STAR-CALIBRATED LUNAR PHOTOGRAPHY BY METHOD OF SEPARATE PLATES FOR A DETERMINATION OF THE COORDINATES OF LUNAR CONTROL POINTS

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Abstract. A photographic determination of the absolute coordinates of lunar features against the stellar background by the method of separated plates is proposed. The description of the technique and the equation for the plates reduction is given. The determination of the stars positions by new technique is given in the description of the method.

1. Introduction

The first catalogue of the coordinates of lunar landmarks obtained by means of photography was prepared by Franz. With this method he calculated the 150 craters coordinates (Franz, 1901). Somewhat later there appeared Saunder's catalogue of 2885 points (Saunder, 1911). At present there are about two dozen catalogues of the lunar objects position based generally on the lunar photographic measurements made with long-focus refractors of large apertures. The plate constants determination against the lunar points with the known selenodetic coordinates is however, the weak point of these works.

The more precise method is the one based on the plate constants determinations with respect to the stars. The reduction of the plates against the stars provides the lunar photographs with a calibration which is independent of previous selenodetic systems, related to the geometrical lunar centre, since all of them are based on the position of the crater Mösting A, obtained by the lunar limb measurements. With the stars as reference points we can obtain craters coordinates with respect to the centre of lunar mass with the accuracy attainable by our knowledge of the Moon's motion, and of the coordinates of the control stars. In practice we meet however, with some difficulties connected with the necessity of simultaneous photography of the Moon and stars on the same plates. These difficulties are increased if we use long-focus refractors which, as a rule, image only a relatively small field. Attempts to solve the problem of the large-scale lunar photograph calibration have recently been made. Thus in preparing a catalogue of 48 reference craters Arthur (1965) used lunar photographs with star trails to determine the orientation of coordinates system. Moutsoulas (1970) worked out a technique by which one can photograph the Moon and the star field in turn.

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According to this method the faint stars as far as 14^m and then the Moon are photographed with the same hour angle and the same declination on the plate with the 2 hr interval. Methods of lunar and stars photography with the aid of occulting disc (Habibullin, 1958) or by Markowitz camera (cf. Rizvanov, 1971) are used to determine the position of the Moon. Since the observations with these methods are usually made with medium focal length telescopes, we cannot determine the selenodetic coordinates of individual craters with sufficient precision because of the insufficient plate scales.

The need of simultaneous observations of the Moon and the faint stars (up to 9–10 mag.) surrounding it makes it necessary to use an objective imaging a field of sufficient size, preferably not less than $2\overset{\circ}{.}5 \times 2\overset{\circ}{.}5$.

The wide-angle astrograph with the focus of 7–10 m would be the best for this purpose. Practically the effective field of $2\overset{\circ}{.}5 \times 2\overset{\circ}{.}5$ can be achieved with a doublet objective by a corresponding decrease of the aperture, but with some deterioration in resolution.

In Kazan photographic observations of the Moon and the stars for selenodetic purposes have been carried out with horizontal telescope, the objective of which is a doublet with $f = 8$ m and $D = 20$ cm (cf. Habibullin, 1958; Rizvanov, 1971). The practice shows that Markowitz's method (Markowitz, 1954) and the method of instantaneous exposures (Mikhailov and Potter, 1957) do not give satisfactory results with this telescope (Rizvanov, 1974). Because of the long-focus, Markowitz lunar filter should be adequate, but it gives rise to undesirable aberrations. While observing with the occulting disc by the instantaneous exposures method, we photograph the Moon in the midst of the stars with the exposure of $0\overset{s}{.}1$–$0\overset{s}{.}2$ and, in consequence, there is the influence of the irregularity of telescope driving and atmospheric turbulence. The lunar image moves also noticeably with respect to the surrounding stars. The estimation of the accuracy of the observations showed that the influence of these factors amounts 1″ and 0.5″ respectively (standard deviation).

2. The Separate Plates Method

In 1968 one of the authors proposed a new method of the lunar and stars observations that were carried out in 1969 in reference to horizontal telescope (Bystrov, 1972).

The principal idea of this method is to photograph the Moon and the stars simultaneously on different photographic plates: the stars are photographed on fast plates of 300 × 300 mm and the Moon is photographed on slow plates of 90 × 90 mm. Lunar plate is located in front of the star plate 3 mm closer to the objective. The addition of the optical compensator similar to star filter of Markowitz is undesirable because of the deterioration of the stars images. The difference in the positions of the plates do not influence the quality of the images. But a difference in the plates scales must be taken into account, since for $\Delta f = 3$ mm it corresponds to $0\overset{\prime\prime}{.}01$ mm^{-1}.

The types of both plates are selected experimentally so that the best images of the Moon and the stars can be obtained with equal exposures. In our case good results are achieved with plates ORWO ZU-1, ZU-2 for stars and ORWO FU-5 WU-4 for the Moon. A compensation for the diurnal motion of the stars is accomplished by the coelostat hour drive; therefore, during the observation the star plate is fixed in a plateholder. For the compensation of the Moon's motion the lunar plate moves about the star plate in the direction of the lunar image motion with corresponding velocity. The plateholder has a special device providing the lunar plate movement to the nearest to several microns per minute. The plate can be fixed by the position angle at the nearest to 3′.

One of the main points of the new technique is the problem of mutual reference point of star and lunar plates. It is carried out with the system of 8 calibrating lamps rigidly connected with each other and fixed with respect to the star plate. The lamps form point marks (about 50 μ in diameter) on the emulsion of both plates, four on lunar plate, and four on the star plate. During the observation lamps are turned on periodically every 10 s. The duration of flashes is about 0.1 s. Owing to its movement the lunar plate has several point markings (in four corners from each lamp), the number of which is equal to the number of flashes. As a result the star plate has 4

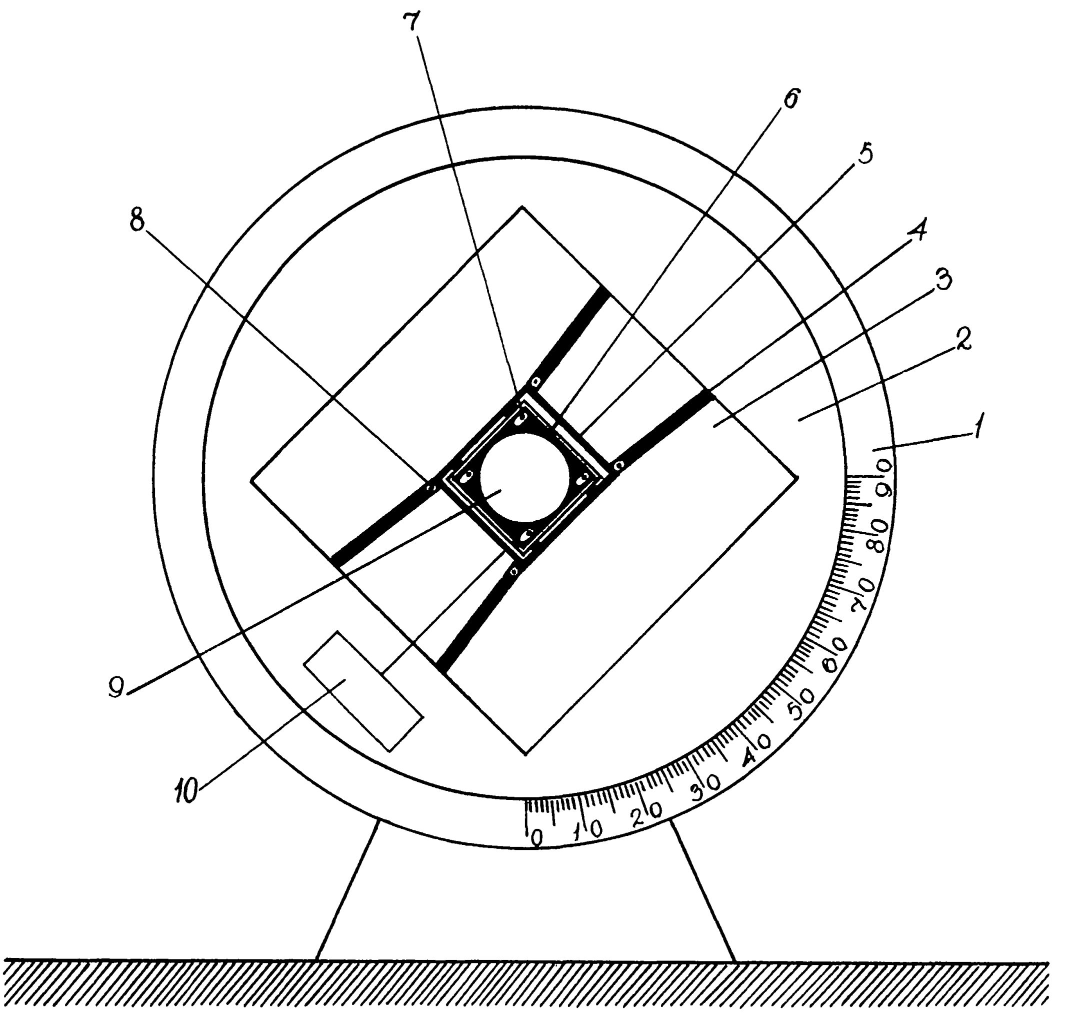

Fig. 1. The general rear view of the plate-holder part of the telescope.

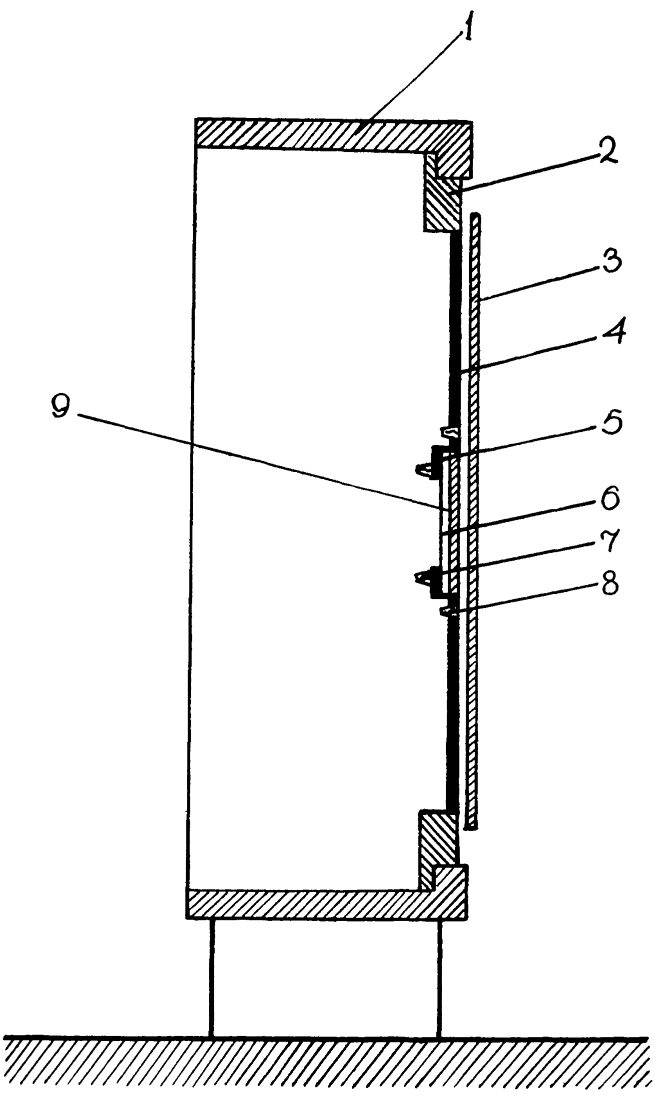

Fig. 2. The lateral view.

point markings and the lunar plate has 4 series of point markings. The distance between markings in the series depends upon the velocity of the lunar plate motion and is 0.16 mm on an average.

Figure 1 shows the general rear view of the telescope plateholder; Figure 2 shows lateral view. The platform (2) on which the star plate (3) is fixed and the lunar camera is attached by the holders (4) is connected to the cylinder (1). The plate-holder is mounted on the cylinder with its bottom fixed on the post. The platform can be rotated with respect to the cylinder by the position angle. The lunar camera consists of the base (5) and movable carriage (6) in which the lunar plate (9) is placed. For impressing the lunar markings there are lamps (7) at the base of the camera, and this camera has an opening for the lunar image. The star marks are made by the lamps (8) fixed at the base of the holders. The carriage is moved by synchronous motor (10). The motor is activated by a vf generator. With digital computer the frequency of generator and the position angle of the platform rotation are calculated in 10 min intervals according to the velocity of the Moon's motion, taking into account the parallax and refraction changes. This is done before carrying out the observations. Topocentric lunar coordinates and their minute changes are calculated simultaneously. To correlate the plates to each other it is necessary to find mutual arrangement of marks on the lunar and star plates in the common coordinate system, which is called a reduced system.

The determination of the marks reduced coordinates can be done in several ways. Let us consider two of them. The bright equatorial star trail with the plate-holder position circle two values of the readings are photographed on the star and lunar plates and the Moon and stars marks are printed. The angle of rotation is about 90°. The star trails permit us to make the mutual reference point of the plates and to define the marks reduced coordinates. But there is the necessity of displacing the intersection point of star trails near the optical centre with the nearest to ± 3 mm.

Number 1 and 2 represent the planes of star and lunar plates respectively (Figure 3). Letters a and b represent intersecting star trails. The angle coefficients and absolute terms of the equations of the straightlines (a, b) are determined from the measured coordinates (x, y) of trails points by the method of least squares i.e.,

$$y_1 = k_{a1}x_1 + c_{a1}\,, \tag{1a}$$

$$y_1 = k_{b1}x_1 + c_{b1}\,, \tag{1b}$$

on the star plate (index 1), and

$$y_2 = k_{a2}x_2 + c_{a2}\,, \tag{2a}$$

$$y_2 = k_{b2}x_2 + c_{b2}\,, \tag{2b}$$

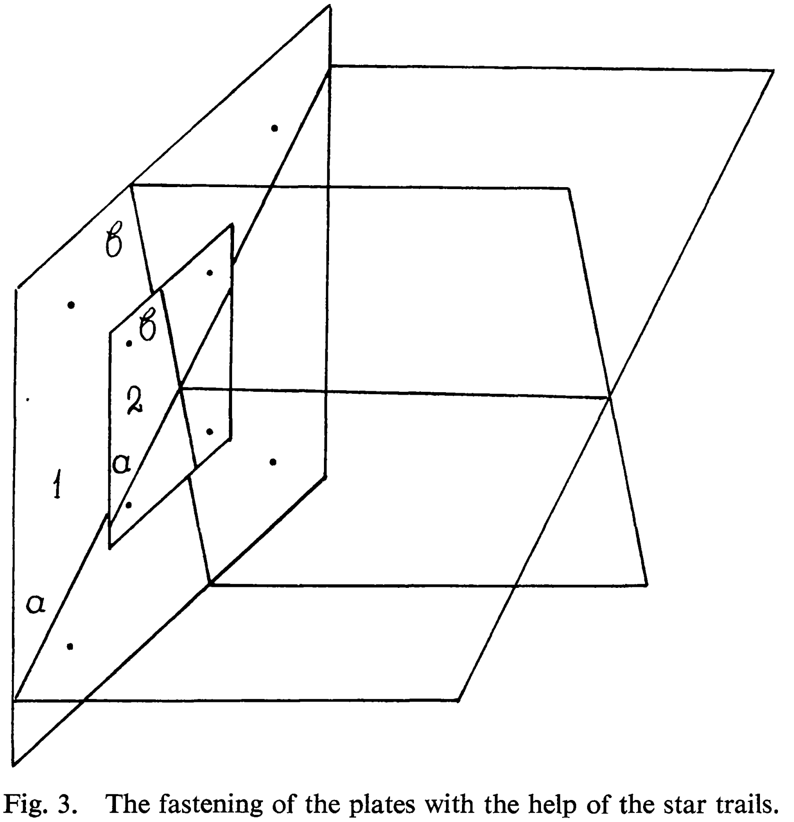

Fig. 3. The fastening of the plates with the help of the star trails.

on the lunar plate (index 2). Points at which the straight lines intersect are found from these equations. Then the equations of the lines passing through these points perpendicular by are calculated, for instance, from

$$y_1 = -\frac{1}{k_{a1}} x_1 + d_{a1} \tag{3}$$

for the star plate, and

$$y_2 = -\frac{1}{k_{a2}} x_2 + d_{a2} \tag{4}$$

for the lunar plate. The Equations (1a), (3) and (2a), (4) are taken for reduced coordinate systems in respect to which the marks locations (X_{1i}, Y_{1i}) on the star plate and (X_{2i}, Y_{2i}) on the lunar plate are calculated by the well-known analytical geometry method $(i=1, 2, 3, 4)$.

In the second method the area rich in stars is photographed simultaneously on the star and lunar plates. Since the coordinates of the optical centre (α_0, δ_0) are the same for both plates, then both standard stars coordinates and standard marks coordinates are referred to one and the same system. Let (x_{1i}, y_{1i}) and (x_s, y_s) $(i=1, 2, 3, 4;$ $s=6, 7, 8,\ldots, n)$ be the measured coordinates of marks and stars on the star plate, (x_{2i}, y_{2i}) and (x_r, y_r), $(r=6, 7,\ldots, m)$ respectively the measured coordinates of marks and stars on the lunar plate.

Standard stars coordinates (X_s, Y_s) and (X_r, Y_r) are calculated from spherical coordinates (α_s, δ_s) of stars on the star plate and (α_r, δ_r) stars on the lunar plate, and common for both plates coordinates of optical centre (α_0, δ_0). Then the plate constants a, b, c, d, e, f are calculated by Turner's method from

$$\begin{aligned} a_1 x_s + b_1 y_s + c_1 &= X_s, \\ d_1 x_s + e_1 y_s + f_1 &= Y_s; \end{aligned} \tag{5}$$

$$\begin{aligned} a_2 x_r + b_2 y_r + c_2 &= X_r, \\ d_2 x_r + e_2 y_r + f_2 &= Y_r. \end{aligned} \tag{6}$$

Then the standard and reduced coordinates of marks are calculated from the equations

$$\begin{aligned} X_{1i} &= a_1 x_{1i} + b_1 y_{1i} + c_1, \\ Y_{1i} &= d_1 x_{1i} + e_1 y_{1i} + f_1, \end{aligned} \tag{7}$$

$$\begin{aligned} X_{2i} &= a_2 x_{2i} + b_2 y_{2i} + c_2, \\ Y_{2i} &= d_2 x_{2i} + e_2 y_{2i} + f_2. \end{aligned} \tag{8}$$

After fixing the position of the lunar and star plate marks in common coordinate system, the transition from the measured coordinate system on the lunar plate to the measured coordinate system on the star plate is made in the following way.

Let (x_*, y_*) and (x_{1i}, y_{1i}) be the coordinates of control stars and marking on the star plate. The lunar objects and marks coordinates measured on lunar plate are respectively $(x_{☾j}, y_{☾j})$ and (x_{2i}, y_{2i}), $(j=1, 2, \ldots, l)$. Since reduced marking coordinates are known, one can relate them with the measured coordinates. For the star plate it can be expressed as

$$\begin{aligned} x_{1i} &= a_{11}X_{1i} + a_{12}Y_{1i} + a_{13}, \\ y_{1i} &= a_{21}X_{1i} + a_{22}Y_{1i} + a_{23}. \end{aligned} \tag{9}$$

The coefficients of these equations of condition determined by the method of least-squares are used for the calculation of lunar mark coordinates in the system of star plates measurements from

$$\begin{aligned} \bar{x}_{1i} &= a_{11}X_{2i} + a_{12}Y_{2i} + a_{13}, \\ \bar{y}_{1i} &= a_{21}X_{2i} + a_{22}Y_{2i} + a_{23}. \end{aligned} \tag{10}$$

Now the connection between systems of star and lunar plates measured coordinates can be established from

$$\begin{aligned} \bar{x}_{1i} &= b_{11}X_{2i} + b_{12}Y_{2i} + b_{13}, \\ \bar{y}_{1i} &= b_{21}X_{2i} + b_{22}Y_{2i} + b_{23}. \end{aligned} \tag{11}$$

The constants derived from it b_{pq} $(p=1, 2; q=1, 2, 3)$ allow us to define the coordinates $(x_{☾j}, y_{☾j})$ in the star plate coordinate system by the method of least-squares by

$$\begin{aligned} \bar{x}_{☾j} &= b_{11}x_{☾j} + b_{12}y_{☾j} + b_{13}, \\ \bar{y}_{☾j} &= b_{21}x_{☾j} + b_{22}y_{☾j} + b_{23}. \end{aligned} \tag{12}$$

Then the star plate constants are derived as usual according to the reference stars coordinates.

Transformation of the lunar plate coordinates to the star plate system can be done in another way. Let us define the dependence

$$\begin{aligned} X_{2i} &= c_{11}x_{2i} + c_{12}y_{2i} + c_{13}, \\ Y_{2i} &= c_{21}x_{2i} + c_{22}y_{2i} + c_{23}. \end{aligned} \tag{13}$$

Then the lunar object coordinates in the system X_2, Y_2 are

$$\begin{aligned} X_{2☾j} &= c_{11}x_{☾j} + c_{12}y_{☾j} + c_{13}, \\ Y_{2☾j} &= c_{21}x_{☾j} + c_{22}y_{☾j} + c_{23}. \end{aligned} \tag{14}$$

Substituting $X_{2☾j}$, $Y_{2☾j}$ in the Equation (9) we obtain the expressions for the determination of $\bar{x}_{☾j}$, $\bar{y}_{☾j}$ in the form

where

$$\begin{aligned} \bar{x}_{☾j} &= d_{11}x_{☾j} + d_{12}y_{☾j} + d_{13}, \\ \bar{y}_{☾j} &= d_{21}x_{☾j} + d_{22}y_{☾j} + d_{23}, \end{aligned} \tag{15}$$

and in which $d_{pq}=a_{p1}c_{1q}+a_{p2}c_{2q}+a_{p3}$, by $q=1, 2$, $a_{p3}=0$.

As to the treatment of the observations, we took the following steps:

(1) First of all, the calculations of the approximate coordinates (ephemerides) of the craters are made in the system of the given measuring instrument. Without these ephemerides it is impossible to identify the craters. The calculations are made according to the specially formulated programme with the aid of the computer.

(2) Then the measurements of the reference stars and craters are made.

(3) The calculations of the observed craters coordinates are made in the following sequence;

(a) The standard reference stars coordinates in the epoch of observations are calculated;

(b) The plate constants are determined by Turner's method;

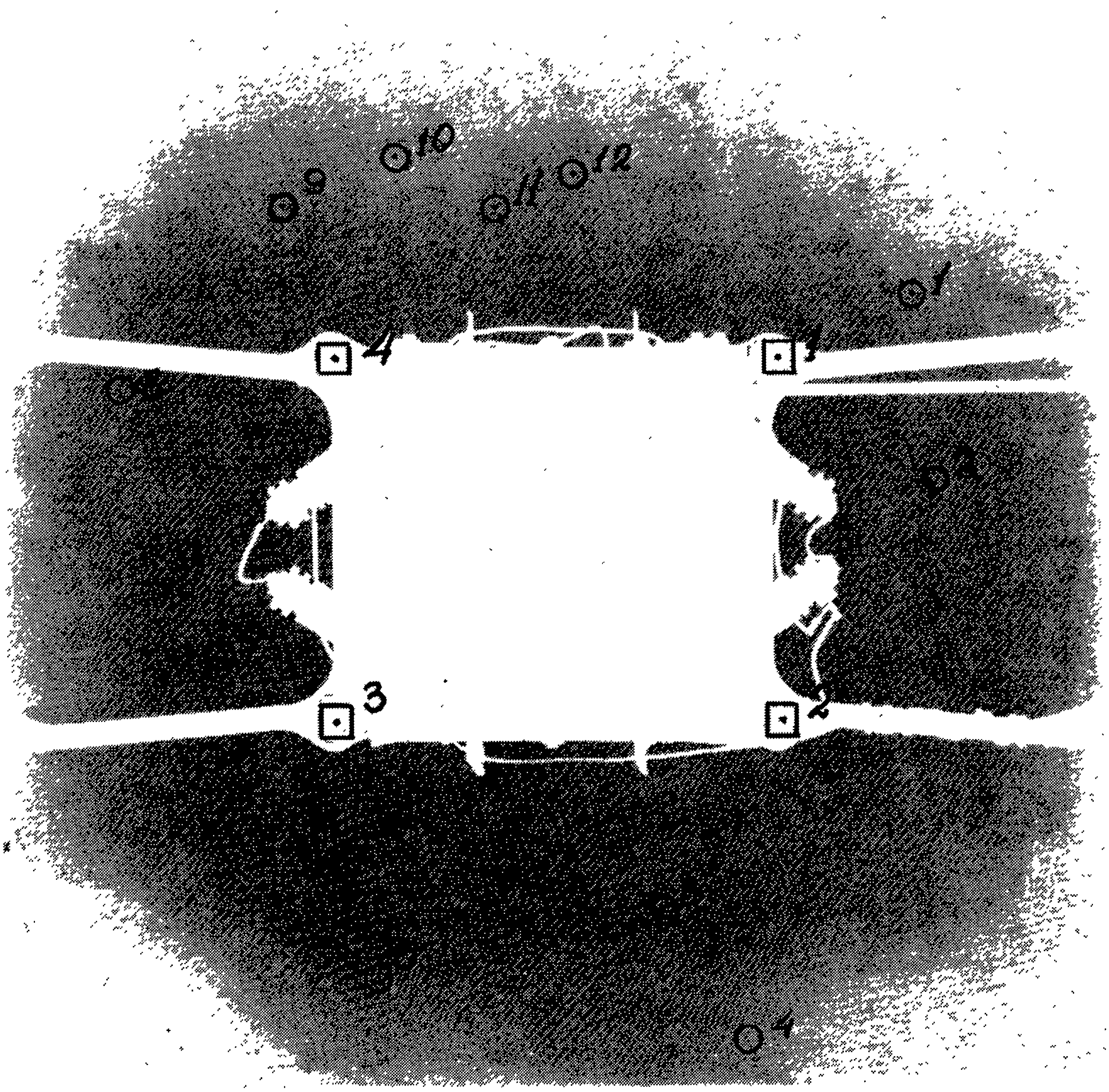

Fig. 4. Star plate.

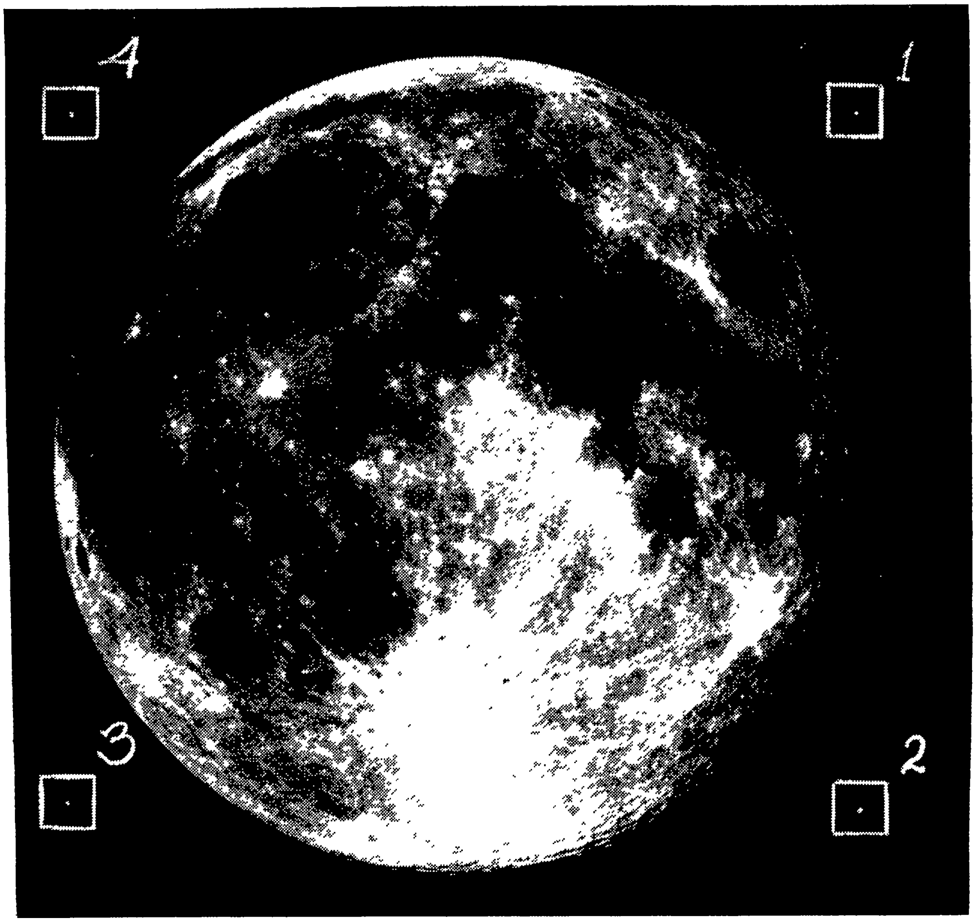

Fig. 5. Lunar plate

(c) The reduction of the measured coordinates of the craters to the system of star plate is made;

(d) Standard crater coordinates are calculated;

(e) The observed topocentric coordinates of the craters are derived.

A determination of the absolute selenodetic coordinates of the craters referred to the lunar mass centre is the final aim of the treatment.

Figures 4 and 5 show the lunar and star plate photographs. The time of observation was November, 1970, 13.86. The duration of exposition was 20 s. Stars were photographed on the plates Kodak Oa-O and the Moon was photographed on the plates Orwo Fu-5. Theoretical topocentric values of equatorial coordinates, libration and the moment of the observation are:

$$\alpha' = 3^{\mathrm{h}}39^{\mathrm{m}}12^{\mathrm{s}}.296, \qquad \delta' = 24^\circ 23' 32''.17,$$
$$l'' = 5^\circ.20, \qquad b'' = -6^\circ.28,$$
$$C'' = -13^\circ.32, \qquad R' = 969''.93.$$

The photographs with the negative star plate image are in the ratio of scaled 1:3. There is a white square in the centre of the plate which marks the lunar camera projection. Markings are located in the corners of the square. The stars images are encircled. The stars are numbered freely. The star numbers in the BD-system are the following: 24°520, 23°483, 23°491, 23°489, 23°499, 23°504, 24°540, 24°537, 24°534, 24°529, 24°528, 24°526. The fig. shows not all stars. There are much more of them on the plates.

Figure 5 shows the actual size image of the lunar plate. Markings can be seen in the corners of the figure. The stars and mark images have been increased on both figures for the contrast by the Indian ink. The lunar and star marks have been placed inside small squares.

3. An Analysis of the Accuracy of the Separate Plates Method

When observing with the proposed method the quality of the photographic lunar image (all other conditions are equal) is defined by the precision with which the lunar plate-holder displacement mechanism can be manufactured. The measurements of the series of lunar plates marks show that the velocity remains constant with the precision to $S=0.5$ mk m^{-1}, where S – is an average quadratic deviation. Since the errors of the lunar movement compensation and the plates reference point do not depend on the focal distance, the precision of the observations in the angular units with new method is better with long-focus instruments.

A transformation of the measured coordinates on the lunar plates (x_2, y_2) to the system of coordinates on the star plates (x_1, y_1) is the crucial point of the Moon and stars observations proposed method.

According to the distances between stars and lunar marks geometrical centres we can judge about the precision of reduced coordinate marks determination. For instance, for two systems of marks they are equal to 0.144 mm and 0.142 mm.

The accuracy of the plate reference point was investigated in the absence of the Moon. It was done by a simultaneous photography of the stars in the Pleiades cluster on the lunar and star plates. The positions of these stars and their proper motions are precisely known (cf. Eichhorn *et al.*, 1970). Some control stars (the craters equivalents) were taken on the lunar plate. Their coordinates were calculated from the reference points distributed on the star plates (the connection between the lunar and star plates). Then they were compared with their catalogue positions (Eichhorn *et al.*, 1970). As the latters are precisely known, then the difference between control stars coordinates (O-C) defines the quality of the lunar plate-to-star plate reference. The coordinates of the star N 1375 were found according to the reference stars, on the lunar plate and compared with their catalogue values for the purpose of control.

Table I gives the results of investigation of 5 stars according to 8 observations. The stars numbers according to the Hertzsprung catalogue (cf. Eichhorn *et al.*, 1970) are given in the first column, photovisual magnitudes of stars are given in the second column, differences in the right ascension and declination are given in the third and the fifth column, mean standard errors of the differences (O-C) in both coordinates are given in the fourth and the sixth columns. The coordinate differences (O-C) and

TABLE I

N	m_{pv}	$\Delta\alpha \cos\delta$	S_α	$\Delta\delta$	S_δ
1284	8.35	−0″.03	0″.14	0″.13	0″.11
1362	8.75	0.14	0.18	−0.04	0.14
1375	6.20	0.07	0.14	0.08	0.14
1397	7.22	0.14	0.13	0.06	0.08
1431	6.77	0.04	0.13	0.00	0.10
1375	6.20	−0.04	0.10	0.06	0.10

their errors defined by the second method are given at the bottom of the table for the star N 1375.

These differences (O-C) and their errors lead to the following two conclusions. First, the horizontal telescope can be used to solve astrometric problems. Second, two plate systems reference point can be made sufficiently precise with the aid of markings.

4. Conclusion

Up to the present a number of methods for a determination of the lunar coordinates have been proposed.

In some methods the scale and the orientation are defined by the lunar points with known selenodetic coordinates (Saunder, 1911; AMS, 1964; etc.). In other methods the photographs orientation is made by imprinted star trails, and the scale is determined by craters (Arthur, 1965). In the method proposed by Moutsoulas (1970) the plate constants are defined with respect to the stars imprinted on the lunar photograph from another part of and sky. The present paper describes a new method of the lunar and surrounding stars observations where the simultaneous photographing on the lunar and star plates is secured.

As far as the authors know the known methods of instantaneous exposures with the occulting discs (cf., Mikhailov and Potter, 1957; Rizvanov, 1971) and Markowitz methods are not used for the determination of the coordinates.

The above-mentioned methods differ from each other, and possess their specific advantages and disadvantages. We think that it is advisable to establish by an international agreement the list of standard craters which should be measured by one of these methods. Such list has, for instance, been proposed by Gurshtain and Slovokhotova (1971).

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