10. Radiative heat transfer

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10.1 The problem of radiative exchange

- Conduction and Convection require a physical medium to transport heat.
- Thermal radiation however requires **NO** medium and can travel through vacuum and the various type of physical mediums.

Applications and examples

- Sun light heating the earth
- The greenhouse effect (both environmental and actual greenhouses)
- Heat dissipation from high temperature processes such as in combustion, high temperature equipment, space heating in buildings, etc
- Cooling of/in space vehicles
- Heat shields for very low temperature processes

10.1 The problem of radiative exchange

The heat exchange problem

Figure 10.1 shows two arbitrary surfaces radiating energy to one another.

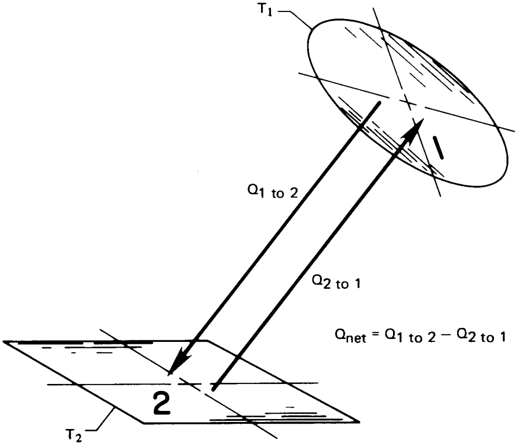


Figure 10.1 Thermal radiation between two arbitrary surfaces.

10.1 The problem of radiative exchange

The net heat exchange from the hotter surface (1) to the cooler surface (2) depends on the following influences :

- T_1 and T_2
- The areas of (1) and (2), A_1 and A_2
- The shape, orientation, and spacing of (1) and (2)
- The radiative properties of the surfaces
- Additional surfaces in the environment, whose radiation may be reflected by one surface to the other
- The medium between (1) and (2) if it absorbs, emits, or "reflects" radiation (When the medium is air, we can usually neglect these effects)

Some definitions

Emittance A real body at temperature T does not emit with the black body emissive power $e - b = \sigma T^4$ but rather with some fraction, ϵ, of e_b.

The same is true of the **monochromatic emissive power**, $e_\lambda(T)$, which is always lower for a real body.

$$\epsilon_\lambda(\lambda, T) = \frac{e_\lambda(\lambda, T)}{e_{\lambda,b}(\lambda, T)} \qquad (10.3)$$

The **total emittance**

$$\epsilon(T) = \frac{e(T)}{e_b(T)} = \frac{\int_0^\infty \epsilon_\lambda(\lambda, T)\, e_{\lambda,b}(\lambda, T)\, d\lambda}{\sigma T^4} \qquad (10.4)$$

For real bodies $0 < \epsilon,\ \epsilon_\lambda < 1$

For black bodies $\epsilon,\ \epsilon_\lambda = 1$

Metals			*Nonmetals*		
Surface	*Temp.* (°C)	ε	*Surface*	*Temp.* (°C)	ε
Aluminum			Asbestos	40	0.93–0.97
Polished, 98% pure	200–600	0.04–0.06	Brick		
Commercial sheet	90	0.09	Red, rough	40	0.93
Heavily oxidized	90–540	0.20–0.33	Silica	980	0.80–0.85
Brass			Fireclay	980	0.75
Highly polished	260	0.03	Ordinary refractory	1090	0.59
Dull plate	40–260	0.22	Magnesite refractory	980	0.38
Oxidized	40–260	0.46–0.56	White refractory	1090	0.29
Copper			Carbon		
Highly polished electrolytic	90	0.02	Filament	1040–1430	0.53
Slightly polished to dull	40	0.12–0.15	Lampsoot	40	0.95
Black oxidized	40	0.76	Concrete, rough	40	0.94
Gold: pure, polished	90–600	0.02–0.035	Glass		
Iron and steel			Smooth	40	0.94
Mild steel, polished	150–480	0.14–0.32	Quartz glass (2 mm)	260–540	0.96–0.66
Steel, polished	40–260	0.07–0.10	Pyrex	260–540	0.94–0.74
Sheet steel, rolled	40	0.66	Gypsum	40	0.80–0.90
Sheet steel, strong rough oxide	40	0.80	Ice	0	0.97–0.98
Cast iron, oxidized	40–260	0.57–0.66	Limestone	400–260	0.95–0.83
Iron, rusted	40	0.61–0.85	Marble	40	0.93–0.95
Wrought iron, smooth	40	0.35	Mica	40	0.75
Wrought iron, dull oxidized	20–360	0.94	Paints		
Stainless, polished	40	0.07–0.17	Black gloss	40	0.90
Stainless, after repeated heating	230–900	0.50–0.70	White paint	40	0.89–0.97
			Lacquer	40	0.80–0.95
Lead			Various oil paints	40	0.92–0.96
Polished	40–260	0.05–0.08	Red lead	90	0.93
Oxidized	40–200	0.63	Paper		
Mercury: pure, clean	40–90	0.10–0.12	White	40	0.95–0.98
Platinum			Other colors	40	0.92–0.94
Pure, polished plate	200–590	0.05–0.10	Roofing	40	0.91
Oxidized at 590°C	260–590	0.07–0.11	Plaster, rough lime	40–260	0.92
Drawn wire and strips	40–1370	0.04–0.19	Quartz	100–1000	0.89–0.58
Silver	200	0.01–0.04	Rubber	40	0.86–0.94
Tin	40–90	0.05	Snow	10–20	0.82
Tungsten			Water, thickness ≥0.1 mm	40	0.96
Filament	540–1090	0.11–0.16	Wood	40	0.80–0.90
Filament	2760	0.39	Oak, planed	20	0.90

Table 10.1 Total emittances for a variety of surfaces.

for a **gray body**, $\epsilon_\lambda = \epsilon$ (independent of λ)

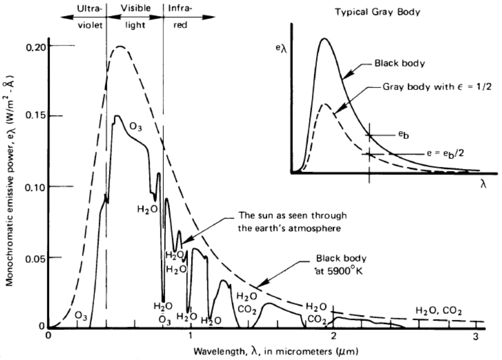
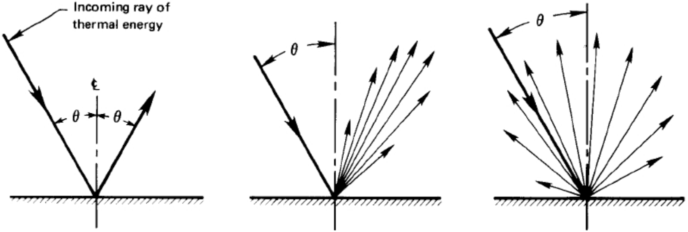


Figure 10.2 Comparison of the sun's energy as typically seen through the earth's atmosphere with that of a black body having the same mean temperature, size, and distance from the earth.

10.1 The problem of radiative exchange

By using it, we can write the emissive power as if the body were gray, without integrating over wavelength (*gray body approximation*)

$$e(T) = \epsilon \sigma T^4 \qquad (10.5)$$



Specular or mirror-like reflection of incoming ray.

Reflection which is between diffuse and specular (a real surface).

Diffuse radiation in which directions of departure are uninfluenced by incoming ray angle, θ.

Figure 10.3 Specular and diffuse reflection of radiation. (Arrows indicate magnitude of the heat flux in the directions indicated.)

Diffuse and specular emittance and reflection

The energy emitted by a non-black surface, together with that portion of an incoming ray of energy that is reflected by the surface, may leave the body diffusely or specularly, as shown in Figure 10.3. That energy may also be emitted or reflected. Black body emission is always diffuse.

Emittance and reflectance depend on wavelength, temperature, and angles of incidence and/or departure. But in this chapter, we shall assume **diffuse behavior** for most surfaces.

Intensity of radiation

To account for the effects of geometry on radiant exchange, we must think about how angles of orientation affect the radiation between surfaces.

If it were non-black but diffuse, the heat flux leaving the surface would again be independent of direction. Thus, the rate at which energy is emitted in any direction from this diffuse element is proportional to the projected area of dA normal to the direction of view, as shown in the upper right side of Figure 10.4.

10.1 The problem of radiative exchange

If an aperture of area dA_a is placed at a radius r and angle θ from dA and is normal to the radius, it will see dA as having an area $cos\theta\, dA$. Radiation that leaves dA within the solid angle $d\omega$ stays within $d\omega$ as it travels to dA_a.

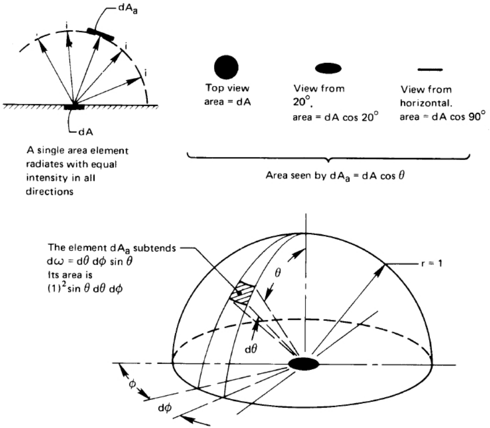


Figure 10.4 Radiation intensity through a unit sphere.

10.1 The problem of radiative exchange

Radiant energy from dA that is intercepted by dA_a

$$dQ_{\mathrm{outgoing}} = (i\, d\omega)(cos\theta\, dA) \quad [\mathrm{steradian}] \qquad (10.6)$$

dividing equation (10.6) by dA and integrating over the entire hemisphere ($r = 1$ and $d\omega = cos\theta\, d\theta\, d\Phi$)

$$q_{\mathrm{outgoing}} = \int_{\Phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} i\, cos\theta\, (sin\theta d\theta\, d\phi) = \pi i \qquad (10.7a)$$

For a black body

$$i_b = \frac{e_b}{\pi} = \frac{\sigma T^4}{\pi} = f(T) \qquad (10.7b)$$

For a given wavelength, the monochromatic intensity is

$$i_\lambda = \frac{e_\lambda}{\pi} = f(T, \lambda) \qquad (10.7c)$$

The problem of predicting α

α depends on the physical properties and temperatures of all bodies involved in the heat exchange process. Kirchhoff's law is an expression that allows α to be determined under certain restrictions.

Kirchhoff's law a body in thermodynamic equilibrium emits as much energy as it absorbs in each direction and at each wavelength

$$\epsilon_\lambda(T,\theta,\Phi) = \alpha_\lambda(T,\theta,\Phi) \qquad (10.8a)$$

For a diffuse body,(emittance and absorptance do not depend on the angles) Kirchhoff's law becomes

$$\epsilon_\lambda(T) = \alpha_\lambda(T) \qquad (10.8b)$$

If, in addition, the body is gray

$$\epsilon(T) = \alpha(T) \qquad (10.8c)$$

It will be accurate either if the monochromatic emittance does not vary strongly with wavelength or if the bodies exchanging radiation are at similar absolute temperatures.

Total absorptance during radiant exchange

Diffuse surfaces.

Consider two plates as shown in Figure (10.5). The plate at T_1 be non-black and that at T_2 be black.

Net heat transfer from plate 1 to plate 2

$$q_{\text{net}} = \underbrace{\int_0^{\infty} \epsilon_{\lambda,1}(T_1)\, e_{\lambda,b}(T_1) d\lambda}_{\text{emitted by plate 1}} - \underbrace{\int_0^{\infty} \alpha_{\lambda,1}(T_1)\, e_{\lambda,b}(T_2) d\lambda}_{\text{radiation form plate 2 absorbed by plate 1}} \tag{10.9}$$

We define the total absorptance, $\alpha_1(T_1, T_2)$,

$$q_{\text{net}} = \underbrace{\epsilon_1(T_1)\,\sigma T_1^4}_{\text{emitted by plate 1}} - \underbrace{\alpha_1(T_1, T_2)\,\sigma T_2^4}_{\text{absorbed by plate 1}} \tag{10.10}$$

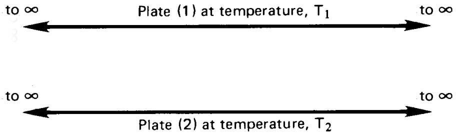


Figure 10.5 Heat transfer between two infinite parallel plates.

Total absorptance depends on T_2 because the spectrum of radiation from plate 2 depends on the temperature of plate 2, and depends on T_1 too because $\alpha_{\lambda,1}$ is a property of plate 1 that may be temperature dependent.

The gray body approximation

If we consider 2 plates. The net heat flux between the plates is

$$q_{\mathrm{net}} = \epsilon_1 \sigma T_1^4 - \alpha_1(T_1, T_2)\sigma T_2^4$$

If plate 1 is a gray body, $\epsilon_1 = \alpha_1$

$$q_{\mathrm{net}} = \epsilon_1 \sigma (T_1^4 - T_2^4) \qquad (10.11)$$

10.3 Radiant heat exchange between 2 finite black bodies

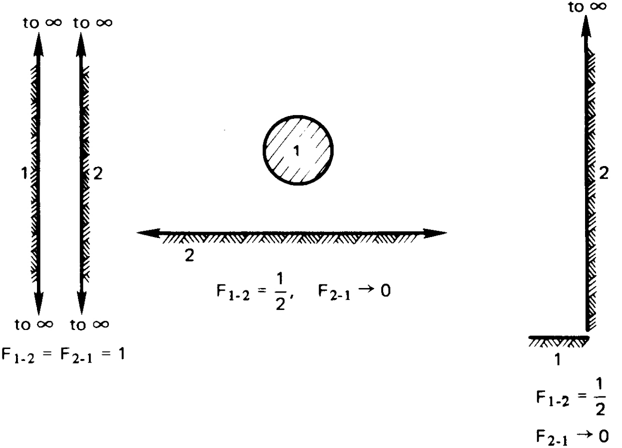


Figure 10.6 Some configurations for which the value of the view factor is immediately apparent.

10.3 Radiant heat exchange between 2 finite black bodies

Some evident results

- F_{1-2} is the fraction of field of view of (1) occupied by (2).

  For an *isothermal* and *diffuse* surface

  F_{1-2} is fraction of energy leaving (1) that reaches (2).
- Conservation of energy

$$1 = F_{1-1} + F_{1-2} + F_{1-3} + ...F_{1-n} \qquad (10.12)$$

  where (2), (3),...,(n) are all of the bodies in the neighborhood of (1).
- $$F_{1-(2+3)} = F_{1-2} + F_{1-3}$$
$$F_{(2+3)-1} \neq F_{2-1} + F_{3-1}$$

10.3 Radiant heat exchange between 2 finite black bodies

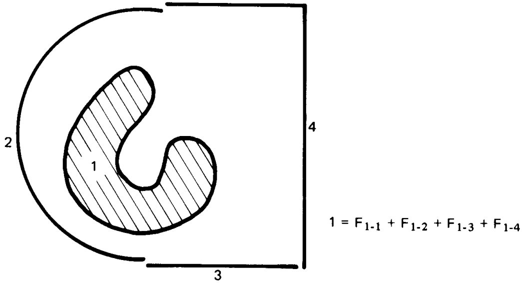


Figure 10.7 A body (1) that views three other bodies and itself as well.

View factor reciprocity

So far, we have referred to the net radiation from black surface (1) to black surface (2) as Q_{net}. Let us refine our notation a bit

$$Q_{\mathrm{net}\,1-2} = A_1 F_{1-2} \sigma (T_1^4 - T_2^4) \qquad (10.13)$$

and

$$Q_{\mathrm{net}\,2-1} = A_2 F_{2-1} \sigma (T_2^4 - T_1^4) \qquad (10.14)$$

With $Q_{\mathrm{net}\,1-2} = -Q_{\mathrm{net}\,2-1}$, we get the **view factor reciprocity**

$$A_1 F_{1-2} = A_2 F_{2-1} \qquad (10.15)$$

Example 10.1

A jet of liquid metal at 2000°C pours from a crucible. It is 3 mm in diameter. A long cylindrical radiation shield, 5 cm diameter, surrounds the jet through an angle of 330°, but there is a 30° slit in it. The jet and the shield radiate as black bodies. They sit in a room at 30°C, and the shield has a temperature of 700°C. Calculate the net heat transfer : from the jet to the room through the slit ; from the jet to the shield ; and from the inside of the shield to the room.

Solution

By inspection, we see that $F_{jet-room} = 30/360 = 0.08333$ and $F_{jet-shield} = 330/360 = 0.9167$. Thus,

$$Q_{\mathrm{net}\,jet-room} = A_{jet}F_{jet-room}\,\sigma(T_{jet}^4 - T_{room}^4) = 1,188\,W/m$$

Likewise,

$$Q_{\mathrm{net}\,jet-shield} = A_{jet}F_{jet-shield}\,\sigma(T_{jet}^4 - T_{shield}^4) = 12,637\,W/m$$

Find the radiation from the inside of the shield to the room. We can find this view factor equating view factors to the room with view factors to the slit ($A_{slit} = \pi * 0.05 * 30/360 = 0.01309\,m^2/m\ \mathrm{length}$).
With equations (10.12) and (10.15),

$$F_{slit-jet} = \frac{A_{jet}}{A_{slit}}F_{jet-room} = 0.06$$

$$F_{slit-shield} = 1 - F_{slit-jet} - \underbrace{F_{slit-slit}}_{\cong 0} = 0.94$$

$$F_{shield-room} = \frac{A_{slit}}{A_{shield}} F_{slit-shield} = 0.08545$$

$$Q_{\mathrm{net}\ shield-room} = A_{shield} F_{shield-room}\, \sigma(T^4_{shield} - T^4_{room}) = 619\, W/m$$

Both the jet and the inside of the shield have relatively small view factors to the room, so that comparatively little heat is lost through the slit.

Calculation of the black-body view factor F_{1-2}

Figure (10.8) shows two bodies dA_1 and dA_2 who are large black bodies and isothermal. Since element dA_2 subtends a solid angle $d\omega$, equation (10.6) gives

$$dQ_{1\ to\ 2} = (i_1 d\omega_1)(cos\beta_1 dA_1)$$

from eqn. (10.7b)

$$i_1 = \frac{\sigma T_1^4}{\pi}$$

Note that because black bodies radiate diffusely, i_1 does not vary with angle ; and because these bodies are isothermal, it does not vary with position.

Solid angle is given by

$$d\omega_1 = \frac{cos\beta_2\, dA_2}{s^2}$$

where s is the distance from (1) to (2) and $cos\beta_2$ enters because dA_2 is not necessarily normal to s.

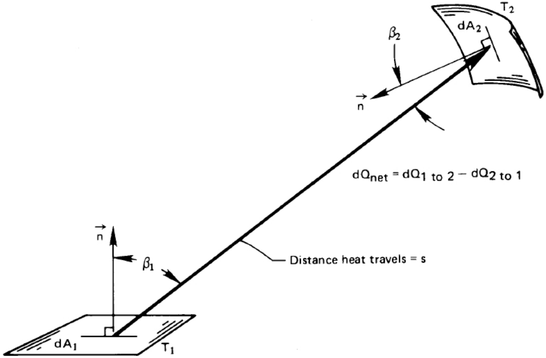


Figure 10.8 Radiant exchange between two black elements that are part of the bodies (1) and (2).

10.3 Radiant heat exchange between 2 finite black bodies

$$dQ_{1\ to\ 2} = \frac{\sigma T_1^4}{\pi}\left(\frac{cos\beta_1 cos\beta_2\, dA_1 dA_2}{s^2}\right)$$

By the same token,

$$dQ_{2\ to\ 1} = \frac{\sigma T_2^4}{\pi}\left(\frac{cos\beta_2 cos\beta_1\, dA_2 dA_1}{s^2}\right)$$

Then

$$Q_{\mathrm{net}\ 1-2} = \sigma(T_1^4 - T_2^4)\int_{A_1}\int_{A_2}\frac{cos\beta_1 cos\beta_2}{\pi s^2}dA_1 dA_2 \qquad (10.16)$$

From equation (10.16). If we compare this result with $Q_{\mathrm{net}\ 1-2} = A_1 F_{1-2}\sigma(T_1^4 - T_2^4)$, we get

$$F_{1-2} = \frac{1}{A_1}\int_{A_1}\int_{A_2}\frac{cos\beta_1 cos\beta_2}{\pi s^2}dA_1 dA_2 \qquad (10.17a)$$

From the inherent symmetry of the problem,

$$F_{2-1} = \frac{1}{A_2}\int_{A_2}\int_{A_1}\frac{cos\beta_2 cos\beta_1}{\pi s^2}dA_2 dA_1 \qquad (10.17b)$$

10.3 Radiant heat exchange between 2 finite black bodies

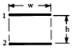
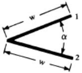
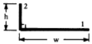
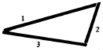
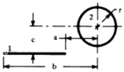
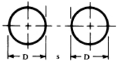
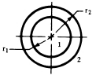
Configuration		Equation
1.		$F_{1\text{-}2} = F_{2\text{-}1} = \sqrt{1 + \left(\frac{h}{w}\right)^2} - \left(\frac{h}{w}\right)$
2.		$F_{1\text{-}2} = F_{2\text{-}1} = 1 - \sin(\alpha/2)$
3.		$F_{1\text{-}2} = \frac{1}{2}\left[1 + \frac{h}{w} - \sqrt{1 + \left(\frac{h}{w}\right)^2}\right]$
4.		$F_{1\text{-}2} = (A_1 + A_2 - A_3)/2A_1$
5.		$F_{1\text{-}2} = \frac{r}{b-a}\left[\tan^{-1}\frac{b}{c} - \tan^{-1}\frac{a}{c}\right]$
6.		Let $X = 1 + s/D$. Then: $F_{1\text{-}2} = F_{2\text{-}1} = \frac{1}{\pi}\left[\sqrt{X^2-1} + \sin^{-1}\frac{1}{X} - X\right]$
7.		$F_{1\text{-}2} = 1, \quad F_{2\text{-}1} = \frac{r_1}{r_2}$, and $F_{2\text{-}2} = 1 - F_{2\text{-}1} = 1 - \frac{r_1}{r_2}$

Table 10.2 View factors (infinite in extent normal to the paper)

10.3 Radiant heat exchange between 2 finite black bodies

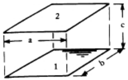
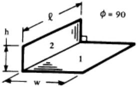
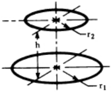
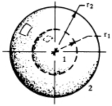
Configuration	Equation
1. 	Let $X = a/c$ and $Y = b/c$. Then: $F_{1\text{-}2} = \frac{2}{\pi XY}\left\{\ln\left[\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}\right]^{1/2} - X\tan^{-1}X - Y\tan^{-1}Y + X\sqrt{1+Y^2}\tan^{-1}\frac{X}{\sqrt{1+Y^2}} + Y\sqrt{1+X^2}\tan^{-1}\frac{Y}{\sqrt{1+X^2}}\right\}$
2. 	Let $H = h/\ell$ and $W = w/\ell$. Then: $F_{1\text{-}2} = \frac{1}{\pi W}\left\{W\tan^{-1}\frac{1}{W} - \sqrt{H^2+W^2}\tan^{-1}\left(H^2+W^2\right)^{-1/2} + H\tan^{-1}\frac{1}{H} + \frac{1}{4}\ln\left\{\left[\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\right] \times \left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2}\left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right\}$
3. 	Let $R_1 = r_1/h$, $R_2 = r_2/h$, and $X = 1 + \left(1+R_2^2\right)/R_1^2$. Then: $F_{1\text{-}2} = \frac{1}{2}\left[X - \sqrt{X^2 - 4(R_2/R_1)^2}\right]$
4. 	Concentric spheres: $F_{1\text{-}2} = 1, \quad F_{2\text{-}1} = (r_1/r_2)^2, \quad F_{2\text{-}2} = 1 - (r_1/r_2)^2$

Table 10.3 View factors for some three-dimensional configurations

10.3 Radiant heat exchange between 2 finite black bodies

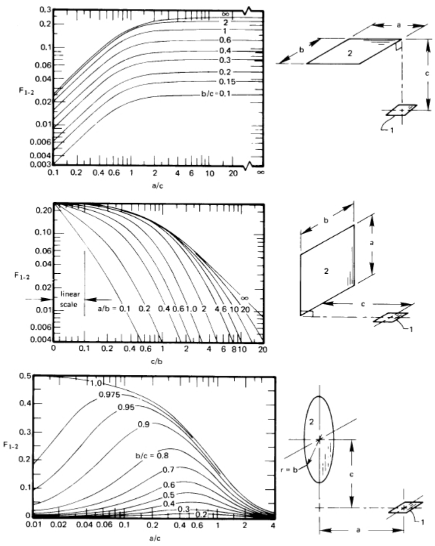


Figure 10.10 View factor for 3 very small surfaces "looking at" 3 large surfaces.

Example 10.2

A heater (h) as shown in Figure 10.11 radiates to the partially conical shield (s) that surrounds it. If the heater and shield are black, calculate the net heat transfer from the heater to the shield.

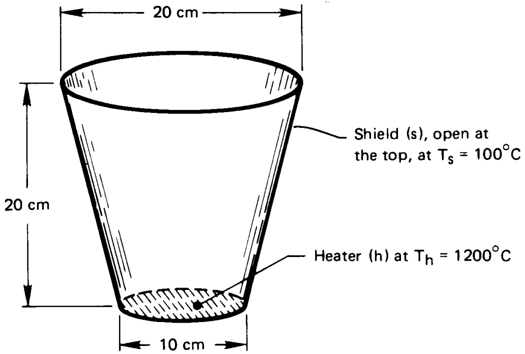


Figure 10.11 Heat transfer from a disc heater to its radiation shield.

Solution

First imagine a plane (i) laid across the open top of the shield :

$$F_{h-s} + F_{h-i} = 1$$

But F_{h-i} can be obtained from Figure 10.9 or case 3 of Table 10.3.

For $R_1 = r_1/h = 0.25$ and $R_2 = r_2/h = 0.5$. The result is $F_{h-i} = 0.192$. Then

$$F_{h-s} = 0.808$$

Thus,

$$Q_{\mathrm{net}\ h-s} = A_h F_{h-s} \sigma (T_h^4 - T_s^4) = 1687\ W$$

Example 10.4

Find F_{1-2} for the configuration of two offset squares of area A, as shown in Figure 10.12.

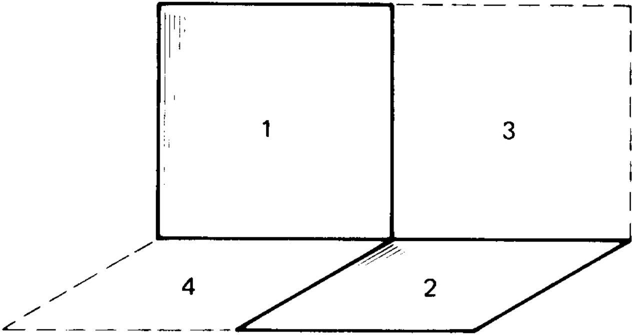


Figure 10.12 Radiation between 2 offset perpendicular squares.

Solution

Consider two fictitious areas 3 and 4 as indicated by the dotted lines. The view factor between the combined areas, (1+3) and (2+4), can be obtained from Figure 10.9. In addition, we can write that view factor in terms of the unknown F_{1-2} and other known view factors.

$$(2A)F_{(1+3)-(4+2)} = AF_{1-4} + AF_{1-2} + AF_{3-4} + AF_{3-2}$$

$$2F_{(1+3)-(4+2)} = 2F_{1-4} + 2F_{1-2}$$

$$F_{1-2} = F_{(1+3)-(4+2)} - F_{1-4}$$

$F_{(1+3)-(4+2)}$ and F_{1-4} can be read from Figure 10.9.

$$F_{1-2} = 0.045$$

Electrical analogy for gray body heat exchange

An electric circuit analogy for heat exchange among diffuse gray bodies was developed by Oppenheim in 1956.

Irradiance, H [W/m^2] is the flux of energy that irradiates the surface.

Radiosity, B [W/m^2] is the total flux of radiative energy away from the surface.

$$B = \rho H + \epsilon e_b \qquad (10.18)$$

Net heat flux leaving any particular surface as the difference between B and H for that surface

$$q_{\mathrm{net}} = B - H = B - \frac{B - \epsilon e_b}{\rho} \qquad (10.19)$$

This can be rearranged as

$$q_{\mathrm{net}} = \frac{\epsilon}{\rho} e_b - \frac{1-\rho}{\rho} B \qquad (10.20)$$

If the surface is opaque ($\tau = 0$), 1-$\rho = \alpha$, and if it is gray, $\alpha = \epsilon$. Then, equation (10.20) gives a form of Ohm's law

$$q_{\rm net} A = Q_{\rm net} = \frac{e_b - B}{(1-\epsilon)/\epsilon A} \qquad (10.21)$$

($e_b - B$) can be viewed as a driving potential for transferring heat away from a surface through an effective surface resistance, $(1-\epsilon)/\epsilon A$.

Now consider heat transfer from one infinite gray plate to another parallel to it. Radiant energy flows past an imaginary surface. If the gray plate is diffuse, its radiation has the same geometrical distribution as that from a black body, and it will travel to other objects in the same way that black body radiation would.

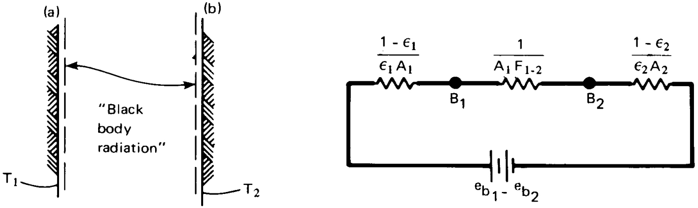


Fig. 10.13 Electrical circuit analogy for radiation between two gray infinite plates.

10.4 Heat transfer among gray bodies

By analogy to equation (10.13)

$$Q_{\mathrm{net}\,1-2} = A_1 F_{1-2}(B_1 - B_2) = \frac{B_1 - B_2}{\frac{1}{A_1 F_{1-2}}} \qquad (10.22)$$

radiosity difference $(B_1 - B_2)$, can be said to drive heat through the geometrical resistance, $1/A_1 F_{1-2}$ that describes the field of view between the two surfaces. When two gray surfaces exchange radiation only with each other, the net radiation flows through a surface resistance for each surface and a geometric resistance for the configuration.

Recalling that $e_b = \sigma T_4$, we obtain

$$Q_{\mathrm{net}\,1-2} = \frac{e_{b,1} - e_{b,2}}{\sum \mathrm{resistances}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\epsilon}{\epsilon A}\right)_1 + \frac{1}{A_1 F_{1-2}} + \left(\frac{1-\epsilon}{\epsilon A}\right)_2} \qquad (10.23)$$

For infinite parallel plates, $F_{1-2} = 1$ and $A_1 = A_2$ with $q_{\mathrm{net}\,1-2} = Q_{\mathrm{net}\,1-2}/A_1$

$$q_{\mathrm{net}\,1-2} = \frac{1}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)} \sigma(T_1^4 - T_2^4) \qquad (10.24)$$

Comparing equation (10.24) with equation (10.2), we may identify

$$\mathcal{F}_{1-2} = \frac{1}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)} \qquad (10.25)$$

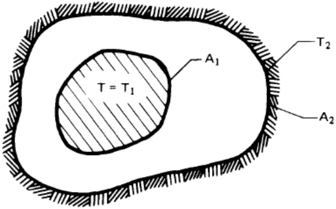
for infinite parallel plates.

If the plates are both black ($\epsilon_1 = \epsilon_2 = 1$)

$$\mathcal{F}_{1-2} = 1 = F_{1-2}$$

Example 10.5 One gray body enclosed by another

Evaluate the $Q_{\mathrm{net}\,1-2}$ and F_{1-2} for 1 gray body enclosed by another.



Solution

The electrical circuit analogy is exactly the same as that shown in Figure 10.13, and F_{1-2} is still unity. Therefore, with equation (10.23)

$$Q_{\text{net }1-2} = A_1 q_{\text{net }1-2} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\epsilon_1}{\epsilon_1 A_1}\right) + \frac{1}{A_1} + \left(\frac{1-\epsilon_2}{\epsilon_2 A_2}\right)} \qquad (10.26)$$

Transfer factor may again be identified by comparison to equation (10.2)

$$Q_{\text{net }1-2} = A_1 \underbrace{\frac{1}{\left(\frac{1}{\epsilon_1}\right) + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}}_{=\mathcal{F}_{1-2}} \sigma(T_1^4 - T_2^4) \qquad (10.27)$$

This calculation assumes that body (1) does not view itself.

Example 10.6 Transfer factor reciprocity

Derive $\mathcal{F}_{2-1}$ for the enclosed bodies shown in the previous figure .

Solution

$$Q_{\text{net}\,1-2} = -Q_{\text{net}\,2-1}$$

$$A_1\mathcal{F}_{1-2}\,\sigma(T_1^4 - T_2^4) = A_2\mathcal{F}_{2-1}\,\sigma(T_2^4 - T_1^4)$$

We obtain the reciprocity relationship for transfer factors.

$$A_1\mathcal{F}_{1-2} = A_2\mathcal{F}_{2-1} \qquad (10.28)$$

with the result of Example 10.5, we have

$$\mathcal{F}_{2-1} = \frac{A_1}{A_2}\mathcal{F}_{1-2} = \frac{1}{\frac{1}{\epsilon_1}\frac{A_2}{A_1} + \left(\frac{1}{\epsilon_2} - 1\right)} \qquad (10.29)$$

Example 10.7 Small gray object in a large environment

Derive $\mathcal{F}_{1-2}$ for a small gray object (1) in a large isothermal environment (2), the result that was given as equation (1.35).

Solution

We may use equation (10.27) with $A_1/A_2 \ll 1$

$$\mathcal{F}_{1-2} = \frac{1}{\frac{1}{\epsilon_1} + \underbrace{\frac{A_1}{A_2}}_{\ll 1}\left(\frac{1}{\epsilon_2} - 1\right)} \cong \epsilon_1 \qquad (10.30)$$

Note that the same result is obtained for any value of A_1/A_2 if the enclosure is black ($\epsilon_2 = 1$). A large enclosure does not reflect much radiation back to the small object, and therefore becomes like a perfect absorber of the small object's radiation - a black body.

Radiation shields

A radiation shield is a surface, usually of high reflectance, that is placed between a high-temperature source and its cooler environment.

Consider a gray body (1) surrounded by another gray body (2) with a thin sheet of reflective material placed between the 2 bodies as a radiation shield. We may put the various radiation resistances in series

$$Q_{\mathrm{net}\,1-2} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1} + \frac{1-\epsilon_2}{\epsilon_2 A_2}\right) + \underbrace{2\left(\frac{1-\epsilon_s}{\epsilon_s A_s}\right) + \frac{1}{A_s}}_{\text{added by shield}}} \tag{10.31}$$

assuming $F_{1-s} = F_{s-2} = 1$. Note that the radiation shield reduces $Q_{\mathrm{net}\,1-2}$ more if its emittance is smaller, i.e., if it is highly reflective.

10.4 Heat transfer among gray bodies

Specular surfaces

If the two gray surfaces in Figure 10.14 are *diffuse emitters but are perfectly specular reflectors* then the transfer factor become.

$$\mathcal{F}_{1-2} = \frac{1}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)} \qquad (10.30)$$

identical to equation (10.25) for parallel plates.

Since parallel plates are a special case of the situation in Figure 10.14, equation (10.25) is true for either specular or diffuse reflection.

Example 10.8

A physics experiment uses liquid nitrogen as a coolant. Saturated liquid nitrogen at 80 K flows through 6.35 mm O.D. stainless steel line ($\epsilon_l = 0.2$) inside a vacuum chamber. The chamber walls are at $T_c = 230$ K and are at some distance from the line. Determine the heat gain of the line per unit length. If a second stainless steel tube, 12.7 mm in diameter, is placed around the line to act as radiation shield, to what rate is the heat gain reduced ? Find the temperature of the shield.

Solution

The nitrogen coolant will hold the surface of the line at essentially 80 K, since the thermal resistances of the tube wall and the internal convection or boiling process are small. Without the shield, we can model the line as a small object in a large enclosure, as in Example 10.7 :

$$Q_{\text{gain}} = (\pi D_l)\epsilon_l \sigma (T_c^4 - T_l^4) = 0.0624\, W/m$$

10.4 Heat transfer among gray bodies

Assuming $A_c \gg A_l$

$$Q_{\text{gain}} = \frac{\sigma(T_c^4 - T_l^4)}{\left(\frac{1-\epsilon_l}{\epsilon_l A_l} + \frac{1}{A_l} + \underbrace{\frac{1-\epsilon_c}{\epsilon_c A_c}}_{\text{neglect}}\right) + \underbrace{2\left(\frac{1-\epsilon_s}{\epsilon_s A_s}\right) + \frac{1}{A_s}}_{\text{added by shield}}} = 0.328\, W/m$$

The radiation shield would cut the heat gain by 47 %.

The temperature of the shield, T_s , may be found using the heat loss and considering the heat flow from the chamber to the shield (small object in a large enclosure)

$$Q_{\text{gain}} = (\pi D_s)\epsilon_s\sigma(T_c^4 - T_s^4) \Rightarrow \; T_s = 213\, K$$

The electrical circuit analogy when more than two gray bodies are involved in heat exchange

Consider a three-body transaction, as pictured Figure 10.15.

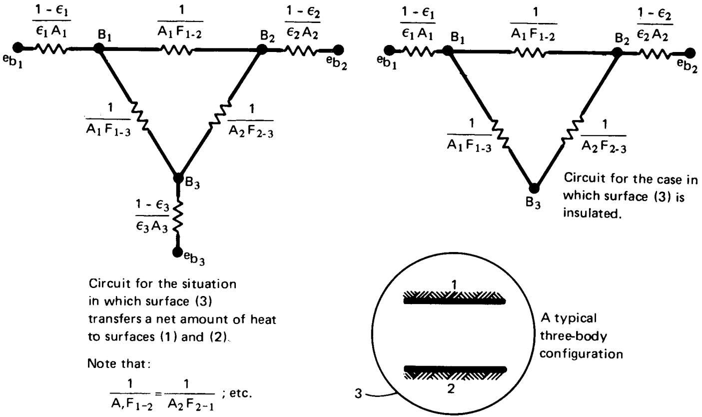


Figure 10.15 Electrical circuit analogy for radiation among 3 gray surfaces.

10.4 Heat transfer among gray bodies

The basic approach is to apply energy conservation at each radiosity node in the circuit, setting the net heat transfer from any one of the surfaces.

$$Q_{\mathrm{net,\,i}} = \frac{e_{b,\,i} - B_i}{\frac{1-\epsilon_i}{\epsilon_i A_i}} \qquad (10.33a)$$

equal to the sum of the net radiation to each of the other surfaces

$$Q_{\mathrm{net,\,i}} = \sum_j \left(\frac{B_i - B_j}{1/A_i F_{i-j}} \right) \qquad (10.33b)$$

For the three body situation shown in Fig. 10.15

$$Q_{\mathrm{net,\,1}},\ \text{at the node } B_1 \ : \ \frac{e_{b,\,1} - B_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}} = \frac{B_1 - B_2}{1/A_1 F_{1-2}} + \frac{B_1 - B_3}{1/A_1 F_{1-3}} \qquad (10.34a)$$

$$Q_{\mathrm{net,\,2}},\ \text{at the node } B_2 \ : \ \frac{e_{b,\,2} - B_2}{\frac{1-\epsilon_2}{\epsilon_2 A_2}} = \frac{B_2 - B_1}{1/A_1 F_{1-2}} + \frac{B_2 - B_3}{1/A_2 F_{2-3}} \qquad (10.34b)$$

$$Q_{\mathrm{net\,3}},\ \text{at the node } B_3 \ : \ \frac{e_{b,\,3} - B_3}{\frac{1-\epsilon_3}{\epsilon_3 A_3}} = \frac{B_3 - B_1}{1/A_1 F_{1-3}} + \frac{B_3 - B_2}{1/A_2 F_{2-3}} \qquad (10.34c)$$

If the temperatures T_1, T_2, and T_3 are known, equations (10.34) can be solved simultaneously for the three unknowns, B_1, B_2, and B_3. the net heat transfer to or from any body (i) is given by equations (10.33).

An insulated wall

If a wall is adiabatic, $Q_{\mathrm{net}} = 0$ at that wall.

If wall (3) in Figure 10.15 is insulated, then equation (10.33b) is

$$e_{b\,3} = B_3$$

We can eliminate one leg of the circuit. The left-hand side of equation (10.34c) equals zero. (all radiation absorbed by an adiabatic wall is immediately reemitted, refractory surfaces)

$$Q_{\mathrm{net}\,1} = \frac{e_{b,\,1} - e_{b,\,2}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{\frac{1}{1/(A_1 F_{1-3}) + 1/(A_2 F_{2-3})} + \frac{1}{1/(A_1 F_{1-2})}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}} \qquad (10.35)$$

A specified wall heat flux

The heat flux leaving a surface may be known (example : electrically powered radiant heater). So the lefthand side of one of equation (10.34) can be replaced with the surface's known Q_{net}, via equation (10.33b).

If surface (1) were adiabatic and had a specified heat flux, then equation (10.35) could be solved for $e_{b,1}$ and the unknown temperature T_1.

Example 10.9

Two very long strips 1 m wide and 2.40 m apart face each other, as shown in Figure below.

(a) Find $Q_{\mathrm{net}\,1-2}$ (W/m) if the surroundings are black and at 250 K.

(b) Find $Q_{\mathrm{net}\,1-2}$ (W/m) if they are connected by an insulated diffuse reflector between the edges on both sides.

Also evaluate the temperature of the reflector in part (b).

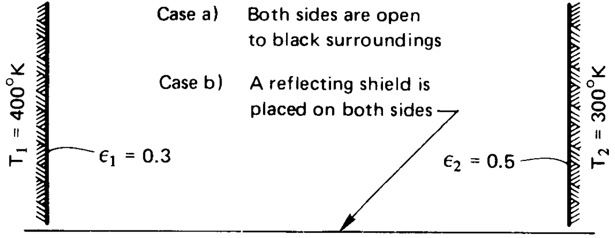


Figure 10.16

Solution

From Table 10.2, case 1, we find $F_{1-2} = 0.2 = F_{2-1}$. In addition, $F_{2-3} = 1 - F_{2-1} = 0.8$, irrespective of whether surface (3) represents the surroundings or the insulated shield. In case (a), the two nodal equations (10.34a) and (10.34b) become

$$\frac{1451 - B_1}{2.333} = \frac{B_1 - B_2}{1/0.2} + \frac{B_1 - B_3}{1/0.8}$$

$$\frac{459.3 - B_2}{1} = \frac{B_2 - B_1}{1/0.2} + \frac{B_2 - B_3}{1/0.8}$$

For black suroundings, equation (10.34c) cannot be used because $\epsilon_3 = 1$ and the surface resistance would be zero.

$$B_3 = \sigma T_3^4 = 221.5\, W/m^2 K$$

can be used directly in the two equations above. Thus

$$B_1 = 612.1\, W/m^2 \qquad B_2 = 379.5\, W/m^2$$

10.4 Heat transfer among gray bodies

The net flow from (1) to (2) is quite small

$$Q_{\mathrm{net}\,1-2} = \frac{B_1 - B_2}{1/A_1 F_{1-2}} = 46.53\,W/m$$

Since each strip also loses heat to the surroundings

$$Q_{\mathrm{net}\,1} \neq Q_{\mathrm{net}\,2} \neq Q_{\mathrm{net}\,1-2}$$

For case (b), with the adiabatic shield in place, equation (10.34c)

$$0 = \frac{B_3 - B_1}{1/0.8} + \frac{B_3 - B_2}{1/0.8}$$

The result is

$$B_1 = 987.7\,W/m^2 \qquad B_2 = 657.4\,W/m^2 \qquad B_3 = 822.6\,W/m^2$$

because surface (3) is adiabatic

$$Q_{\mathrm{net}\,1} = Q_{\mathrm{net}\,1-2}$$

from equation (10.33a)

$$Q_{\mathrm{net}\,1-2} = 198\,W/m$$

Because node (3) is insulated, it is much easier to use equation (10.35)

$$Q_{\mathrm{net}\,1-2} = 198\,W/m$$

The result, of course, is the same. The presence of the reflector increases the net heat flow from (1) to (2).

The temperature of the reflector (3) equation (10.33b) with $Q_{\mathrm{net}\,3} = 0$

$$T_3 = 347\,K$$

Algebraic solution of multisurface enclosure problems

The evaluation of radiant heat transfer among several surfaces proceeds in essentially the same way as for three surfaces. In thise case, the electrical circuit approach is less convenient than a formulation based on matrices.

An enclosure formed by *n* surfaces is shown in Figure 10.17. We will assume that :

- Each surface is **diffuse**, **gray** and **opaque**, so that $\epsilon = \alpha$ and $\rho = 1 - \epsilon$.
- The temperature and net heat flux are uniform over each surface (more precisely, the radiosity must be uniform and the other properties are averages for each surface). Either temperature or flux must be specified on every surface.
- The view factor, F_{i-j}, between any two surfaces i and j is known.
- Conduction and convection within the enclosure can be neglected, and any fluid in the enclosure is transparent and nonradiating.

We will determine the heat fluxes at the surfaces where temperatures are specified, and vice versa.

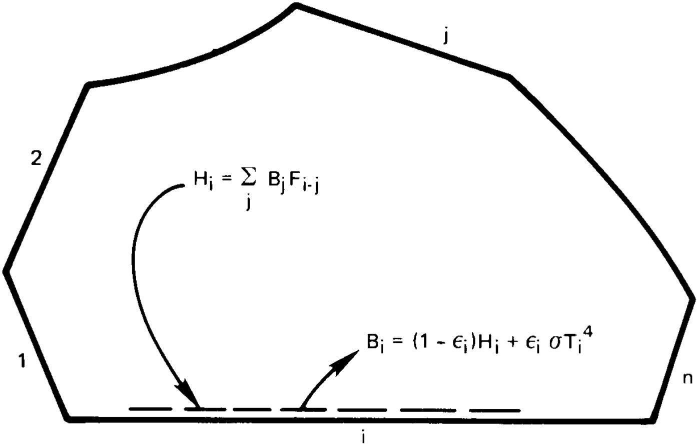


Figure 10.17 An enclosure composed of *n* diffuse, gray surfaces.

10.4 Heat transfer among gray bodies

Equation (10.19) and (10.21) give the rate of heat loss from the ith surface of the enclosure.

$$q_{\mathrm{net},\,i} = B_i - H_i = \frac{\epsilon_i}{1-\epsilon_i}(\sigma T_i^4 - B_i) \qquad (10.36)$$

where

$$B_i = \rho_i H_i + \epsilon_i e_{b,\,i} = (1-\epsilon_i)H_i + \epsilon_i \sigma T_i^4 \qquad (10.37)$$

The irradiating heat transfer incident on surface i, is the sum of energies reaching i from all other surfaces, including itself.

$$A_i H_i = \sum_{j=1}^{n} A_j B_j F_{j-i} = \sum_{j=1}^{n} B_j A_i F_{i-j}$$

Thus

$$H_i = \sum_{j=1}^{n} B_j F_{i-j} \qquad (10.38)$$

from equations (10.37) and (10.38)

$$B_i = (1-\epsilon_i)\sum_{j=1}^{n} B_j F_{i-j} + \epsilon_i \sigma T_i^4 \qquad \text{for } i = 1, \ldots, n \qquad (10.39)$$

10.4 Heat transfer among gray bodies

If all the surface temperatures are specified, the result is a set of n linear equations for the n unknown radiosities.

We introduce the Kronecker delta to rearrange equation (10.39)

$$\sum_{j=1}^{n} \underbrace{\left[\delta_{ij} - (1-\epsilon_i) F_{i-j}\right]}_{C_{ij}} B_j = \epsilon_i \sigma T_i^4 \qquad \text{for } i = 1, ..., n \qquad (10.41)$$

The radiosities are then found by inverting the matrix C_{ij}.

The rate of heat loss from the ith surface can be obtained from equation (10.36).

For those surfaces where heat fluxes are prescribed, we can eliminate the $\epsilon_i \sigma T_i^4$ term in equation (10.39) or (10.41) using equation (10.36).

Finally, equation (10.36) is solved for the unknown temperature of surface. matrix equation for the n unknown values of $Q_{\mathrm{net},\, i}$:

$$\sum_{j=1}^{n} \left[\frac{\delta_{ij}}{\epsilon_i} - \frac{(1-\epsilon_j)}{\epsilon_j A_j} A_i F_{i-j}\right] Q_{\mathrm{net},\, j} = \sum_{j=1}^{n} A_i F_{i-j} (\sigma T_i^4 - \sigma T_j^4) \qquad (10.42)$$

Example 10.10

Two sides of a long triangular duct, as shown in Figure 10.18, are made of stainless steel ($\epsilon = 0.5$) and are maintained at 500.C. The third side is of copper ($\epsilon = 0.15$) and has a uniform temperature of 100°C.

Calculate the rate of heat transfer to the copper base per meter of length of the duct.

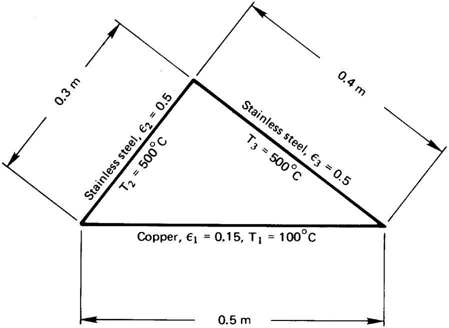


Figure 10.18

Solution

Assume the duct walls to be gray and diffuse and that convection is negligible. The view factors can be calculated from configuration 4 of Table 10.2

$$F_{1-2} = 0.4$$

Similary

$$F_{2-1} = 0.67,\ F_{1-3} = 0.6,\ F_{3-1} = 0.75,\ F_{2-3} = 0.33,\ F_{3-2} = 0.25$$

$$F_{1-1} = F_{2-2} = F_{3-3} = 0$$

From equation (10.39)

$$B_1 = (1-\epsilon_1)(F_{1-1}B_1 + F_{1-2}B_2 + F_{1-3}B_3) + \epsilon_1\sigma T_1^4$$

$$B_2 = (1-\epsilon_2)(F_{2-1}B_1 + F_{2-2}B_2 + F_{2-3}B_3) + \epsilon_2\sigma T_2^4$$

$$B_3 = (1-\epsilon_3)(F_{3-1}B_1 + F_{3-2}B_2 + F_{3-3}B_3) + \epsilon_3\sigma T_3^4$$

We find

$$B_1 = 0.232\sigma T_1^4 + 0.319\sigma T_1^4 + 0.447\sigma T_3^4$$

10.4 Heat transfer among gray bodies

Equation (10.36) gives the rate of heat loss by surface (1)

$$Q_{\mathrm{net},\,1} = A_1 \frac{\epsilon_1}{1-\epsilon_1}(\sigma T_1^4 - B_1) = -1294\, W/m$$

The negative sign indicates that the copper base is gaining heat.

Enclosures with nonisothermal or nongray surfaces

If the primary surfaces in an *enclosure are nonisothermal*, they may be subdivided into a larger number of smaller surfaces (approximately isothermal). Then either equation may be used to calculate the heat exchange among the set of smaller surfaces.

When the *gray surface approximation cannot be applied*, equations (10.41) and (10.42) may be applied on a monochromatic basis, since equation (10.8b) remains valid.