
				1.4994		0	2	0	0	0	0	0	0	0	TAU
				-0.5162		0	3	0	0	0	0	0	0	0	TAU
				0.4937		0	1	0	0	0	0	0	1	0	TAU
				-0.3150		0	0	0	0	0	0	0	1	0	TAU
				-0.3015		0	2	0	0	0	0	0	1	0	TAU
				-0.0767		1	0	0	0	0	0	0	0	0	TAU
				-0.0668		0	1	0	1	0	0	0	0	0	TAU
				-0.0531		0	1	0	0	0	0	0	2	0	TAU
				-0.0506		0	3	0	0	0	0	0	1	0	TAU
				0.0412		0	0	0	1	0	0	0	0	0	TAU
				0.0406		0	2	0	1	0	0	0	0	0	TAU
				0.0331		1	1	0	0	0	0	0	0	0	TAU
				-0.0317		0	2	0	0	0	0	0	2	0	TAU
				0.0285		0	0	0	0	0	0	0	2	0	TAU
				-0.0181		0	4	0	0	0	0	0	0	0	TAU
				0.0151		0	1	0	1	0	0	0	1	0	TAU
				0.0134		1	3	0	0	0	0	0	0	0	TAU
				0.0132		1	0	0	0	0	0	0	1	0	TAU
				-0.0129		1	2	0	0	0	0	0	0	0	TAU
				0.0108		1	2	0	0	0	0	0	1	0	TAU
				-0.0097		0	0	0	1	0	0	0	1	0	TAU
				-0.0083		1	1	0	0	0	0	0	1	0	TAU
				-0.0075		0	1	0	0	0	0	0	3	0	TAU
				0.0052		0	2	0	1	0	0	0	1	0	TAU
				-0.0029		2	0	0	0	0	0	0	0	0	TAU
				0.0028		1	1	0	0	0	0	0	2	0	TAU
				0.0028		0	3	0	1	0	0	0	0	0	TAU
				-0.0025		0	0	0	0	0	0	0	3	0	TAU
				0.0021		0	4	0	0	0	0	0	1	0	TAU
				0.0020		0	1	0	1	0	0	0	2	0	TAU
				0.0015		0	3	0	0	0	0	0	2	0	TAU
				0.0013		0	0	0	1	0	0	0	2	0	TAU
				0.0013		0	0	0	0	0	2	0	0	0	TAU
				-0.0011		1	0	0	0	0	0	0	2	0	TAU
				-0.0011		1	2	0	1	0	0	0	0	0	TAU
				0.0011		0	1	0	0	0	0	0	0	2	TAU
				0.0011		2	1	0	0	0	0	0	0	0	TAU
				-0.0010		0	1	0	2	0	0	0	0	0	TAU
				0.0008		0	5	0	0	0	0	0	0	0	TAU
				0.0007		0	0	0	2	0	0	0	0	0	TAU
				-0.0007		0	2	0	0	0	0	0	0	2	TAU
				0.0006		1	0	0	1	0	0	0	0	0	TAU
				-0.0006		1	1	0	1	0	0	0	1	0	TAU
				-0.0006		1	3	0	0	0	0	0	1	0	TAU
				-0.0006		1	4	0	0	0	0	0	0	0	TAU
				-0.0006		0	0	0	0	0	0	0	4	0	TAU
				0.0005		0	2	0	0	0	0	0	3	0	TAU
2	0	0	-4												
				0.0219		0	0	0	0	0	0	0	0	0	TAU
				0.0006		0	1	0	0	0	0	0	0	0	TAU
2	0	0	-3												
				-0.0014		0	0	0	0	0	0	0	0	0	TAU
2	0	0	-2												
					0.0570	0	0	0	0	0	0	0	0	1	TAU
					-0.0110	0	0	0	0	1	0	0	0	0	TAU
				9.9995		0	0	0	0	0	0	0	0	0	TAU

Table I (*Continued*)

J_1	J_2	J_3	J_4	SIN	COS	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	
				0.2710		0	1	0	0	0	0	0	0	0	TAU
				0.0739		0	0	0	0	0	0	0	1	0	TAU
				-0.0087		0	0	0	1	0	0	0	0	0	TAU
2	0	0	-1	-0.0029		0	0	0	0	0	0	0	0	0	TAU
2	0	0	0		-0.0012	0	0	0	0	0	0	0	0	1	TAU
				-0.4475		0	0	0	0	0	0	0	0	0	TAU
				-0.0117		0	1	0	0	0	0	0	0	0	TAU
				-0.0037		0	0	0	0	0	0	0	1	0	TAU
2	0	0	2	-0.0058		0	0	0	0	0	0	0	0	0	TAU
2	0	2	-2	0.0009		0	0	0	0	0	0	0	0	0	TAU
2	1	-2	0	0.0120		0	0	0	0	0	0	0	0	0	TAU
				0.0009		0	1	0	0	0	0	0	0	0	TAU
				0.0006		0	0	0	0	0	0	0	1	0	TAU
2	1	0	-4	0.0022		0	0	0	0	0	0	0	0	0	TAU
2	1	0	-2		0.0009	0	0	0	0	0	0	0	0	1	TAU
				0.1558		0	0	0	0	0	0	0	0	0	TAU
				0.0041		0	1	0	0	0	0	0	0	0	TAU
				0.0010		0	0	0	0	0	0	0	1	0	TAU
2	1	0	0	0.0058		0	0	0	0	0	0	0	0	0	TAU
2	2	0	-2	0.0030		0	0	0	0	0	0	0	0	0	TAU
3	-1	0	-2	0.0006		0	0	0	0	0	0	0	0	0	TAU
3	-1	0	0	-0.0005		0	0	0	0	0	0	0	0	0	TAU
3	0	-3	0		-0.0007	0	0	0	0	0	1	0	0	0	TAU
3	0	-2	0	0.0006		0	0	0	0	0	0	0	0	0	TAU
3	0	0	-4	-0.0011		0	0	0	0	0	0	0	0	0	TAU
3	0	0	-2	0.0338		0	0	0	0	0	0	0	0	0	TAU
				0.0009		0	1	0	0	0	0	0	0	0	TAU
3	0	0	0	-0.0224		0	0	0	0	0	0	0	0	0	TAU
				-0.0006		0	1	0	0	0	0	0	0	0	TAU
3	1	0	-2	0.0010		0	0	0	0	0	0	0	0	0	TAU
4	0	-2	-2	-0.0014		0	0	0	0	0	0	0	0	0	TAU
4	0	0	-4	-0.0043		0	0	0	0	0	0	0	0	0	TAU
4	0	0	-2	0.0015		0	0	0	0	0	0	0	0	0	TAU
4	0	0	0	-0.0013		0	0	0	0	0	0	0	0	0	TAU

$$E_6 = \frac{\eta * C_{32}}{5 * 10^{-6}},$$

$$E_7 = \frac{\eta * S_{32}}{2 * 10^{-6}},$$

$$E_8 = \frac{\eta * C_{33}}{3 * 10^{-6}},$$

$$E_9 = -\frac{\eta * S_{33}}{2 * 10^{-6}}.$$

The accuracy is 0.″001 for all values of the parameters such as

$$|E_i| \leqslant 1, \qquad i = 1, \ldots, 9,$$

and the epoch of the solution is 2000.

4. Comparisons with Other Theories

In order to compare the results presented in Table I with those of Migus (Migus, 1980) and Eckhardt (1981), we adopt the Eckhardt's 500 libration set of parameters.

TABLE II

Differences (Migus–Moons)

Series	Term	Migus	Moons	Moons–Migus
p_1	$\sin F$	5562.506	5562.459	−0.047
	$\sin (l - F)$	124.474	124.483	0.009
	ind. term	−80.652	−80.644	0.008
	$\cos (l' + D)$	–	−0.006	−0.006
	$\sin (l' + D)$	–	−0.006	−0.006
p_2	$\cos F$	5540.379	5540.331	−0.048
	$\sin (l' + D)$	–	0.006	0.006
	$\cos (l' + D)$	–	−0.006	−0.006
τ	$\sin (2l - 2F)$	17.139	17.020	−0.119
	$\sin (2l' - 2F + 2D)$	–	−0.025	−0.025 *
	$\cos (l - F)$	−6.617	−6.597	0.020
	$\sin l$	−16.780	−16.795	−0.015
	$\cos F$	1.070	1.084	0.014
	$\sin l'$	90.692	90.704	0.012
	$\sin (l - l' - D)$	−1.143	−1.152	−0.009 *
	$\sin (2l - 2l' - 2D)$	0.399	0.408	0.009 *
	$\sin (l - 2F)$	−0.418	−0.426	−0.008

The general agreement between the three theories is good, as shown in Figures 2 to 4 which are plots of the differences (superior to 0.″001) between the solutions in pairs. The amplitude of the differences, evaluated for 2400 days from the Julian day 2444605, never

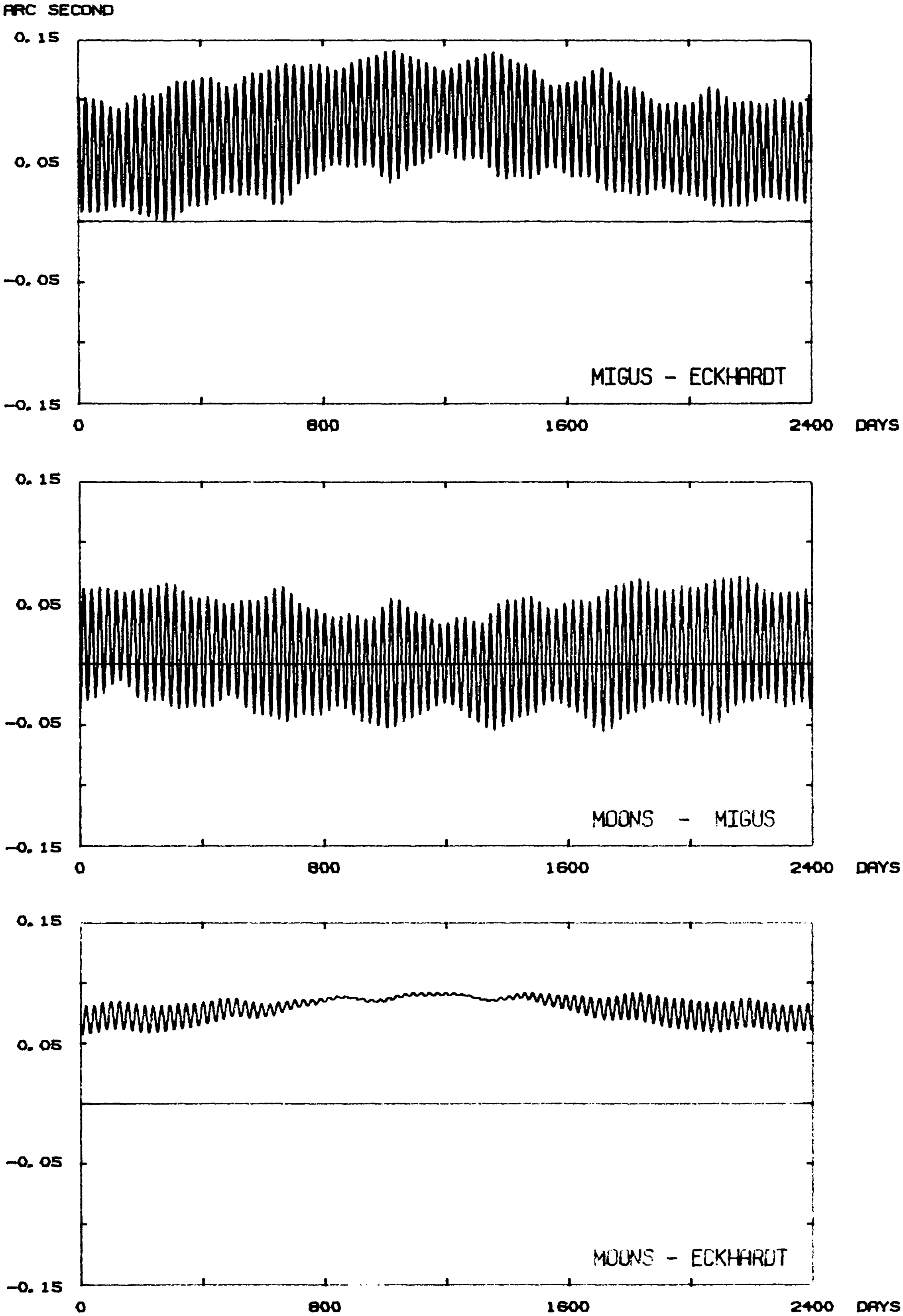

Fig. 2. Differences in p_1.

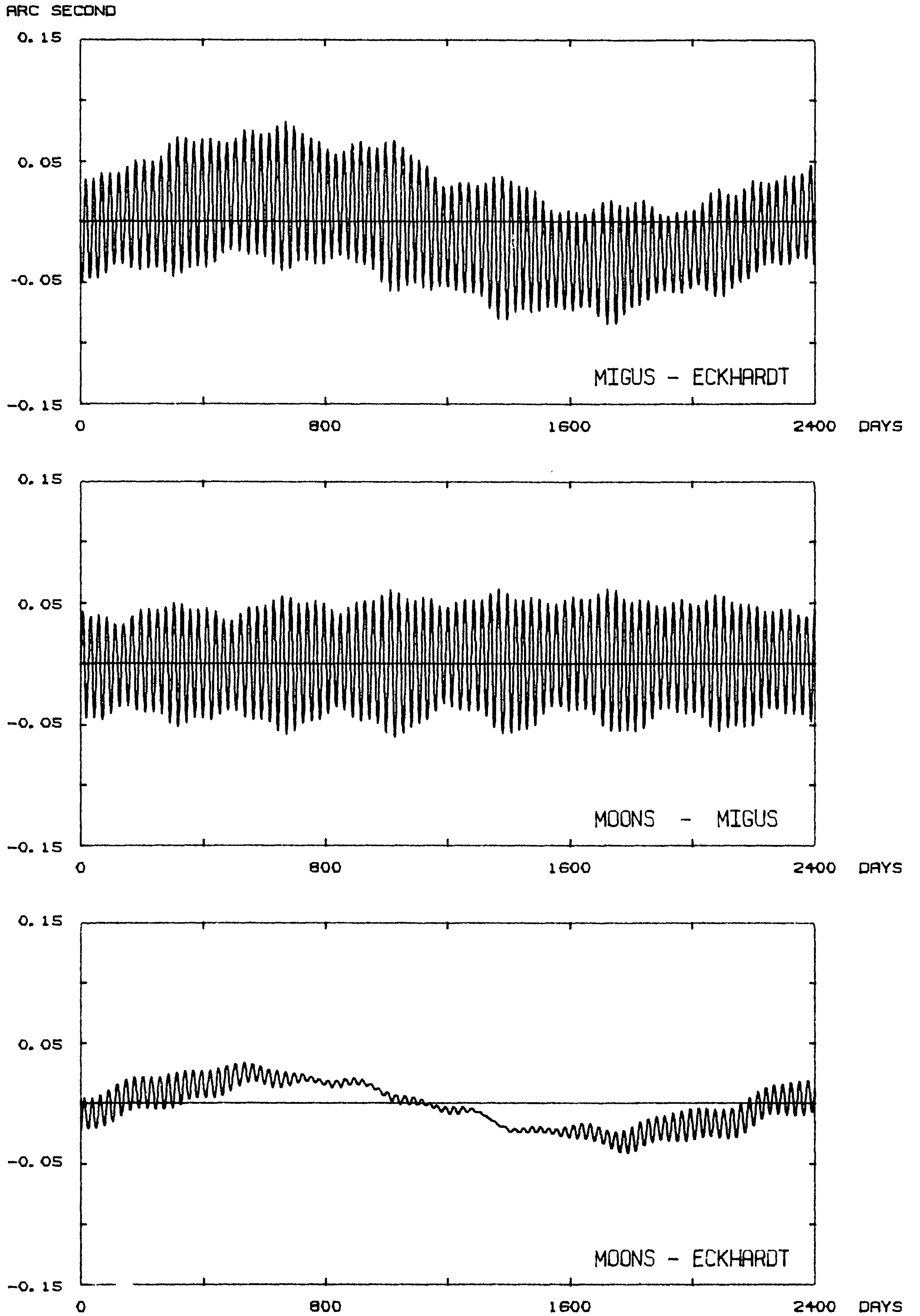

Fig. 3. Differences in p_2.

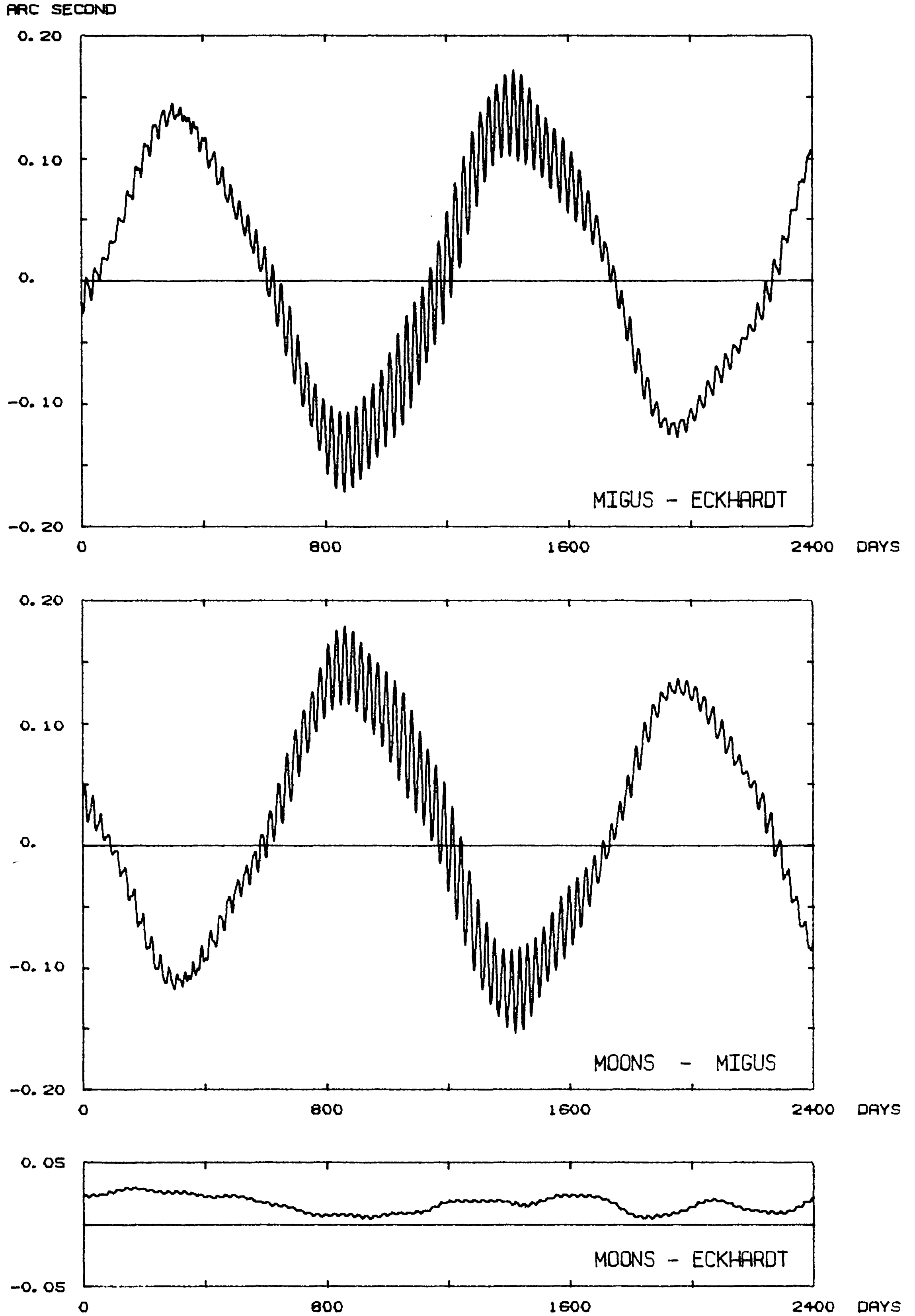

Fig. 4. Differences in τ.

exceeds 0.″2. Except for an offset of 0.″08 on the independent term of p_1, the discrepancy between our solution and Eckhardt's one does not exceed 0.″05.

In Table II, we give the terms for which our solution and Migus' one differ from one another by more than 0.″005. Table III is similar but with respect to Eckhardt's solution.

TABLE III
Differences (Eckhardt–Moons)

Series	Term	Eckhardt	Moons	Moons–Eckhardt
p_1	ind. term	−80.724	−80.644	0.080
	$\sin (l - F)$	124.492	124.483	−0.009
	$\cos F$	5.746	5.752	0.006
	$\sin (2l' - F + 2D)$	0.018	0.024	0.006
p_2	$\cos (l - F)$	−75.458	−75.433	0.025
	$\sin F$	−5.769	−5.775	−0.006
	$\cos (2l' - F + 2D)$	0.018	0.024	0.006
τ	ind. term	214.170	214.187	0.017
	$\sin (2l - 2F)$	17.014	17.020	0.006

Some of the discrepancies come probably from the fact that the three authors use different theories of the motion of the Moon's center of mass (it is certainly the case for the terms denoted by * in Table II). Moreover, the epoch of the theories used by Migus and Eckhardt is 1900 and they had to adjust them to 2000.

In spite of our efforts, we cannot point out with any assurance to a specific source for most of the discrepancies.

5. The Free Libration

The method we use permits us also to obtain an analytical expression of the free libration of the Moon, oscillation which depends on the *history* of the Moon (impact of meteorites, motions of masses, . . .) and on the dissipation processes inside the Moon.

In other theories, the free libration is disregarded. It is presented here as series depending analytically on the lunar gravitational field parameters but also on variables whose values must be determined by the observations. These series are tabulated in Table IV where each term is of the form

$$\mathrm{COEFF} * 10^5 * \mathrm{FACTOR} * {}^{\sin}_{\cos}(j_1 l + j_2 l' + j_3 D + j_4 F + j_5 p + j_6 q + j_7 r),$$

where j_5, j_6, j_7 are integer coefficients, p, q, r are the free libration angles, and

$$\mathrm{FACTOR} = \prod_{i=1}^{12} (E_i)^{k_i}$$

with $E_{10} = \sqrt{2P}$, $E_{11} = \sqrt{2Q}$ and $E_{12} = \sqrt{2R}$ the free libration amplitudes, k_{10}, k_{11}, k_{12} being integers.

TABLE IV

Series of the free libration

J_1	J_2	J_3	J_4	J_5	J_6	J_7	SIN	COS	K_1K_2	$K_3K_4K_5K_6K_7K_8K_9$	$K_{10}K_{11}K_{12}$	
0	0	0	0									
				0	1	0		0.0018	0 0	0 0 0 0 0 0 1	0 1 0	P1
				0	1	0	-1.3069		0 0	0 0 0 0 0 0 0	0 1 0	P1
				0	1	0	0.0040		0 1	0 0 0 0 0 0 0	0 1 0	P1
				0	1	0	-0.0017		1 0	0 0 0 0 0 0 0	0 1 0	P1
0	0	1	0									
				0	0	1		0.0030	0 0	0 0 0 0 0 0 1	0 0 1	P1
				0	0	1	2.0626		0 0	0 0 0 0 0 0 0	0 0 1	P1
				-1	0	0	-0.1826		0 0	0 0 0 0 0 0 0	1 0 0	P1
				1	0	0	0.1632		0 0	0 0 0 0 0 0 0	1 0 0	P1
				0	0	-1	-0.0035		0 0	0 0 0 0 0 0 0	0 0 1	P1
				-1	0	0	-0.0021		1 0	0 0 0 0 0 0 0	1 0 0	P1
				1	0	0	0.0021		1 0	0 0 0 0 0 0 0	1 0 0	P1
1	0	-1	0									
				0	0	-1	0.0139		0 0	0 0 0 0 0 0 0	0 0 1	P1
				1	0	0	0.0015		0 0	0 0 0 0 0 0 0	1 0 0	P1
1	0	0	0									
				0	-1	0	-0.0136		0 0	0 0 0 0 0 0 0	0 1 0	P1
				0	1	0	-0.0087		0 0	0 0 0 0 0 0 0	0 1 0	P1
2	0	-3	0									
				0	-1	0		0.0042	0 2	0 0 0 1 0 0 0	0 1 0	P1
				0	-1	0		0.0021	0 1	0 0 0 1 0 1 0	0 1 0	P1
				0	-1	0	-0.0055		0 2	0 0 0 0 1 0 0	0 1 0	P1
				0	-1	0	-0.0028		0 1	0 0 0 0 1 1 0	0 1 0	P1
2	0	-2	0									
				1	0	0		-0.0332	0 2	0 0 0 1 0 0 0	1 0 0	P1
				1	0	0		-0.0169	0 1	0 0 0 1 0 1 0	1 0 0	P1
				1	0	0		0.0024	1 1	0 0 0 1 0 0 0	1 0 0	P1
				1	0	0		-0.0021	0 0	0 0 0 1 0 2 0	1 0 0	P1
				1	0	0		0.0019	0 1	0 1 0 1 0 0 0	1 0 0	P1
				1	0	0	0.0438		0 2	0 0 0 0 1 0 0	1 0 0	P1
				1	0	0	0.0220		0 1	0 0 0 0 1 1 0	1 0 0	P1
				1	0	0	-0.0037		0 1	0 0 0 0 1 0 0	1 0 0	P1
				1	0	0	-0.0031		1 1	0 0 0 0 1 0 0	1 0 0	P1
				1	0	0	0.0028		0 0	0 0 0 0 1 2 0	1 0 0	P1
				1	0	0	-0.0025		0 1	0 1 0 0 1 0 0	1 0 0	P1
2	0	-1	0									
				0	-1	0		0.0046	0 2	0 0 0 1 0 0 0	0 1 0	P1
				0	-1	0		0.0024	0 1	0 0 0 1 0 1 0	0 1 0	P1

J_1	J_2	J_3	J_4	J_5	J_6	J_7	SIN	COS	K_1K_2	$K_3K_4K_5K_6K_7K_8K_9$	$K_{10}K_{11}K_{12}$	
				0	-1	0	-0.0061		0 2	0 0 0 0 1 0 0	0 1 0	P1
				0	-1	0	-0.0031		0 1	0 0 0 0 1 1 0	0 1 0	P1
0	0	0	0									
				0	1	0		3.2471	0 0	0 0 0 0 0 0 0	0 1 0	P2
				0	1	0		0.0100	0 1	0 0 0 0 0 0 0	0 1 0	P2
				0	1	0		-0.0044	1 0	0 0 0 0 0 0 0	0 1 0	P2
				0	1	0		-0.0021	0 0	0 1 0 0 0 0 0	0 1 0	P2
				0	1	0	-0.0044		0 0	0 0 0 0 0 0 1	0 1 0	P2
0	0	1	0									
				0	0	1		2.0607	0 0	0 0 0 0 0 0 0	0 0 1	P2
				-1	0	0		-0.1831	0 0	0 0 0 0 0 0 0	1 0 0	P2
				1	0	0		0.1636	0 0	0 0 0 0 0 0 0	1 0 0	P2
				0	0	-1		-0.0035	0 0	0 0 0 0 0 0 0	0 0 1	P2
				-1	0	0		-0.0022	1 0	0 0 0 0 0 0 0	1 0 0	P2
				1	0	0		0.0021	1 0	0 0 0 0 0 0 0	1 0 0	P2
				0	0	1	-0.0030		0 0	0 0 0 0 0 0 1	0 0 1	P2
0	1	-1	1									
				0	0	-1		-0.0013	0 0	0 0 0 0 0 0 0	0 0 1	P2
1	0	-1	0									
				0	0	-1		-0.0073	0 0	0 0 0 0 0 0 0	0 0 1	P2
				1	0	0		-0.0063	0 0	0 0 0 0 0 0 0	1 0 0	P2
				-1	0	0		-0.0020	0 0	0 0 0 0 0 0 0	1 0 0	P2
1	0	0	0									
				0	-1	0		-0.0139	0 0	0 0 0 0 0 0 0	0 1 0	P2
				0	1	0		-0.0086	0 0	0 0 0 0 0 0 0	0 1 0	P2
2	0	-3	0									
				0	-1	0		0.0055	0 2	0 0 0 0 1 0 0	0 1 0	P2
				0	-1	0		0.0028	0 1	0 0 0 0 1 1 0	0 1 0	P2
				0	-1	0	0.0042		0 2	0 0 0 1 0 0 0	0 1 0	P2
				0	-1	0	0.0021		0 1	0 0 0 1 0 1 0	0 1 0	P2
2	0	-2	0									
				1	0	0		-0.1093	0 2	0 0 0 0 1 0 0	1 0 0	P2
				1	0	0		-0.0550	0 1	0 0 0 0 1 1 0	1 0 0	P2
				1	0	0		0.0092	0 1	0 0 0 0 1 0 0	1 0 0	P2
				1	0	0		0.0078	1 1	0 0 0 0 1 0 0	1 0 0	P2
				1	0	0		-0.0069	0 0	0 0 0 0 1 2 0	1 0 0	P2
				1	0	0		0.0064	0 1	0 1 0 0 1 0 0	1 0 0	P2
				1	0	0		0.0023	0 0	0 0 0 0 1 1 0	1 0 0	P2
				1	0	0		0.0020	1 0	0 0 0 0 1 1 0	1 0 0	P2

Table IV (*Continued*)

J_1	J_2	J_3	J_4	J_5	J_6	J_7	SIN	COS	K_1K_2	$K_3K_4K_5K_6K_7K_8K_9$	$K_{10}K_{11}K_{12}$	
				1	0	0		0.0016	0 0	0 1 0 0 1 1 0	1 0 0	P2
				1	0	0		0.0014	0 3	0 0 0 0 1 0 0	1 0 0	P2
				1	0	0		0.0011	0 2	0 0 0 0 1 1 0	1 0 0	P2
				1	0	0	-0.0829		0 2	0 0 0 1 0 0 0	1 0 0	P2
				1	0	0	-0.0421		0 1	0 0 0 1 0 1 0	1 0 0	P2
				1	0	0	0.0060		1 1	0 0 0 1 0 0 0	1 0 0	P2
				1	0	0	-0.0053		0 0	0 0 0 1 0 2 0	1 0 0	P2
				1	0	0	0.0048		0 1	0 1 0 1 0 0 0	1 0 0	P2
				1	0	0	0.0018		0 1	0 0 0 1 0 0 0	1 0 0	P2
				1	0	0	0.0015		1 0	0 0 0 1 0 1 0	1 0 0	P2
				1	0	0	0.0012		0 0	0 1 0 1 0 1 0	1 0 0	P2
				1	0	0	0.0011		0 3	0 0 0 1 0 0 0	1 0 0	P2
2	0	-1	0									
				0	-1	0		-0.0061	0 2	0 0 0 0 1 0 0	0 1 0	P2
				0	-1	0		-0.0031	0 1	0 0 0 0 1 1 0	0 1 0	P2
				0	-1	0	-0.0047		0 2	0 0 0 1 0 0 0	0 1 0	P2
				0	-1	0	-0.0024		0 1	0 0 0 1 0 1 0	0 1 0	P2
0	0	0	0									
				0	1	0		0.0016	0 0	0 0 0 1 0 0 0	0 1 0	TAU
				1	0	0	12.8044		0 0	0 0 0 0 0 0 0	1 0 0	TAU
				0	0	1	0.0935		0 0	0 0 0 0 0 0 0	0 0 1	TAU
				1	0	0	-0.0834		0 1	0 0 0 0 0 0 0	1 0 0	TAU
				1	0	0	-0.0212		0 0	0 0 0 0 0 1 0	1 0 0	TAU
				1	0	0	0.0027		0 0	0 1 0 0 0 0 0	1 0 0	TAU
				1	0	0	0.0014		0 2	0 0 0 0 0 0 0	1 0 0	TAU
				0	1	0	-0.0013		0 0	0 0 0 0 1 0 0	0 1 0	TAU
0	0	1	0									
				0	1	0	0.0305		0 0	0 0 0 0 0 0 0	0 1 0	TAU
				0	-1	0	0.0129		0 0	0 0 0 0 0 0 0	0 1 0	TAU
0	1	0	0									
				-1	0	0	0.0010		0 0	0 0 0 0 0 0 0	1 0 0	TAU
1	0	-1	0									
				0	-1	0	-0.0273		0 0	0 0 0 0 0 0 0	0 1 0	TAU
				0	1	0	0.0058		0 0	0 0 0 0 0 0 0	0 1 0	TAU
2	0	-2	0									
				0	-1	0		0.3262	0 2	0 0 0 1 0 0 0	0 1 0	TAU
				0	-1	0		0.1657	0 1	0 0 0 1 0 1 0	0 1 0	TAU
				0	-1	0		-0.0238	1 1	0 0 0 1 0 0 0	0 1 0	TAU
				0	-1	0		0.0210	0 0	0 0 0 1 0 2 0	0 1 0	TAU
				0	-1	0		-0.0189	0 1	0 1 0 1 0 0 0	0 1 0	TAU
				0	-1	0		-0.0070	0 1	0 0 0 1 0 0 0	0 1 0	TAU
				0	-1	0		-0.0064	0 3	0 0 0 1 0 0 0	0 1 0	TAU
				0	-1	0		-0.0060	1 0	0 0 0 1 0 1 0	0 1 0	TAU
				0	-1	0		-0.0048	0 0	0 1 0 1 0 1 0	0 1 0	TAU
				0	-1	0		-0.0038	0 2	0 0 0 1 0 1 0	0 1 0	TAU
				0	-1	0		-0.0020	0 1	1 0 0 0 0 0 0	0 1 0	TAU
				0	-1	0		-0.0018	0 0	0 0 0 1 0 1 0	0 1 0	TAU
				0	-1	0	-0.4303		0 2	0 0 0 0 1 0 0	0 1 0	TAU
				0	-1	0	-0.2165		0 1	0 0 0 0 1 1 0	0 1 0	TAU
				0	-1	0	0.0362		0 1	0 0 0 0 1 0 0	0 1 0	TAU
				0	-1	0	0.0306		1 1	0 0 0 0 1 0 0	0 1 0	TAU
				0	-1	0	-0.0272		0 0	0 0 0 0 1 2 0	0 1 0	TAU
				0	-1	0	0.0250		0 1	0 1 0 0 1 0 0	0 1 0	TAU
				0	-1	0	0.0091		0 0	0 0 0 0 1 1 0	0 1 0	TAU
				0	-1	0	0.0084		0 3	0 0 0 0 1 0 0	0 1 0	TAU
				0	-1	0	0.0077		1 0	0 0 0 0 1 1 0	0 1 0	TAU
				0	-1	0	0.0064		0 2	0 0 0 0 1 1 0	0 1 0	TAU
				0	-1	0	0.0063		0 0	0 1 0 0 1 1 0	0 1 0	TAU
				0	0	-1	0.0018		0 0	0 0 0 0 0 0 0	0 0 1	TAU
				0	-1	0	0.0014		0 1	0 0 0 0 1 2 0	0 1 0	TAU
				0	-1	0	-0.0013		1 0	0 0 0 0 1 0 0	0 1 0	TAU
				0	-1	0	-0.0010		0 0	0 1 0 0 1 0 0	0 1 0	TAU
2	0	0	-2									
				-1	0	0	0.0014		0 0	0 0 0 0 0 0 0	1 0 0	TAU

The other quantities have the same meaning as for the forced libration series.

The amplitudes and phases of the free librations must be determined by observations. Their frequencies are given in Table V. Let us notice at the end that the series presented in Table IV, are truncated at the first order in $\sqrt{2P}$, $\sqrt{2Q}$ and $\sqrt{2R}$ ($k_{10} + k_{11} + k_{12} = 1$) since these quantities are very small.

TABLE V
Frequencies

$$
\begin{aligned}
10^2 * n_p/n = {} & 2.594\,995\,3 \\
& + 0.033\,822\,2 * E_2 \\
& + 0.008\,607\,6 * E_8 \\
& - 0.001\,095\,9 * E_4 \\
& - 0.000\,221\,1 * (E_2)^2 \\
& - 0.000\,112\,6 * E_2 * E_8 \\
& - 0.000\,050\,0 * E_1 \\
& + 0.000\,019\,8 * (E_9)^2 \\
& + 0.000\,015\,0 * E_2 * E_4 \\
& - 0.000\,014\,3 * (E_8)^2 \\
& + 0.000\,003\,8 * E_4 * E_8 \\
& + 0.000\,002\,9 * (E_2)^3 \\
& + 0.000\,002\,8 * E_5 * E_9 \\
& + 0.000\,002\,3 * (E_6)^2 \\
& + 0.000\,002\,2 * (E_2)^2 * E_8 \\
& - 0.000\,001\,1 * E_2 * (E_9)^2
\end{aligned}
$$

$$
\begin{aligned}
10^4 * n_q/n = {} & 9.982\,07 \\
& + 0.120\,79 * E_1 \\
& - 0.014\,01 * E_2 \\
& - 0.012\,33 * E_4 \\
& + 0.002\,96 * E_8 \\
& - 0.000\,19 * (E_2)^2 \\
& + 0.000\,16 * E_1 * E_2
\end{aligned}
$$

$$
\begin{aligned}
10^3 * n_r/n = {} & -3.096\,310 \\
& + 0.008696 * E_1 \\
& + 0.004\,326 * E_2 \\
& - 0.003\,149 * E_4 \\
& + 0.000\,731 * E_8
\end{aligned}
$$

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