ON THE DEVELOPMENT OF OUR KNOWLEDGE OF THE MOTION OF THE MOON AROUND ITS CENTRE OF MASS

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Abstract. This paper presents historical stages of the development of methods concerning observation and theory of the Moon's rotation around its centre of mass from the beginning of the 17th century up to the present day. The accuracy and usefulness of these methods is estimated and a critical attitude is assumed towards the results concerning the values of constants of the Moon's physical libration.

Since the days when Galileo Galilei, after directing the then constructed telescope towards the Moon, noticed that the spots on the lunar disk changed their position, approaching once the eastern, then the western, limb of the Moon, the problem of the mapping of the Moon became strictly connected with the study of our satellite's rotation around its centre of mass. The famous selenographer of Gdańsk, John Hevelius (1647), noted similar oscillations of the features of the Moon from north to south, and Isaac Newton (1686a) explained these librations rightly as possessing an optical character and ascribed to the Moon a uniform rotation around an axis inclined to the plane of its orbit while its revolution about the Earth is uneven. Soon afterwards, J. D. Cassini specified the explanation of the optical libration by formulating three empirical laws named after him, which were published posthumously by his son, Giacomo Cassini, in 1721. The first of these laws stated that the Moon rotated eastward, on a fixed axis, with a constant angular velocity, and the period of rotation was exactly the same as that of the Moon's sidereal revolution around the Earth. The next two laws stated that the inclination of the Moon's axis of rotation to the ecliptic was constant and that the poles of the Moon's equator, of the ecliptic and of the Moon's orbit, were lying in one great circle in the just given successive order. It is perhaps worth noting that according to Delambre (*Histoire de l'Astronomie moderne* **II**, 733) the three laws mentioned above were already known to Kepler. Cassini also determined the inclination of the Moon's equator to the ecliptic I, as $2\frac{1}{2}^\circ$, which was important from the selenographic point of view.

Tobias Mayer and J. A. Lalande carried out observations, which confirmed Cassini's laws on the understanding that if there existed deviations from these laws, they would have to be insignificant. T. Mayer introduced, moreover, the conception of 'first radius' and 'zero meridian' used to this day both in the theory of the Moon's rotation and its cartography. For the inclination I he obtained $2^\circ 45'$. In the second half of the 18th century, Lalande carried out micrometric measurements of the position of crater Manilius and obtained for the inclination I a value not much differing from that determined in modern times: namely, $I = 1^\circ 43'$.

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Along with these observational researches of the Moon's rotation theoretical studies were also carried out. D'Alembert was the first to discuss the rotation of a rigid body and Euler derived equations for that rotation. But it was Lagrange (1780) to whom we owe a full treatment of the problem of the Moon's rotation – on the assumption that it is a rigid body. Lagrange explained the empirical laws of Cassini on the basis of Newton's law of universal gravitation, for which he was awarded a prize of the Paris Academy. The works of Laplace (1798) and Poisson constitute some supplement to Lagrange's researches. An extraordinarily clear exposition of this theory can be found in the second volume of Tisserand's (1891) celestical mechanics. As late as the year 1880, Hartwig (1880) wrote in his doctoral thesis, which contained a reduction of his Strassburg series of libration observations, that the theory of Lagrange and Laplace was so thorough that "in spite of the rapid development of science, even today [i.e., in 1880] one does not perceive in it drawbacks from the point of view of observational astronomy".

Already Newton (1686b) was speaking of the Moon's physical libration resulting from the attraction by the Earth and by the Sun of the lunar globe deviating from the spherical shape. According to the Lagrange – Laplace theory, the physical libration consists of a forced libration and a free one, the forced libration being a reflection of the periodical terms of the Moon's orbital motion and the amplitudes of its individual terms depending on the inward mass distribution in the Moon. While the free libration, not yet satisfactorily determined, consists of terms possessing a very small amplitude, phase and period depending on the mass distribution within the Moon.

Thus, it was now the first task to obtain from observation the constants characterizing the physical libration. This task was undertaken in 1806 by Bouvard and Arago on the basis of observations of the crater Manilius. Nicollet (1823) supplemented their observations and reduced the whole series of observations thus obtained, which yielded for the inclination I the value $1° 28' 45''$, and for the Moon's mechanical ellipticity $f = B(C-B)$: $A(C-A)$, the value 0.055, where A, B, C, denote the lunar globe's principal moments of inertia. Soon afterwards Kreil (1837) and Stambucchi, who determined the position of the crater Bode relatively to the Moon's limb obtained for $I = 1° 35' 48''$, and for $f = 0.005$. The values of the inclination of the Moon's equator to the ecliptic I in the two enterprises do not in principle differ from each other and they are comparable with present-day values, but the quantities of the Moon's mechanical ellipticity wholly remain out of any discussion and do not even constitute a first approximation. In the 1830's Beer and Mädler expressed the opinion that a crater as large as Manilius is unfit for investigation of the Moon's libration and, moreover, the said crater is lying rather far from the disk's centre.

About 140 years ago, the problem was tackled in a promising manner by the founder of modern astrometry, the astronomer of Königsberg, Bessel (1839). For this purpose he used a heliometer, new at the time and improved by himself, which since that time became the standard instrument serving for the determination of the Moon's physical libration constants. Moreover, Bessel prescribed a method of reduction of these observations elaborated in detail, and his disciples Schlüter and Wichmann (1846, 1847) left two series of observations, the first of which has been elaborated as many as four times. Bessel set up the problem of determination of the physical libration constants in such a way that among them there was not only the Moon's mechanical ellipticity f characterizing the differences of its moments of inertia and the inclination I of the lunar equator to ecliptic, but also the selenographic coordinates of the observed crater, for which Bessel chose the crater Mösting A in conformity with the above-mentioned suggestion of Beer and Mädler that it should be a small crater near the centre of the Moon's disk. At the first Selenodetic Conference, which took place in April 1960 in the High Pyrénées at the foot of Pic du Midi in Bagnères-de-Bigorre (cf. Kopal and Finlay, 1960), crater Mösting A was chosen as a first-order point in the triangulation network on the Moon (Kozieł,1964). Thus, since Bessel's days, the problem of the Moon's motion around its centre of mass has been very closely connected with selenodetic problems and, consequently, also with lunar cartography.

Schlüter did not himself reduce his observations mentioned above from the years 1841–1843. The reduction was undertaken by Franz (1889), who obtained $f = 0.488 \pm 0.028$, and subsequently by Stratton (1908), who found $f = 0.50 \pm 0.03$, by Naumann (Hayn, 1914), who obtained $f = 0.71 \pm 0.08$, and, recently, by the author of the present paper (Kozieł, 1979). Even from this brief account it can be seen that the results of reductions of heliometric observations carried out using Bessel's method leave a lot to be desired. Similarly, Wichmann, in the reduction of his own observations from the years 1844–1846, obtained $f = 0.48 \pm 0.08$ and stated in his paper (Wichmann, 1847) that the difficulties in obtaining accurate results, as far as the libration constants were concerned, mainly result from the irregularities of the Moon's limb. Recently, the author of the present paper (Kozieł, 1979) reduced these observations fundamentally afresh. In the second half of the 19th century heliometric observations of the Moon with Bessel's method were undertaken by that indefatigable observer E. Hartwig (1880). He took Wichmann's above-mentioned remark so much to heart that even during the observation he endeavoured at his telescope to smooth the irregularities of the lunar limb. However, such a procedure is dangerous, because it introduces some subjective factors into the otherwise excellent observations of Hartwig. This astronomer observed the Moon for almost half a century and left us in his scientific heritage three series of observations. The first one, the Strassburg series, from the years 1877–1879, containing 42 observation evenings was reduced by Hartwig himself, who published it in his doctor's thesis (1880); then it was reduced by Hayn (1907) and by Kozieł (1967). The second one, the Dorpat series from the years 1884–1885 consisting of 36 evenings has been elaborated by K. Kozieł (1948–1949) in Cracow, and the third one – the longest series ever carried out by one observer – consisting of 266 evenings over the period 1890–1922 was in part worked out and published by Naumann (1939) of Leipzig. The latter series has also been re-reduced by Schrutka-Rechtenstamm (1955) and its first half by Masłowski (1968) and Kozieł (1967).

At the turn of the 19th century, investigations on the problem of the Moon's motion around its centre of mass were undertaken by the astronomer of Leipzig, Hayn. Apart from theoretical research, of which we shall speak somewhat later, he carried out many micrometrical measurements of several lunar craters (Hayn, 1904). In particular, he referred to the disk's limb the crater Mösting A generally used since Bessel's times. As a novelty, Hayn introduced in the reduction of observations a third coordinate of this crater, namely its distance from the Moon's centre of mass h as an unknown beside the Moon's radius R_0. This made it possible to determine the height above the mean lunar level of crater Mösting A, which is important not only from the selenodetic point of view, but also from that of lunar cartography.

The heliometric method of libration observations, consisting in measurements of the angular distances s_0 (s observatum) of crater Mösting A situated near the lunar disk's centre from the illuminated lunar limb in chosen position angles p, possesses besides its undoubted advantages also some drawbacks. The most serious among them, as was already stated by Wichmann, is the fact that the lunar limb does not constitute any mathematical line, but reveals some irregularities. Thus, one of the fundamental achievements of Hayn in this domain are his charts of these irregularities (Hayn, 1914) giving the elevations and depressions in relation to the mean lunar level. They surpass in precision all charts published later, i.e. the atlas of Weimer (1952), the charts of Nefediev (1958) and those of Watts (1963).

A fair chapter in the domain of investigations on the Moon's rotation was written by the Engelhardt Observatory in Kazan, where heliometric observations of the Moon for the determination of physical libration constants were carried out, with short interruptions, from 1895 to 1963 by the following astronomers: Krasnov (1895–1898), Michajlowski (1898–1905), Banachiewicz (1910–1915), Yakovkin (1916–1931), Belkovič (1932–1942) and Nefediev (1936–1963). Over this time interval they collected the impressive number of over 1,100 published observation evenings. All this vast observational meterial has been elaborated in Kazan by Yakovkin (1928, 1939, 1945), Belkovič (1936, 1949) and Nefediev (1951, 1955, 1970), and the series of Banachiewicz, moreover, by Mietelski (1968) and Kozieł (1967) in Cracow.

An attempt at tackling the problem of the determination of the constants of the Moon's physical libration by means of photography – undertaken in Paris by Puiseux (1925), and who from among 6,000 photographs of the Moon taken over the period of 15 years chose 40 of the best ones – proved a failure because the reduction of these observations contained grave computation errors. Only Weimer (1949) contrived – after unsuccessful attempts by Chandon (1941) – to obtain, on the basis of Puiseux's photographic observations, results comparable with the results of heliometric observations. Recently, the constants of the Moon's physical libration were also successfully obtained by photographic means by Š. T. Habibullin (1958) of Kazan. However, in the bulk of libration investigations the photographic method has not played an important role.

In recent years, with the growing importance of selenodetic problems, several new observational methods for the determination of libration constants have been developed which endeavour to do without observations of the Moon's limb. Let us mention here Yakovkin's method of measuring position angles of the directions from crater Mösting A to other craters lying near the lunar limb, cited by K. Kozieł (1962), or referring crater Mösting A immediately to the stars, as suggested by Šakirov (1963), or else the method of measuring the shadows of lunar mountains proposed by Witkowski and mentioned in the description of his scientific achievements (Kozieł 1968). Unfortunately, it should be stressed that so far none of these methods has given results even approximately comparable with the accuracy of those obtained by the aid of Bessel's heliometric method.

As well as the accumulation of the libration observation material, the methods of reduction of these observations were developing. Since Bessel's days, the heliometric libration observations were traditionally adjusted in two stages; first there were determined the auxiliary unknowns of the problem, i.e. the corrections to the plane rectangular coordinates of crater Mösting A on the disk, and only the second adjustment yielded the proper unknowns of the problem, i.e. the corrections to the spacial coordinates of crater Mösting A and the corrections to the libration constants I and f. Such an approach to the problem afforded from the very beginning of its application serious difficulties in the selection of weights of the right-hand sides of observation equations in the second adjustment. In consequence of Banachiewicz's (1950) remark that the procedure mentioned above, leading in its second stage to an adjustment of dependent equations, is not strictly correct from the point of view of the least-squares method and, consequently, no choice of weights can secure here any correct solution, Kozieł (1949a) gave a method of adjustment of libration observations of the Moon enabling it to be done at one stroke, and derived new formulae for the necessary differential coefficients (Kozieł, 1949b). Recently, along with a joint and uniform adjustment of four libration series, namely the Strassburg and Dorpat series of Hartwig, the first half of the Bamberg series of Hartwig and the Kazan series of Banachiewicz from the years 1879–1915, the present writer (Kozieł 1956, 1967) published a new method of adjustment of heliometric observations. This method is simple from the mathematical point of view and easy for programming an electronic computer, with observation equations containing on their left-hand side directly the principal unknowns of the problem, i.e. the corrections to the libration constants, and on their right-hand side the independent quantities $s_0 - s_c$ (s observatum $- s$ calculatum), which enables the adjustment to be carried out in accordance with the principles of the least-squares method and, in practice, it gives much smaller mean errors of the unknowns than it was possible by the help of the existing methods. Moreover, the form of the observation equations given by the author enables use to be made even of the so-called 'quite incomplete observation eventings', i.e. such as in the extreme case of only a single referring of the crater Mösting A to the lunar disk's illuminated limb. A further advantage of the method discussed is the possibility of introducing to the right-hand sides of the observation equations corrections to s_0 for the irregularities of the Moon's limb and then carry out a parallel adjustment without changing the left-hand sides of these equations.

At the turn of the 19th century, a repeated elaboration of the theory of the Moon's physical libration was undertaken by Hayn, because, as he writes in his paper *Selenographische Koordinaten I* (1902), "it is not customary in astronomy to augment the uncertainties of observations by the inaccuracy of computations". He was of the opinion that the developments of physical libration obtained till then by a method of a rather geometrical character, more appealing to imagination, do not suffice here, but he endeavoured to attain the accuracy of $0\overset{''}{.}01$ geocentrically by analytical-numerical means. On this occasion he stated that the developments of the Moon's physical libration used till then do not contain some terms which, on account of their periods, in spite of a small amplitude can play a role in the integration of the equations of rotation. Unfortunately, Hayn's discussion of the problem in question did not avoid some errors. A few years later there appeared a paper by A. Jönsson (1917) who was the first to apply rectangular coordinates to the investigation of the Moon's rotation. His paper confirms, in the main, Hayn's results, but as concerns the terms formerly neglected, it contains results contradictory to those of Hayn. Thus it is no wonder that, three years later, Hayn (1920) published a revision of his former calculations, in which, however, he could not confirm his results from the year 1902 and at the same time he denied also the results obtained in this respect by Jönsson. This state of affairs can be best illustrated by considering theoretical values of the coefficient at the term sin 2ω – where ω denotes the distance of the Moon's perigee to the ascending node of its orbit – in the development of the physical libration in longitude for the mechanical ellipticity of the Moon $f = 0.5$ (Kozieł, 1948, p. 81):

Hayn (in the year 1902)	$-20\overset{''}{.}2$
Jönsson (in the year 1917)	$+\ 6.9$
Hayn (in the year 1920)	$-\ 6.2.$

In this state of affairs the present writer (Kozieł, 1948) rediscussed the theory of the Moon's physical libration right from its basis and, in particular, he devoted his attention to the third equation of Euler enabling the determination of the physical libration in longitude, in the development of which there appeared the discrepancies given above. The following value was obtained for the coefficient in question:

Kozieł (in the year 1948)	$-7\overset{''}{.}5.$

In any case, the sign of this coefficient, the same as with Hayn (1920) cancels the result obtained in this respect by Jönsson. Kozieł also confirmed Hayn's results as concerns the remaining coefficients in the development of libration in longitude.

In 1955, H. Jeffreys (1955) published a paper containing an outline of a solution of the problem concerning the theory of the Moon's physical libration in terms of rectangular coordinates. Jeffreys did not notice, as it seems, the above-cited paper by A. Jönsson, written some forty years earlier, who not only did the same, i.e. treated the problem of the Moon's physical libration in rectangular coordinates using Lagrange's equations, but above all, went much further, because he obtained final numerical values of the quantities required.

The problem of physical libration in longitude associated with the integration of Euler's third equation for the rotation of the Moon has lately been the subject of interest to Makower (1962), Masłowski and Mietelski (1963), Habibullin (1966) and Moutsoulas (1967). The developments of physical libration in longitude, inclination and node were recalculated using a computer by D. H. Eckhardt (1965), who confirmed the results obtained in this field by Hayn and Kozieł. Sharing the opinion of investigators in this domain, such as G. Schrutka-Rechtenstamm (1955) and Š. T. Habibullin (1958), we come to the final conclusion that Hayn–Kozieł's theory of the Moon's physical libration satisfies, for the time being, present-day requirements, as concerns both observational techniques and the needs of computation and reduction.

In conclusion, it is worthwhile to reflect on the present state of our knowledge of the Moon's motion around its centre of mass and, in particular, on our knowledge of the constants of the Moon's physical libration characterizing that motion. A review of recent literature in this domain can hardly dispose the reader optimistically.

Thus, for the two fundamental libration constants – i.e., the mechanical ellipticity of the Moon f and for the inclination I of the Moon's axis of rotation to the perpendicular to the ecliptic – values have been obtained in recent years ranging from $f = 0.50$ (Habibullin, 1958; p. 103) and $f = 0.91$ (Nefediev, 1970; p. 88). But, according to theory, f must lie within the limits $0 < f < 1$. Similarly, for I there have been obtained values ranging from $I = 1° \, 30' \, 54''$ (Šakirov, 1963) to $I = 1° \, 33' \, 50''$ (Watts, 1955). When comparing these values of fundamental constants of physical libration with those obtained in the course of the past 130 years, we might come to the discouraging conclusion that the efforts to solve the problem of the Moon's physical libration have ended in a deadlock and have not advanced for the past 130 years.

The present author (Kozieł, 1949c) has shown in his paper dealing with the reduction of the Dorpat heliometric series of Hartwig that a mathematical discussion of the problem of the Moon's physical libration admits of two solutions for f lying on both sides of the critical value $f_0 = 0.662$. The Dorpat series, however, was not extensive enough to settle the question which of the two solutions corresponds to reality. Only a joint and uniform elaboration by Kozieł (1967, 1970) of four heliometric series covering the period from 1877 to 1915 with the application of the new method of adjustment of libration observations mentioned above makes it possible to state that the only value for the mechanical ellipticity of the Moon is

$$f = 0.632 \pm 0.0088, \tag{1}$$

which lies below the critical point f_0. Jeffreys (1971) rediscussed these results of Kozieł and confirmed them.

Recently the present writer's result (1) has found a splendid confirmation on the basis of a quite different method of measurement. As is known, the expeditions of Apollo and the Soviet Lunochod have placed on the Moon at diverse points laser mirrors enabling the distance of these mirrors from observation points on the Earth to be determined with enormous accuracy of the order of less than 1 m. These laser measurements yielded (Bender *et al.*, 1973) for the Moon's mechanical ellipticity the value

$$f = 0.642 \pm 0.005, \tag{2}$$

wholly consistent with Kozieł's value (1) within the limits of their mean errors. It should be added that Eckert (1956) in his investigation on the Moon's orbital motion – thus independently of both heliometric measurements connected with the Moon's rotation around its axis and laser measurements – obtained

$$f = 0.638 \tag{3}$$

in full conformity with the two values (1) and (2).

Thus it is clearly seen that the mechanical ellipticity of the Moon is at present known up to two significant figures

$$f = 0.63$$

and that the f values lying in the region 0.8–0.9 published by some authors (Gorynia, 1965; Kisljuk, 1969; Nefediev, 1970) only create confusion and give the impression that the problem of the determination of f were only in the preliminary stages, which evidently is not the case considering the full agreement of the values (1), (2) and (3).

Finally, as concerns the constants of free libration, their correct determination requires observation series covering sufficiently long time periods, because some periodical terms in which they are present, have periods of order of 75 years. Thus earlier attempts in this direction often based on short observation series could not give any good results. Recently, the author of present review determined the constants of free libration in longitude on the basis of a joint and uniform discussion of 4 heliometric series extending from 1877 to 1915, i.e. over a period of 38 years (Kozieł, 1967, 1970). On the basis of laser observations, certainly much more accurate than the heliometric ones, O. Calame (1976) determined the free libration constants. Having, however, at his disposal merely a 5-year observation period, this must rouse at least some doubts as to his results. While the present writer (Kozieł, 1979) has now at his disposal the first approximation of an adjustment of 8 heliometric series, other than the four series cited above, covering the period from 1841 to 1945 – i.e., 104 years; in this adjustment the author determines simultaneously all the libration constants together: i.e. in particular the force libration constants simultaneously with all the six free libration constants; and in this paper he confirms his results from the year 1967 concerning the constants of free libration in longitude.

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