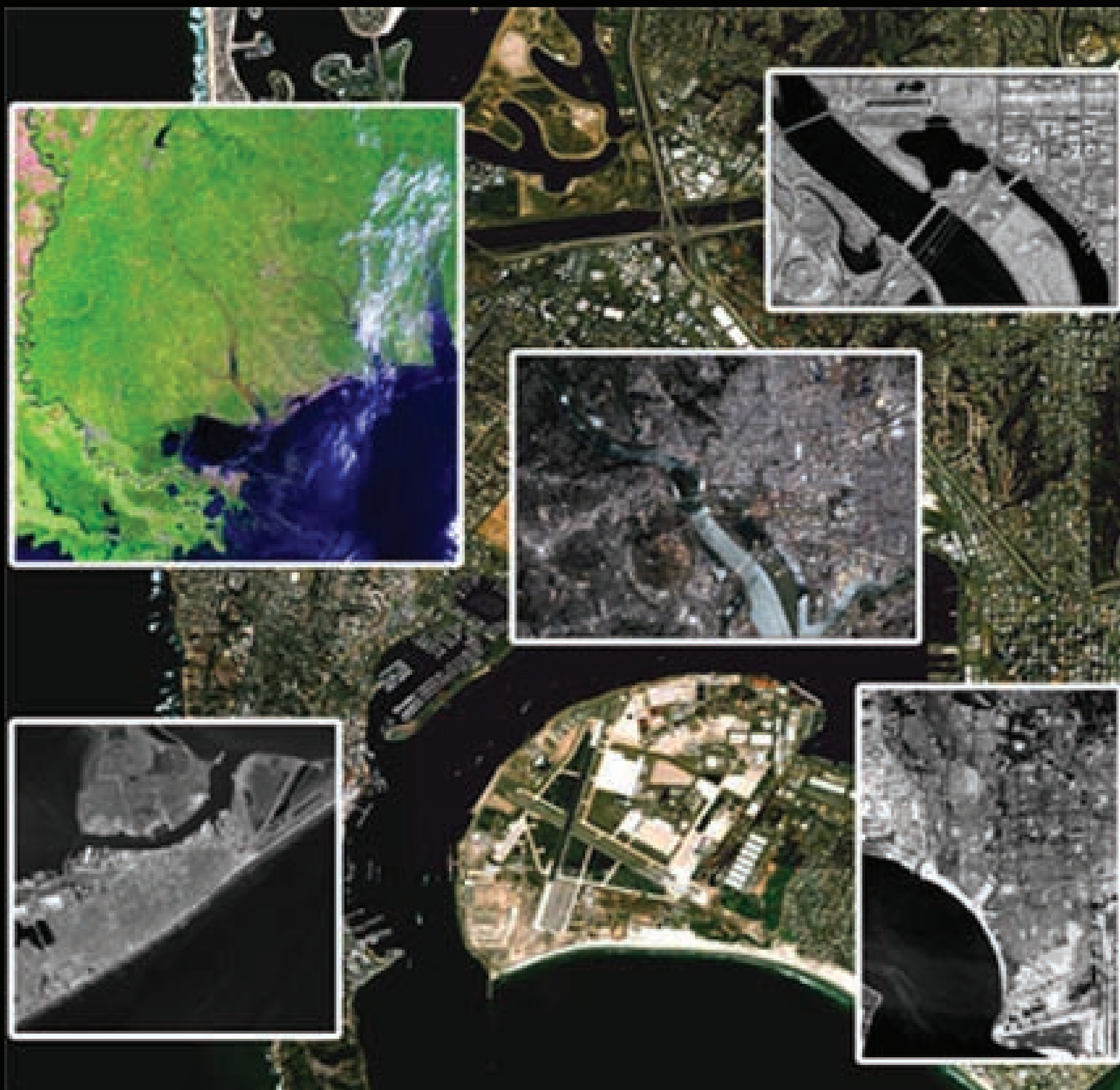
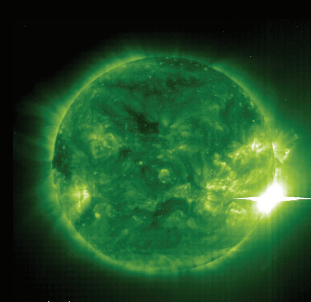
National Aeronautics and Space Administration

# Remote Sensing Math





# Remote Sensing


Remote sending of a dust devil on the surface of mars - NASA/Mars Rover/JPL

A Brief Mathematical Guide

Dr. Sten Odenwald
NASA

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2009-2010 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 9 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

**Acknowledgments:**

We would like to thank Dr. Michael Wilson (University of Maryland) for his careful reading of the draft of this book, his extensive and efficient error-detection algorithm, and the many constructive comments made about problem content and level of difficulty. Dr. Wilson is a research associate for the Goddard Earth Sciences and Technology Center at the University of Maryland, Baltimore County. Mike received his Ph.D. in 2009 in Atmospheric Sciences from the University of Illinois at Urbana-Champaign, with a focus on remote sensing in the polar regions. His current work focuses on cloud radiative modeling and cloud detection in Landsat imagery.

We would also like to thank Ms. Jeannette Allen and Anita Davis (NASA Goddard Eath Sciences Division) for their enthusiastic support of this resource and many helpful comments and suggestions.

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

To suggest math problem or science topic ideas, contact the Author, Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

**<u>Front and back cover credits</u>:** Front: Collage (USGS); Earth globe (MODIS); Messier 82 (Spitzer/Hubble); Sun(SOHO/EIT) Back: Oman (Landsat); Mykolaiv Ukraine(Landsat) Interior: Mars Dust Devil (NASA/JPL)

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# Alignment with Standards

The following benchmarks were extracted from 'AAAS Project: 2061: Benchmarks'.

**(3-5)** - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1

**(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1

**(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

# Mathematics Topic Matrix

| Topic | Problem Numbers | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
| Inquiry | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Technology, rulers | X | X | X | | | | | | | | | | | | | | | X | | | | | | | | | | | | | |
| Numbers, patterns, percentages | X | | X | X | X | | | X | X | X | X | | X | | X | | | | | | | | | | | | | | | | |
| Averages | | | | | X | | | | | | | | | | | | | | X | X | X | | | | | X | | | | | |
| Time, distance, speed | | X | | | | | | | | | | | | X | | | X | X | | | | | | | | | X | X | | | |
| Areas and volumes | | | | | | | | | | | | X | | | | | | | | | | | | | | | | | | | |
| Scale drawings | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Geometry | X | | | | | | | | | | | | | X | | | | | | | | | | | | X | | | X | X | X |
| Probability, odds | | | | | | | | | | | | | | | | | | | X | X | X | | | | | | | | | | |
| Scientific Notation | | | | | | | | | | | | | | | | X | X | | | | | X | | | X | | | X | | | |
| Unit Conversions | | X | | | | | | | | | | | | | X | X | X | | | | | X | | X | X | | X | X | | | |
| Fractions | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | X | X |
| Graph or Table Analysis | | | | X | X | | | | | | | | | | | | | X | | | | | X | X | | | | | | | |
| Pie Graphs | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Linear Equations | | | | | | | | | | | | | | | | | | | | | | | X | X | | | | | | | |
| Rates & Slopes | | | | X | | X | | | | X | X | | | | | X | X | | | | | | X | X | X | | | | | | |
| Solving for X | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | X |
| Evaluating Fns | | | | | | | | | | | | | | | | | | | | X | X | X | X | X | | | X | | | | |
| Modeling | | | | | | | | | | | | | | | | | | | | | | | | | | | X | | | | |
| Trigonometry | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Logarithms | | | | | | X | | | | | | | | | | | | | | X | X | | | | | | | | | | |
| Calculus | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Arrays or matrices | | | | X | X | | X | X | X | | | | | | | | | | | | | | | | | | | | | | |
| Conics | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

# Mathematics Topic Matrix (cont'd)

| Topic | Problem Numbers | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 |
| Inquiry | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Technology, rulers | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Numbers, patterns, percentages | | | | | | | | | | | X | | | | | | | | | | | | X | X | X | X | X | X | | | |
| Averages | | | | | X | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Time, distance, speed | | | | | X | | | | | X | | | X | | | | | X | | | | | | | | | | | | | |
| Areas and volumes | | X | X | | | | | X | | X | X | | | | | | | | | | X | | X | X | | | | | X | X | X |
| Scale drawings | X | X | X | X | X | X | X | X | X | X | X | | | | | X | X | X | X | X | | | | | | | X | | | | |
| Geometry | X | X | X | X | X | X | X | X | X | X | X | | X | | X | X | X | X | X | X | | | X | X | X | X | X | | | | |
| Probability, odds | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Scientific Notation | | | | | | | | | | | | X | | X | X | | X | X | | | | | | | | | | | X | X | X |
| Unit Conversions | | | | | | | | | | | | | | X | X | | X | X | | | | | | | | | | | X | X | X |
| Fractions | | | | | | | | | | | | | | | | | | X | | | | | | | | | | | | | |
| Graph or Table Analysis | | | | | | | | | | | | | | | X | | | | | | X | | | X | X | X | X | X | | | |
| Pie Graphs | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Linear Equations | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rates & Slopes | | | | | | | | X | | X | | | | | | | | | | | | | | | | | | | | | |
| Solving for X | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Evaluating Fns | | | | | | | | | | | | X | | X | X | | X | | | | | X | | | | | | | | | |
| Modeling | | | | | | | | | | | | | | | | | | | | | X | | | X | X | X | X | X | | | |
| Trigonometry | | | | | | | | | | | | | | | | | | | | X | | | | | | | | | | | |
| Logarithms | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Calculus | | | | | | | | | | | | | | | | | | | | | | X | | | | | | | | | |
| Arrays or Matrices | | | | | | | | | | | | | | | | | | | | | | | | X | X | | | | | | |
| Conics | | | | | | | | | | | | X | | | | | | | | | | | | | | | | | | | |

# Mathematics Topic Matrix (cont'd)

| Topic | Problem Numbers | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 |
| Inquiry | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Technology, rulers | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Numbers, patterns, percentages | | | | | | | | | | | | | | | | | | | | | | | | X | X | X | X | | | | |
| Averages | | | | | | | | | | | | | | | | | | | X | | X | | | | | | | | | | |
| Time, distance, speed | | | | | X | | X | X | X | X | X | X | X | X | X | X | X | | | | | | | | X | | | | X | | |
| Areas and volumes | | | | X | | | | | | | | | | | X | | | X | X | X | | | X | | X | | X | | | | |
| Scale drawings | X | X | X | X | X | X | X | X | X | X | X | X | X | | X | X | X | X | X | | | X | X | | | | | X | X | X | |
| Geometry | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X | | X | X | X | X | |
| Probability, odds | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Scientific Notation | | | | | | X | X | | | | | | | | | | | X | | | | | X | | | | | | | | |
| Unit Conversions | X | X | X | X | X | X | X | X | X | X | X | X | | | | X | X | X | X | X | X | | X | | X | X | X | | | | |
| Fractions | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | X |
| Graph or Table Analysis | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | X | X |
| Pie Graphs | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Linear Equations | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rates & Slopes | | | X | | | | | X | | | | | | | | X | | | | | X | | | | | | | | | | |
| Solving for X | | | | | | | | | | | | | | | | | X | | | | | | | | | | | | | | |
| Evaluating Fns | | | | | | X | | | | | | | | | | | X | | | | | | | | | | | | | | |
| Modeling | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Trigonometry | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Logarithms | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Calculus | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Arrays or matrices | | | | | | | | | | | | | | | | | | | | | | | | X | | | | | | | |
| Conics | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

# Mathematics Topic Matrix (cont'd)

X

| Topic | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 |
|---|---|---|---|---|---|---|---|---|---|---|
| Inquiry | | | | | | | | | | |
| Technology, rulers | | | | | | | | | | |
| Numbers, patterns, percentages | | | | | | | X | X | X | |
| Averages | | | | | | | | | | |
| Time, distance, speed | | | | | X | | | | X | X |
| Areas and volumes | | | X | | | | | | | |
| Scale drawings | X | | | | X | X | X | | | X |
| Geometry | X | | X | X | X | X | X | X | X | X |
| Probability, odds | | | | | | | | | | |
| Scientific Notation | | | | | | | | | | |
| Unit Conversions | | | | | | | | | | |
| Fractions | X | X | | | | | | | | |
| Graph or Table Analysis | X | X | X | X | | X | X | X | X | X |
| Pie Graphs | | | | | | | | | | |
| Linear Equations | | | | | | | | | | |
| Rates & Slopes | | | | | | | | | | |
| Solving for X | | | | | | | | | | |
| Evaluating Fns | | | | | | | | | | |
| Modeling | | | | | X | | | | | |
| Trigonometry | | | | | | | | | | |
| Logarithms | | | | | | | | | | |
| Calculus | | | | | | | | | | |
| Arrays or Matrices | | | | | | | | | | |
| Conics | | | | | | | | | | |

# How to Use this Book



Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. This book covers a single topic **Remote Sensing Math**.

**Remote Sensing Math** is designed to be used as a supplement for teaching mathematical topics; the problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery and also as a supplement in the science classroom, it is a good source as a complete study for remote sensing and mathematical models. Concepts from physics and chemistry, insights from history, mathematical ways of thinking, and ideas about the role of technology in exploring the universe all contribute to a grasp of the character of the cosmos.

*Increasingly sophisticated technology is used to learn about the universe. Visual, radio, and X-ray telescopes collect information from across the entire spectrum of electromagnetic waves; computers handle data and complicated computations to interpret them; space probes send back data and materials from remote parts of the solar system; and accelerators give subatomic particles energies that simulate conditions in the stars and in the early history of the universe before stars formed. 4A/H3 Mathematical models and computer simulations are used in studying evidence from many sources in order to form a scientific account of the universe. 4A/H4 (Benchmarks-Physical Setting-Universe)*

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Remote Sensing Math**. Read the scenario that follows:

Ms. Smith has been teaching math in high school for 10 years and noticed that students did far better when they used their math skills in real world situations. She worked with the science teacher to determine when the skills would be needed in science so that students became aware that the math was very important in their science class to meet success in both subjects. The teachers used Remote Sensing Math in the math and science classrooms to enhance what they were teaching and demonstrate how math is part of science for mastery of math skills.

**Remote Sensing Math** can be used as a classroom challenge activity, assessment tool, and enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

# Teacher Comments

"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than “Consider a particle of mass ‘m’ and speed ‘v’ that…” (Associate Professor of Physics)

"Space Math has more up-to-date applications than are found in any textbook. Students enjoy real-world math problems for the math they have already learned. Doing Space Math problems has encouraged some of my students to take pre-calculus and calculus so they can solve the more advanced problems. I learned about Space Math through an email last year. I was very impressed with the problems. I assigned some of the problems to students in my Physics classes, printing them out to put in their interactive notebooks. I displayed other problems for group discussion, assigned some for homework and used some for group class work. I like the diversity, the color format and having the solutions. I expect to use them even more next year in our new space science class. We will have 50 students in two sections." (Alan. High School Science Teacher)

"It took time for them to make the connection between the math they learned in math class and applying it in the science classroom. Now I use an ELMO to project them. I have used them for class work and/or homework. The math activities were in conjunction with labs and science concepts that were being presented. The math helped "show" the science. Oftentimes students were encouraged to help and teach each other. Students began to see how math and science were connected. I knew the students were making the connections because they would comment about how much math they had to do in science. Their confidence in both classes increased as they were able practice the concepts they learned in math in my science class." (Brenda, Technology Resource Teacher)

# Introduction to Remote Sensing

I might be watching the soccer match between Greece and Argentina on TV during the 2010 World Cup exhibition in South Africa. You might be talking to a friend on your cell phone. Both of these activities are common examples of remote sensing that we encounter every day. You are located far from the source of the particular event, yet through technology, you can sense exactly what is happening. Over the decades, we have created a whole host of technologies that bring us information from far beyond the limits of our immediate surroundings and senses, from remote quasars in the universe, to the innermost structure of matter at the atomic scale. For some needs, this information can be as simple as a conversation or a weather forecast. For scientists, remote sensing can be an extremely complex task involving enormous telescopes, expensive satellites, or robotic activity on a distant planet.

Remote sensing is often patterned upon the operation of the human senses. For example, light enters your eye (imaging system), passes to the retina where it is detected (sensing system). The intensity of the light is noted and this information is passed through the optic nerve to the brain (telemetry), where it is analyzed by the visual cortex (calibrated and interpreted). In this guide, we will examine this end-to-end process and see how simple mathematics can be used to help us better understand this basic scientific process in detail.

## 1.0 Digital Picture Basics

As the saying goes, 'One picture is worth a thousand words'. In remote sensing this is literally true! Most of us are familiar with digital cameras. They are so common that most cell phones have them, to the delight of teenagers around the world!

Each image consists of individual 'pixels' that define the light intensity at a particular spot in the camera's viewfinder. Digital cameras are often ranked in terms of the number of these pixels that are present. The larger the number, the more detail you will see in your picture, and the larger the dimensions of the picture will be. For example, a typical cell phone camera has 2 million pixels (2 megapixels), which means that the picture it takes can have a format that is 640 pixels wide and 480 pixels tall. Professional cameras such as the Nikon d3000 have 10.7 megapixels with picture formats of 3872 x 2592 pixels. This, however, is a long way from ordinary film cameras. Although film-based cameras do not have pixels, the total information that film can store in one 35mm negative defined in terms of 'lines per mm' equals about 35 megapixels for the best color films!

The pixels in a digital image come from the format of the Charged-Coupled Device (CCD) electronics 'chip' that forms the heart of the camera system. Each pixel corresponds to one 'well' on the CCD chip that converts the incoming light photons into individual electrons. These electrons are stored in each well during the timed exposure. When the exposure is completed the electrons are counted electronically, and this number is stored in a mathematical array of numbers; one number for each pixel. The array is stored in compressed 'jpeg' format then when you are ready, it is transferred to your computer to be edited by a software program (Photoshop etc).

All cameras perform a number of internal steps in processing this image; steps that are hidden from view, but that must be carried out in order for your image to look spectacular! In scientific imaging, these steps have to be carried out by the scientist in order to 'clean' the images and remove many different kinds of electronic artifacts that corrupt the image.

**2.0 Photons and Digital Imaging**

Digital photography would not be possible if it were not for a critical piece of technology: The CCD chip. This device relies on the ability of certain elements and compounds to produce electrons when light shines upon them. A single photon of light enters the device, and interacts with electrons bound up in the atomic structure of the element germanium. The interaction gives an electron enough energy that it leaves the atom and enters the conduction region of the pixel well. Once enough of these 'photoelectrons' have accumulated, the electrons are read-out and counted by a separate set of electrical components. Like the human retina, CCD pixels can usually detect individual photons of light, counting them one by one. This leads to some interesting problems when very low light levels, or very faint objects, are being studied.

Usually, there are lots of photons to work with. In fact, it is easier to measure light intensities in different physical units than photons. During a typical sunny day, the amount of light energy falling on the ground is about 500 watts in every square-meter. This irradiance level also corresponds to about $2.0 \times 10^{21}$ photons/sec/meter$^2$, which is a far more cumbersome unit to remember!

When images produced from a small numbers of photons are being studied, scientists have to be mindful of the fact that in each pixel, you may only have a few dozen photons being registered. As you know if you were to survey your friends what they wanted to do today, you will get a wider range of votes from a small group than from a large group. This is what statisticians call sampling error. The same thing happens with photons! If you take two consecutive images of the same dimly-lit scene and compare them, you will discover that corresponding pixels do not have exactly the same numbers of photons detected.

**3.0 Digital Calibration and Photometry**

When we use our cell phone cameras to photograph a scene, we only care about the number of shades of color in each pixel. We do not care about the exact number of photons, or the pixel irradiance, that goes along with the image. When scientists take photographs of some distant object or phenomenon, they almost always want to know exactly how bright a particular pixel was in physical units such as irradiance. This quantitative information can be used to determine the temperature, speed, mass or some other physical quantity related to the object.

To make this connection between the numbers stored in the image array, and real physical units such as watts/meter$^2$ or watts/meter$^2$/steradian/Hertz, you have to calibrate the imaging system. Calibration is the mathematical procedure of relating one set of numbers that you can easily measure to some other measuring scale that is more convenient. For example, mercury expands when heated. By calibrating the height of a thin mercury column against a temperature standard, we can use a thermometer to measure temperature, even though 'millimeters' have nothing to do with degrees Centigrade.

By imaging a set of standard objects with known brightness units, we can calibrate the numbers in each pixel to measure brightness or some other radiometric quantity.

### 4.0 Spatial Resolution

A basic feature of any optical system is how well it can discern small features in the scene being photographed. The resulting digital image is usually described in terms of 'meters per pixel' or even 'light years per pixel' depending on the subject matter.

The angular resolution of an image can be easily determined with a little geometry, by knowing the distance to the object and its physical size. This property of an imaging system is also determined by the size and type of optical system in the camera, and the format of the CCD chip at the focus of the system.

The spatial resolution of an image describes what physical length can just be resolved by the CCD pixels in the optical system. This depends on both the angular resolution of the optical system, and the distance from the CCD to the object being imaged.

Spatial resolution allows a remote sensing system to take images of an object at a resolution high enough for scientists to conduct an investigation of contents, scale, and surface details in the target. The target can be the surface of Earth, the sun, a planet, or features within a distant nebula, star cluster or galaxy. Usually, the desired spatial resolution is known in advance, and the optical system is designed to meet this resolution. Sometimes, major discoveries in science can be made just by having an image clear enough to reveal previously hidden details. These also turn out to be among the most visually interesting images you will find will find in science.

### 5.0 Multiwavelength Imaging

The electromagnetic spectrum consists of radiation at many different wavelengths. By selecting which of these wavelengths to image, scientists can use filters to study various aspects of distant objects and phenomena. This is usually accomplished by taking an image in one filter and comparing it against identical images taken through other filters. By comparing the images, changes in temperature, the locations of various elements or compounds, and even magnetic fields can be measured and mapped.

A simple astronomical 'filter band' study might use three filters spanning the visible band to classify the thousands of stars in a star cluster according to their temperature. A more complex 'multispectral' study involving dozens of individual filters spanning the visible and infrared spectrum might map the surface of Earth or a distant planet to classify surface forms in terms of ice, rock, sand, or plant life among hundreds of other forms that have distinct spectral fingerprints.

### 6.0 Temporal Studies

Although spatial studies focus on mapping techniques to determine where things are in space, temporal studies may use images taken at different times to determine how features change in time.

By comparing two images over a span of time, features that change their pixel location can be used to determine how fast they are moving through space. For example, Solar Dynamics Observatory images of the sun can be used to determine the speed of the ejected matter in flares, and how the different parts of the flare are moving.

By comparing two images in which no features are shifting location, the radiometric quantities can be used to determine how fixed features are changing in time. For example, Landsat or Terra satellite images of the same geographic region can be used to monitor the progress of deforestation or glacial retreat.

**7.0 Remote Sensing from Around the Cosmos**

Remote sensing data can also be placed in 'image' form, even though the information was not created by an actual imaging CCD system. There are literally thousands of different kinds of 'image' data that represent some physical property that has been geometrically mapped into a data format that is image-like. For example, the GRACE satellite measures the strength of Earth's gravity as it passes over a specific geographic longitude and latitude. This gravity data can be mapped onto an Earth globe to show us the unseen gravity of Earth, rendered not by photons on a CCD chip, but by the dips in the orbiting satellite motion.

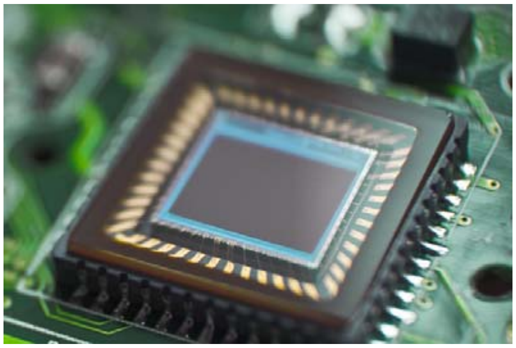
**8.0 Special Studies**

A few examples show how imaged data is created from the raw information gathered by orbiting satellites. The exact steps that must be carried out in translating raw data into a finished image differ greatly from satellite to satellite. The most difficult steps often involve the calibration process. Without calibrated data, scientific imaging is no more useful than the photos you take with your own digital camera.

**9.0 Satellite Design**

When scientists design a $200 million satellite to study a specific phenomenon or object, they have to start from a specific list of requirements that state the minimum acceptable resolution and photometric sensitivity at each wavelength being studied. Sometimes a satellite cannot be designed in exactly the way needed because of the ultimate cost of the satellite, its maximum power or mass. Because all aspects of satellite design have to obey specific Laws of Physics, design elements and economic tradeoffs are tightly linked together. In some ways it resembles the construction of a complicated mobile that must exactly balance when finally assembled.

# Image Formats: Taking a Digital Picture



Most satellites that 'take pictures' of the sun, moon, earth and distant stars use a camera (called an imaging system) based on a Charge-Coupled Device (CCD) chip. These chips, like the one shown to the left, are similar to the ones used inside the common digital camera and consist of millions of individual sensors called 'pixels' in a square format.

CCD cameras are described according to the number of pixels they contain in multiples of one million pixels (1 megapixel). They are also described by their format in rows (M) and columns (N) as containing MxN pixels. The total number of pixels is usually rounded to the nearest power of 2 as is the row and column format.

For example, a CCD with a format of 1024x1024 pixels has a total of 1,048,576 pixels. In the digital camera industry, this is called 1 megapixel. A 4 megapixel CCD has a format of 2048x2048 pixels or 4,194,304 pixels.

**Problem 1** - A digital camera created a 1024x2048 pixel image. What was the format of this image and how many megapixels did it contain?

**Problem 2** - A new digital camera produces square images containing 16 megapixels. What is the likely format for the image, and the actual number of pixels in the image?

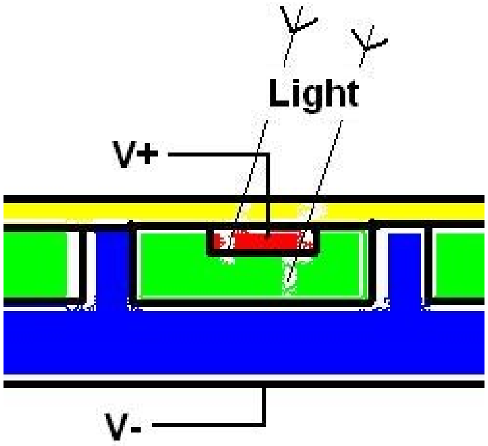
**Problem 3** - The CCD and camera optics are designed so that, from an orbiting satellite, the picture will have a resolution of 1 meter per pixel. What are the dimensions of the total area that can be recorded in square 4 megapixel image in kilometers?

**Problem 1** - A digital camera created a 1024x2048 pixel image. What was the format of this image and how many megapixels did it contain?
Answer: There were **1024 rows and 2048 columns** for a total of 1024x2048 = 2,097,152 pixels or **2 megapixels**.

**Problem 2** - A new digital camera produces square images containing 16 megapixels. What is the likely format for the image, and the actual number of pixels in the image?
Answer; For a square image, one side will have a length of m = n = $(16 \text{ million})^{1/2}$ so m=n=4,000 pixels. Since m and n are specified as powers of 2, the format is **4096 x 4096** .The actual total number of pixels is 4096x4096 = **16,777,216 pixels**.

**Problem 3** - The CCD and camera optics are designed so that, from an orbiting satellite, the picture will have a resolution of 1 meter per pixel. What are the dimensions of the total area that can be recorded in a 4 megapixel image?

Answer: The format of the camera image is m=n=$(4 \text{ million})^{1/2}$ = 2,000 . The nearest power of 2 is $2^{11}$ = 2048, so the actual format is 2048 pixels x 2048 pixels. At 1 meter/pixel, the image covers about **2.0 kilometers x 2.0 kilometers.**




Simplified Story: Light photons strike pixel region (green) liberating electrons. Electrons accumulate in pixel 'well' until exposure is completed, then voltage polarity is reversed so that electrons forced out of pixel wells and conducted to electronics that counts them.

Each pixel in a CCD chip is a complicated electronic device called a photodiode. When a photon of light falls on the surface of the pixel, it interacts with the material in the pixel and causes an electron to be freed from its atomic 'prison'. This electron is then stored within the electronic components of the pixel.

After a period of time, called the exposure or readout time, the accumulated electrons within the pixel are counted electronically, and this information is stored.

After all the electrons have been counted, the CCD chip is now ready to accumulate the next batch of electrons for the next image.

CCD camera pixels can accumulate up to 10 million electrons in a pixel 'well' before no more can be generated. The pixel becomes 'saturated' when more than this number of electrons are generated, so the pixel must be read-out before this limit is reached.

**Problem 1** - Suppose that a camera was photographing a bright daytime scene that produced 5 billion electrons/sec. If the saturation limit is 10 million electrons per pixel, for how many milliseconds could the camera shutter be left open before the CCD was saturated?

**Problem 2** - An astronomer wants to photograph a faint nebula whose light produces 1000 electrons/sec in each CCD pixel. If the saturation limit is 10 million electrons, to the nearest tenth of an hour, what is the longest exposure that the astronomer can use before image saturation occurs?

**Problem 3** - An astronaut wants to photograph the Space Shuttle as it approaches the International Space Station, but wants to also see stars in the sky. The white paint on the Space Shuttle has a brightness equivalent to 3 million electrons/sec and the stars have a brightness equivalent to 30,000 electrons/sec. If the camera takes an exposure of 1/30 second, how many electrons will be accumulated for the pixels covering the Space Shuttle, and how many will be accumulated for the stars?

**Problem 1** - Suppose that the camera was photographing a bright daytime scene that produced 5 billion electrons/sec. If the saturation limit is 10 million electrons per pixel, for how many milliseconds could the camera shutter be left open before the CCD was saturated?

Answer: T = 10 million electrons / (5 billion electrons/s) so
T = 0.002 seconds or
**T = 2 milliseconds.**

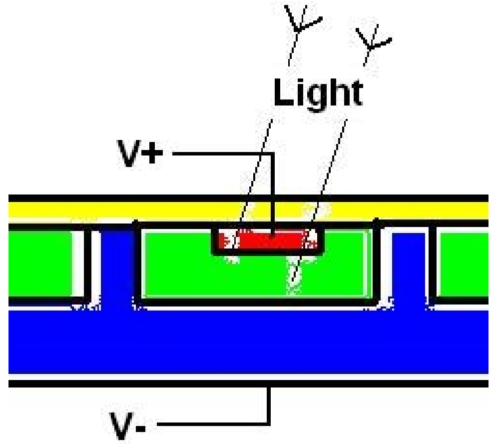
**Problem 2** - An astronomer wants to photograph a faint nebula whose light produces 1000 electrons/sec in each CCD pixel. If the saturation limit is 10 million electrons, to the nearest tenth of an hour, what is the longest exposure that the astronomer can use before image saturation occurs?

Answer: T = 10 million electrons / (1000 electrons/s) so
T = 10,000 seconds or about **2.8 hours**.

**Problem 3** - An astronaut wants to photograph the Space Shuttle as it approaches the International Space Station, but wants to also see stars in the sky. The white paint on the Space Shuttle has a brightness equivalent to 3 million electrons/sec and the stars have a brightness equivalent to 30,000 electrons/sec. If the camera takes an exposure of 1/30 second, how many electrons will be accumulated for the pixels covering the Space Shuttle, and how many will be accumulated for the stars?

**Answer:** Space Shuttle: 3 million/sec x 1/30 sec = **100,000 electrons**. Stars: 30,000 electrons/sec x 1/30 sec = **1,000 electrons**.

# Storing Pixel Data



Pixel well (green) where electrons are accumulated.

After the electrons that have accumulated in a pixel 'well' are counted at the conclusion of each exposure, the numbers counted in each pixel have to be stored and processed by a computer.

The number of electrons counted in a pixel is stored as a binary number based on powers of 2. This number is called a digital or 'data' word (DN), and its size determines how many megabytes of memory are needed to store the information in each image after each exposure.

For example, an 8-bit data word can store numbers with values up to $2^8 = 256$. A 10-bit data word can store numbers up to $2^{10} = 1024$, and so on.

**Problem 1** - An engineer designs a CCD that can count up to 10 million electrons in each pixel. What is the minimum number of bits needed in each data word in order to store the electron number counts?

**Problem 2** - An astronomer wants to take a picture of a distant galaxy for which the brightness ratio between the faintest and the brightest features is 1/100,000. What is the minimum data word size, in bits, that can accommodate this 'dynamic range' of 100,000?

| Bits | Value | Bits | Value | Bits | Value |
|---|---|---|---|---|---|
| 1 | 2 | 11 | 2048 | 21 | 2,097,152 |
| 2 | 4 | 12 | 4096 | 22 | 4,194,304 |
| 3 | 8 | 13 | 8192 | 23 | 8,388,608 |
| 4 | 16 | 14 | 16,384 | 24 | 16,777,216 |
| 5 | 32 | 15 | 32,768 | 25 | 33,554,432 |
| 6 | 64 | 16 | 65,536 | 26 | 67,108,864 |
| 7 | 128 | 17 | 131,072 | 27 | 134,217,728 |
| 8 | 256 | 18 | 262,144 | 28 | 268,435,456 |
| 9 | 512 | 19 | 524,288 | 29 | 536,870,912 |
| 10 | 1024 | 20 | 1,048,576 | 30 | 1,073,741,824 |

**Problem 1** - An engineer designs a CCD that can count up to 10 million electrons in each pixel. What is the minimum number of bits needed in each data word in order to store the electron number counts?

Answer: Using powers of 2, a 23-bit data word has a capacity of $2^{23}$ =8,388,608 .This is smaller than 10 million, so to store N = 10 million you need a **24-bit data word** which can store counts up to N=16,777, 216.

**Problem 2** - An astronomer wants to take a picture of a distant galaxy for which the brightness ratio between the faintest and the brightest features is 1/100,000. What is the minimum data word size, in bits, that can accommodate this 'dynamic range' of 100,000?

Answer: From powers of 2, the nearest ones to the value of 100,000 are $2^{17}$ = 131,072 and $2^{16}$ = 65,536. To include the number 100,000 a **17-bit data word** would be required.

# Images as Arrays of Numbers

$$I = \begin{pmatrix} 10387 & 29876 & 40987 \\ 20345 & 30987 & 50328 \\ 40923 & 50876 & 70987 \end{pmatrix}$$

$$I = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

When a digital image is generated by a CCD chip, it consists of an array of numbers. Each number gives the number of electrons that were counted in each specific pixel.

The information to the left gives the number of electrons counted in a CCD image that consists of 3 rows and 3 columns of pixels. Each pixel is labeled by its row and column number starting in the upper left cell element. For example '$P_{23}$' is name of the pixel located in row 2, column 3 of the CCD array.

**Problem 1** - How many electrons were counted in pixel $P_{32}$?

**Problem 2** - Which pixel counted the smallest number of electrons?

**Problem 3** - In which direction is the brightness of the image increasing?

**Problem 4** - What is the average brightness of the pixels in this image?

**Problem 5** - The image was exposed for 0.001 seconds. To two significant figures, what was the brightness of the brightest portion of scene being photographed in electrons/sec?

**Problem 1** - How many electrons were counted in pixel $P_{32}$?

Answer: **50,876**

**Problem 2** - Which pixel counted the smallest number of electrons?

Answer: $\mathbf{P_{11}}$

**Problem 3** - In which direction is the brightness of the image increasing?

Answer: **From the top of the image down towards pixel $\mathbf{P_{33}}$ in the lower right corner**

**Problem 4** - What is the average brightness of the pixels in this image?

Answer:

$$I = \begin{pmatrix} 10387 & 29876 & 40987 \\ 20345 & 30987 & 50328 \\ 40923 & 50876 & 70987 \end{pmatrix}$$

The sum of all 9 pixels is 345,696 so the average is A = 345,696/9 = **38,411**

**Problem 5** - The image was exposed for 0.001 seconds. To two significant figures, what was the brightness of the brightest portion of scene being photographed in electrons/sec?

**Answer:** Brightness = 70,987 electrons / 0.001 seconds
= 70,987,000 million electrons/sec.
**= 71 million electrons/sec**

Note: It is very important to note that image arrays are simply tables of numbers. Although the values of specific pixels can be represented by a notation such as $a_{ij}$, they do not represent mathematical objects called matrices such as having an inverse, or obeying the matrix rules of multiplication. Arrays can only be added and subtracted on a pixel by pixel basis such as $\mathbf{c_{ij} = a_{ij} + b_{ij}}$

# Image Math - Hot Pixels

$$I = \begin{pmatrix} 23 & 22 & 24 & 23 & 24 & 23 & 22 & 24 \\ 24 & 83 & 24 & 25 & 22 & 25 & 23 & 23 \\ 22 & 22 & 23 & 23 & 24 & 22 & 24 & 23 \\ 22 & 24 & 23 & 24 & 79 & 25 & 23 & 24 \\ 23 & 22 & 22 & 23 & 24 & 23 & 25 & 22 \\ 24 & 23 & 24 & 22 & 25 & 22 & 23 & 23 \\ 23 & 24 & 23 & 65 & 22 & 24 & 24 & 22 \\ 99 & 24 & 23 & 25 & 22 & 22 & 24 & 25 \end{pmatrix}$$

When a CCD imager creates an image, it is unavoidable that defects in the CCD chip are also recorded in the data. The first of these defects are 'hot pixels' in which the counting data has been corrupted by cosmic ray 'hits' or because of manufacturing problems when the electronics for a specific pixel were fabricated.

Hot pixels are easily identified because their count values are very different than those in neighboring pixels, or because the counts change erratically between exposures.

In the problems below, use the pixel naming method $I_{mn}$ where m = row and n = column. For example, in the array above, $I_{45}$ , is the value found at row=4, column=5, which is the value 79.

**Problem 1** - The above array of pixel values shows a portion of an image after a single readout of the CCD array. Identify all of the hot pixels in the field.

**Problem 2** - Based on the average values of surrounding pixels, what may have been the actual counts that should have appeared at each hot pixel?

**Problem 3** - An astronomer wants to clean-up his images by eliminating hot pixels. What approach would you suggest using that preserves the over-all image quality?

**Problem 4** - Approximately what is the average number of electrons in the pixels across this image?

**Problem 1** - The above array shows a portion of an image after a single readout of the CCD array. Identify all of the hot pixels in the field.

Answer: The hot pixels are located at

**$I_{22}$ = 83**

**$I_{45}$ = 79**

**$I_{74}$ = 65 and**

**$I_{81}$ = 99**

**Problem 2** - Based on the average values of surrounding pixels, what may have been the actual counts that should have appeared at each hot pixel?

Answer:

**$I_{22}$ = 23**

**$I_{45}$ = 24**

**$I_{74}$ = 23 and**

**$I_{81}$ = 24**

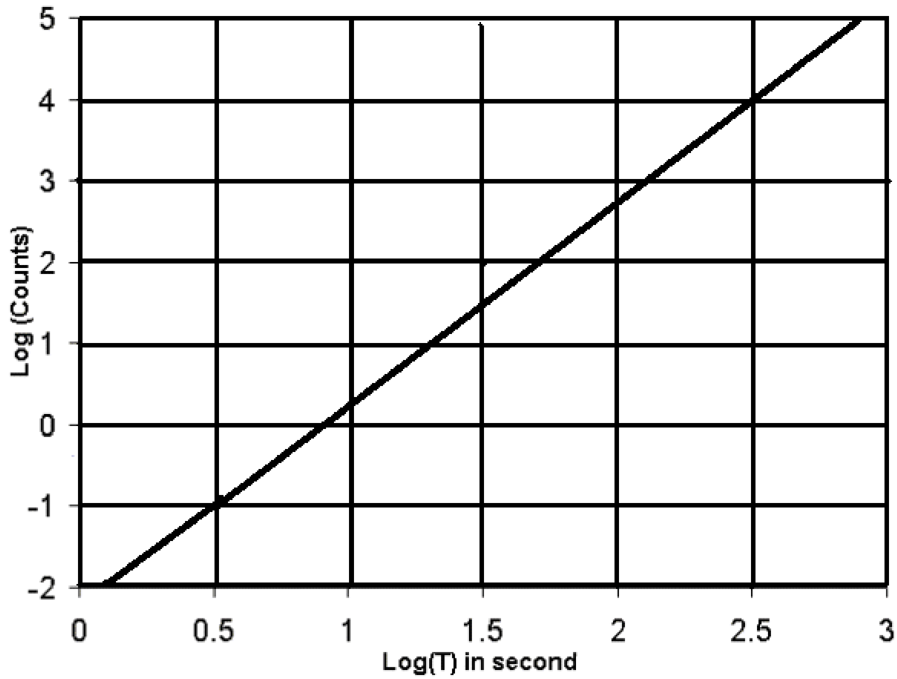
**Problem 3** - An astronomer wants to clean-up his images by eliminating hot pixels. What approach would you suggest using that preserves the over-all image quality?

Answer:
**Replace the hot pixel values by the average values of the neighboring pixels.**

**Problem 4** - Approximately what is the average number of electrons in the pixels across this image?

**Answer:** There are 64 pixels in this image, which is too tedious to add up and average by hand, but from a small sample of pixels in a typical 3x3 patch of pixels the average is **about 23 electrons.**

# Image Math - Dark Current




A CCD chip is an electrical device in which electrons are counted in each pixel in response to the amount of light applied. However, electrons can be generated inside each pixel even though no light is applied and the camera is in a perfectly dark location with its shutter closed. This happens because the camera CCD is warm and the jiggling of atoms can shake loose electrons that get trapped in each pixel over time. This phenomenon is called 'dark current' and the data counted after each exposure has to be corrected for this effect in order to get an accurate count of the actual scene brightness. If the dark current is too high, and the exposure time is too long, a CCD can easily accumulate so many dark current electrons that there is no room left over to count the actual electrons before the pixel becomes saturated!

**Problem 1** - The graph above gives the $Log_{10}$ of the number of dark current electrons generated in three pixels after the amount of time has elapsed, which is also given in $Log_{10}$ units for a selection of pixels in an array. For example, $Log_{10}(T)=1.5$ means 31.6 seconds. The value on the y-axis is $log_{10}(E) = 2$ so E = 100 electrons. A) What is the maximum number of dark current electrons defined by the vertical axis? B) What is the range of exposure times in seconds represented along the horizontal axis?

**Problem 2** - From the graph above, and rounded to the nearest integer, how many dark current electrons are accumulated by Hot Pixel Number 1 after an exposure time of about A) 10 seconds? B) 316 seconds? C) 10 minutes?

**Problem 1** - The graph above gives the $Log_{10}$ of the number of dark current electrons generated after the amount of time has elapsed, which is also given in $Log_{10}$ units for a selection of pixels in an array. A) What is the maximum number of dark current electrons defined by the vertical axis? B) What is the range of exposure times in seconds represented along the horizontal axis?

Answer: A) Log(E) = 5 so **E = 100,000 electrons**
B) Log(T) = 0 to 3 so **T = 1 second to 1,000 seconds**.

**Problem 2** - From the graph, and rounded to the nearest integer, how many dark current electrons are accumulated by the Hot Pixel Number 1 after an exposure time of about A) 10 seconds? B) 316 seconds? C) 10 minutes?

**Answer: A)** Log(10 seconds) = +1.0 and so
Log(E) = +0.2 so
$E = 10^{0.2} = 1.58$ **= 2 electrons**

B) Log(316 seconds) = +2.5 and
Log(E) = +4.0 and **E = 10,000 electrons**

C) T = 10 minutes x (60 sec/minute) = 600 seconds
Log(600) = +2.8 and
Log(E) = +5.0 so E = **100,000 electrons**

# Image Math - Flat Fielding

$$A = \begin{pmatrix} 24 & 28 & 37 \\ 25 & 30 & 21 \\ 26 & 38 & 42 \end{pmatrix} \quad B = \begin{pmatrix} 1.33 & 1.14 & 0.86 \\ 1.28 & 1.07 & 1.52 \\ 1.23 & 0.84 & 0.76 \end{pmatrix}$$

Because each pixel is a separate electronic device, no two pixels respond exactly the same way to the same intensity of light that falls on them. One pixel may count 1234 electrons while its neighbor counts 1267 electrons for the same light intensity. The CCD array can be corrected for this effect by photographing a perfectly uniform scene that has the same overall brightness as the scene you want to study.

Astronomers often use a photograph of the twilight sky before stars 'come out' (called a Sky Flat) or the inside of the dome at the observatory (called a Dome Flat). Both of these scenes have very smooth and constant brightness so by photographing them the pixel counts can be corrected for this effect. This process also removes actual geometric distortions in the optics of the camera so it is often called flat-fielding.

**Problem 1** - Array **A** above is the raw data for a small section of an image. Array **B** is the Sky Flat array that corresponds to the same pixels in Array **A**. To 'flatten' Array **A**, create Array **C** defined so that $c_{ij} = a_{ij} \times b_{ij}$, where I and j are the row and column numbers for the pixel. (Example $c_{12} = a_{12} \times b_{12}$ so for the above arrays, $c_{12} = 28 \times 1.14$ and $c_{12} = 32$.)

**Problem 2** - What can you conclude about the image that was taken by this portion of the CCD camera?

$$A = \begin{pmatrix} 24 & 28 & 37 \\ 25 & 30 & 21 \\ 26 & 38 & 42 \end{pmatrix} \quad B = \begin{pmatrix} 1.33 & 1.14 & 0.86 \\ 1.28 & 1.07 & 1.52 \\ 1.23 & 0.84 & 0.76 \end{pmatrix}$$

**Problem 1** - Array **A** above is the raw data for a small section of an image. Array **B** is the Sky Flat array that corresponds to the same pixels in Array **A**. To 'flatten' Array **A**, create Array **C** defined so that $c_{ij} = a_{ij} \times b_{ij}$, where I and j are the row and column numbers for the pixel.

**Answer:** $$C = \begin{pmatrix} 32 & 32 & 32 \\ 32 & 32 & 32 \\ 32 & 32 & 32 \end{pmatrix}$$

**Problem 2** - What can you conclude about the image that was taken by this portion of the CCD camera?

Answer: The corrected image, Array C, has the same number of electrons in each pixel, so the scene that was imaged has a constant brightness across this portion of the image.

$$R = \begin{pmatrix} 24 & 28 & 37 \\ 25 & 30 & 21 \\ 26 & 38 & 42 \end{pmatrix} \qquad B = \begin{pmatrix} 15 & 15 & 15 \\ 15 & 15 & 15 \\ 15 & 15 & 15 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix} \qquad F = \begin{pmatrix} 1.25 & 0.91 & 0.48 \\ 1.43 & 1.28 & 2.50 \\ 1.00 & 0.50 & 0.40 \end{pmatrix}$$

After an image is collected, called reading-out the CCD, the raw pixel counts have to be corrected for dark current counts and for flat fielding. There may also be a constant number of electrons in each well which will 'bias' the numbers up or downwards. Correcting for array bias, dark current and flat-fielding is done mathematically.

If the raw image read-out produces Array **R**, the dark current counts recorded for an identical exposure time is given by Array **D**, the bias counts are defined by Array **B**, and the flat-fielding correction is given by Array **F**, then the final, corrected image will be **C** = (**R - D - B**) x **F**. For example, for pixel (1,3) we would have $C_{13} = (R_{13} - D_{13} - B_{13})xF_{13}$, so that if $R_{13} = 37$, $B_{13} = 15$, $D_{13} = 1$ and $F_{13} = 0.48$ we would have $C_{13} = (37-15-1)x0.48$ and so $C_{13} = 10$.

**Problem 1** - Based on the array values given above, what are the values for Array **C** after all of the corrections have been applied?

**Problem 2** - The astronomer was hoping to detect a faint star in this image. In what pixels do you think the star is located?

**Problem 3** - How much brighter do you think this star is compared to the background sky?

$$R = \begin{pmatrix} 24 & 28 & 37 \\ 25 & 30 & 21 \\ 26 & 38 & 42 \end{pmatrix} \qquad B = \begin{pmatrix} 15 & 15 & 15 \\ 15 & 15 & 15 \\ 15 & 15 & 15 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix} \qquad F = \begin{pmatrix} 1.25 & 0.91 & 0.48 \\ 1.43 & 1.28 & 2.50 \\ 1.00 & 0.50 & 0.40 \end{pmatrix}$$

**Problem 1** - Based on the array values given above, what are the values for Array **C** after all of the corrections have been applied?

**Answer:** **C** = (**R - D - B**)x**F** **see below**

$$C = \begin{pmatrix} 10 & 10 & 10 \\ 10 & 18 & 10 \\ 10 & 10 & 10 \end{pmatrix}$$

**Problem 2** - The astronomer was hoping to detect a faint star in this image. In what pixels do you think the star is located?

Answer: The corrected image array C shows that the pixel $C_{22}$ is distinct from all the other 'background' pixels so the star is probably located at the position of $\mathbf{C_{22}}$ in the sky.

**Problem 3** - How much brighter do you think this star is compared to the background sky?

**Answer:** The sky background is probably represented by the pixel values of '10' so the star is brighter than the sky by 18 - 10 = 8 units.

# Telemetry: Transmitting data back to Earth

$$A = \begin{pmatrix} 23 & 25 & 28 & 26 \\ 27 & 29 & 26 & 28 \\ 25 & 29 & 21 & 22 \end{pmatrix}$$

A = {23, 25, 28, 26, 27, 29, 26, 28, 25, 29, 21, 22}

Once the data for each pixel have been read-out, this information needs to be transmitted back to Earth from the satellite. To do this, the individual numbers that represent the individual pixel counts have to be transmitted sequentially in a carefully defined stream of data. In the example to the left, a 3x3 image is converted into a string of numbers.

To properly encode and decode the data string, the format of the image must be known (3 x 4) along with the reading sequence {a11, a12, a13, a14, a21, a22, a23, a24, a31, a32, a33, a34}.

**Problem 1** - An image is obtained by a satellite sensor and reduced to the data string {11, 14, 12, 12, 18, 15, 21, 16, 17, 25, 19, 17, 4, 8, 13, 16, 5, 9, 20, 32, 12, 7, 19, 13, 11, 13, 14, 21, 16, 8} If the format is a 5x6 image, what will the array look like when it is recovered from the data string?

**Problem 2** - During transmission, the 13th data word in the string in Problem 1 was corrupted. Which pixel in the image was damaged during transmission?

**Problem 3** - What is the data string that corresponds to the following image array?

$$I = \begin{pmatrix} 23 & 22 & 24 & 23 & 24 \\ 24 & 83 & 24 & 25 & 22 \\ 22 & 22 & 23 & 23 & 24 \\ 22 & 24 & 23 & 24 & 79 \\ 23 & 22 & 22 & 23 & 24 \\ 24 & 23 & 24 & 22 & 25 \\ 23 & 24 & 23 & 65 & 22 \\ 99 & 24 & 23 & 25 & 22 \end{pmatrix}$$

**Problem 4** - In which positions in the data stream sequence are the pixels $I_{22}$, $I_{45}$, $I_{27}$ and $I_{18}$ found, and what are their values?

**Problem 1** - An image is obtained by a satellite sensor and reduced to the data string {11, 14, 12, 12, 18, 15, 21, 16, 17, 25, 19, 17, 4, 8, 13, 16, 5, 9, 20, 32, 12, 7 ,19, 13, 11, 13, 14, 21, 16, 8} If the format is a 6x5 image, what will the array look like when it is recovered from the data string?

Answer:

$$I = \begin{pmatrix} 11 & 14 & 12 & 12 & 18 \\ 15 & 21 & 16 & 17 & 25 \\ 19 & 17 & 4 & 8 & 13 \\ 16 & 5 & 9 & 20 & 32 \\ 12 & 7 & 19 & 13 & 11 \\ 13 & 14 & 21 & 16 & 8 \end{pmatrix}$$

**Problem 2** - During transmission, the 13th data word in the string was corrupted. Which pixel in the image was damaged during transmission?

Answer: **$I_{33} = 4$**

**Problem 3** - What is the data string that corresponds to the following image array?

$$I = \begin{pmatrix} 23 & 22 & 24 & 23 & 24 \\ 24 & 83 & 24 & 25 & 22 \\ 22 & 22 & 23 & 23 & 24 \\ 22 & 24 & 23 & 24 & 79 \\ 23 & 22 & 22 & 23 & 24 \\ 24 & 23 & 24 & 22 & 25 \\ 23 & 24 & 23 & 65 & 22 \\ 99 & 24 & 23 & 25 & 22 \end{pmatrix}$$

Answer: **{23, 22, 24, 23, 24, 24, 83, 24, 25, 22, 22, 22, 23, 23, 24, 22, 24, 23, 24, 79, 23, 22, 22, 23, 24, 24, 23, 24, 22, 25, 23, 24, 23, 65, 22, 99, 24, 23, 25, 22}**

**Problem 4** - In which positions in the data stream sequence are the pixels $I_{22}$, $I_{54}$, $I_{72}$ and $I_{81}$ found, and what are their values?

**Answer: At positions 7, 24, 32 and 36 with values of 83, 23, 24 and 99**.

| Data | Binary | Data | Binary |
|---|---|---|---|
| 1 | 0001 | 10 | 1010 |
| 2 | 0010 | 11 | 1011 |
| 3 | 0011 | 12 | 1100 |
| 4 | 0100 | 13 | 1101 |
| 5 | 0101 | 14 | 1110 |
| 6 | 0110 | 15 | 1111 |
| 7 | 0111 | | |
| 8 | 1000 | | |
| 9 | 1001 | | |

The data for each pixel that describes the number of electrons counted in an image are represented as a string of numbers. To transmit these numbers from the satellite to Earth, they have to be translated into a binary string of data consisting of a pattern of '1' and '0' called binary numbers.

The scheme to the left shows how normal numbers are represented in this way.

**Problem 1** - The pattern above shows how the numbers 1-15 are represented in a 4-bit data word. How would you write the same numbers in an 8-bit data word?

**Problem 2** - What is the largest number you can write in a 16-bit data word?

**Problem 3** - How would you write the number 149 in an 8-bit data word?

**Problem 4** - In designing a satellite telemetry set up, you agree to send the pixels in a 2x3 image as a sequence of 8-bit data words. If the image is given by the array of numbers below, what is the A) sequence of pixel numbers in normal form and B) the sequence of numbers rendered as a 4-bit telemetry stream?

$$A = \begin{pmatrix} 5,\ 7,\ 12 \\ 13,\ 6,\ 2 \end{pmatrix}$$

**Problem 5-** In the 4-bit binary telemetry string for the array in Problem 4, the 10th binary number is changed from a '1' to a '0' by a telemetry error during transmission. Which pixel was affected, and what is its new, incorrect, value?

**Problem 1** - The pattern above shows how the numbers 1-15 are represented in a 4-bit data word. How would you write the same numbers in an 8-bit data word? Answer:

| Data | Binary | Data | Binary |
|---|---|---|---|
| 1 | **00000001** | 10 | **00001010** |
| 2 | **00000010** | 11 | **00001011** |
| 3 | **00000011** | 12 | **00001100** |
| 4 | **00000100** | 13 | **00001101** |
| 5 | **00000101** | 14 | **00001110** |
| 6 | **00000110** | 15 | **00001111** |
| 7 | **00000111** | | |
| 8 | **00001000** | | |
| 9 | **00001001** | | |

**Problem 2** - What is the largest number you can write in a 16-bit data word?
Answer: Each place value in a binary data word is a factor of 2 larger than the previous position to the left. The largest binary value of a 16-bit binary data word is '1111111111111111'. The decimal value is found by adding up all of the factors:

$1 \times 2^0 = 1$ $1 \mathrm{x} 2^1 = 2$ $1 \mathrm{x} 2^2 = 4$ ….. $1 \times 2^{15} = 32{,}768$ so

$1+2+4+8+16+32+64+\ldots+65{,}536 = 2^{16}-1 =$ **65,535**

**Problem 3** - How would you write the number 149 in an 8-bit data word? Answer:
The place values for an 8-bit data word are

| | X | x | x | x | x | x | x | x |
|---|---|---|---|---|---|---|---|---|
| | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| 149 = 128+16+4+1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |

So the binary data word is **'10010101'**

**Problem 4** - In designing a satellite telemetry set up, you agree to send the pixels in a 2x3 image as a sequence of 8-bit data words. If the image is given by the array of numbers below, what is the A) sequence of pixel numbers in normal form and B) the sequence of numbers rendered as a 4-bit telemetry stream?

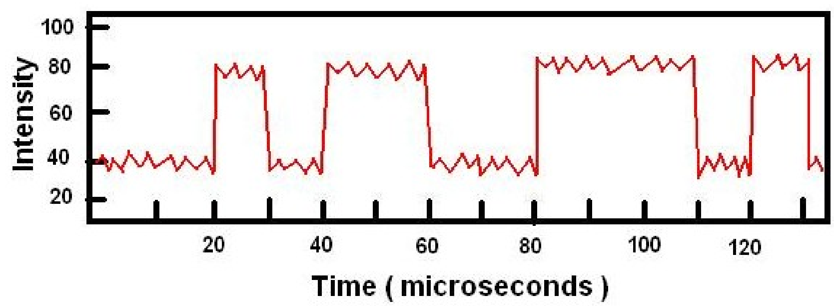
**Answer:** A) $\{a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}\}$=**{5, 7, 12, 13, 6, 2}**
B) From the table, 5 = 0101; 7=0111; 12=1100; 13 = 1101; 6=0110 and 2=0010 so the telemetry string will be **{010101111100110101100010}**

**Problem 5-** In the 4-bit binary telemetry string for the array in Problem 4, the 10th binary number is changed from a '1' to a '0' by a telemetry error during transmission. Which pixel was affected and what is its new, incorrect, value?
Answer: **The numbers are coded in 4-bit binary numbers so it was the third pixel value ($A_{13}$) that was corrupted from '1100' to '1000' which means the correct value of '12' was changed to an incorrect value of '8'.**

# Interplanetary Communication

The giant telescopes of NASA's Deep Space Network can detect the radio signals from Voyager's 40-watt transmitter at a distance of over 12 billion kilometers. But to make the signal detectable at Earth, Voyager has to greatly slow-down the rate at which it downloads its binary information. Even satellites like STEREO operating far closer to Earth have to reduce their 'data rates' to insure that their data is accurately received. Let's see how this works!

**Problem 1** – The STEREO transmitter relays 100,000 bits/sec of information to Earth. (There are 8 bits in one computer 'byte' of information called a 'word'). A) How long does it take, in microseconds, to transmit one bit of information at this rate? B) The DSN recorded the signal shown below from a satellite. What is the pattern of bits (1s and 0s) that the transmitted signal corresponded to if each bit takes 10 microseconds to transmit?




**Problem 2** – As the distance to the satellite doubles, the intensity of the transmitted signal decreases by (1/2)x(1/2) = ¼. This is called the 'Inverse-Square Law'. Draw a graph similar to the one above, but for a satellite located twice as far away as in Problem 2.

**Problem 3** –The table below shows the same weak 5-bit signal transmitted four times. Each bit lasted 10 microseconds. If '1' represents an intensity value greater than or equal to 41 units; A) What is the string of bits for Transmission 1? B) Average the values for the four transmissions and enter the result in 'Average' column. Then in 'Bit Value' column give the bit values. C) How long did it take to transmit each bit for the new string in Column 7? D) How many bits were transmitted per second at this new time interval? E) Can you explain how lowering the bit rate improves the accuracy of receiving the data?

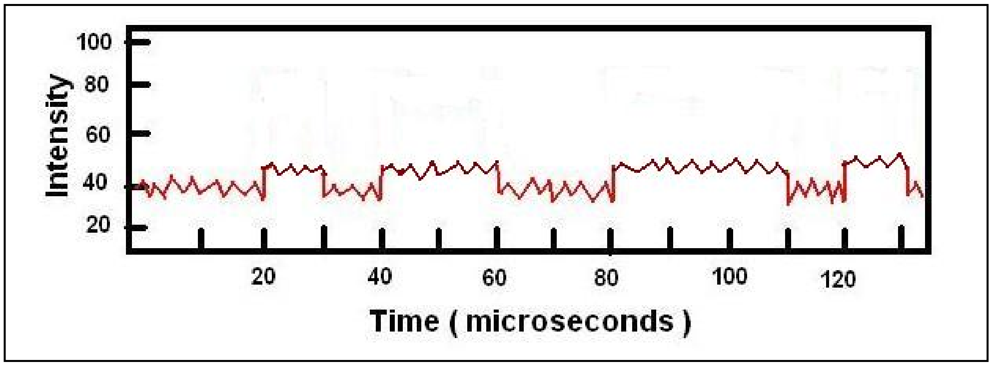
| Time | Transmission 1 | Transmission 2 | Transmission 3 | Transmission 4 | Average | Bit value |
|---|---|---|---|---|---|---|
| Bit 1 | 40 | 43 | 39 | 41 | | |
| Bit 2 | 42 | 39 | 41 | 39 | | |
| Bit 3 | 43 | 42 | 44 | 41 | | |
| Bit 4 | 39 | 39 | 40 | 43 | | |
| Bit 5 | 43 | 39 | 38 | 40 | | |

# Answer Key

**Problem 1** – Answer: It takes 1/100,000 = 0.00001 seconds or 10 microseconds for each bit.

**Problem 2** – Answer: The ‘0’ level is near 20 intensity units. The ‘1’ is near 80 intensity units, so the pattern each 10 microseconds is ‘0 0 1 0 1 1 0 0 1 1 1 0 1 0’

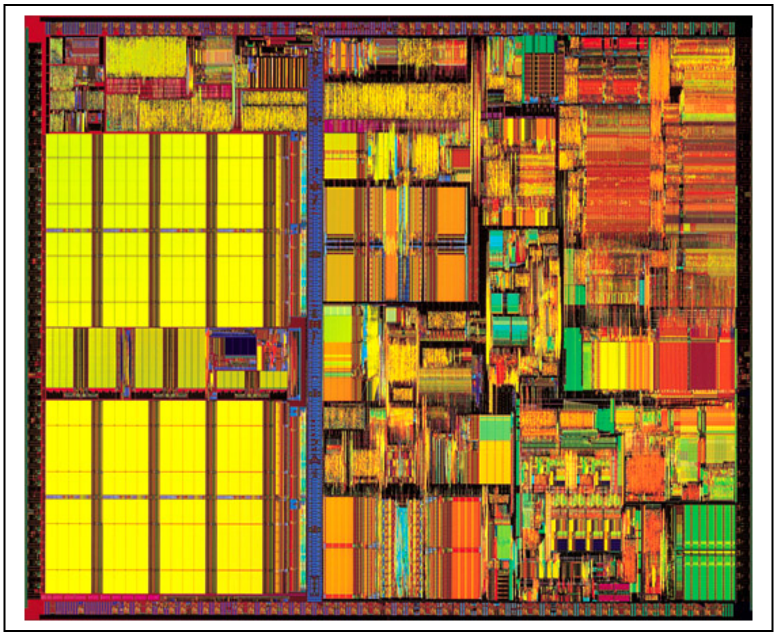
**Problem 3** – Students should note that ‘0’ means no signal detected, and that ‘1’ means that the signal was detected. The difference in intensity between 0 and 1 is 80 – 40 = 40 intensity units. At twice the distance, the new intensity of the ‘1’ signal is only ¼ as great or 40/4 = 10 intensity units above the ‘0’. That means the new signal will look like this:




**Problem 4** – A) Answer: 0,1,1,0,1 B) See table below. C) Answer: Each of the 4 bit transmissions takes 10 microseconds, so to get all 4 transmissions takes 40 microseconds. D) How many bits were transmitted per second at this new time interval? Answer: If each averaged bit took 40 microseconds or 0.000040 seconds, then in one second there were 1/0.00004 = 25,000 bits/sec. E) Lowering the bit rate lets you make multiple measurements on the transmitted message to improve the accuracy of the final message. As satellites move away from Earth, engineers steadily lower the transmission rate to improve signal detection. This also means that it takes longer and longer to receive a message of a fixed size such as a high-resolution picture.

| Time | 1 | 2 | 3 | 4 | Average | Bit value |
|---|---|---|---|---|---|---|
| 10 | 40 | 43 | 39 | 41 | 41 | 1 |
| 20 | 42 | 39 | 41 | 39 | 40 | 0 |
| 30 | 43 | 42 | 44 | 41 | 43 | 1 |
| 40 | 39 | 39 | 40 | 43 | 40 | 0 |
| 50 | 43 | 39 | 38 | 40 | 40 | 0 |

# Computer Memory and Glitches



Computers are used in space, but that means they can get clobbered by radiation and develop 'glitches' in the way they work. Sometimes these glitches cause the satellite to fail, or transmit corrupted data.

This is a photo of the Pentium III microprocessor board, which is about 15 cm wide. The solid, colored rectangular areas are memory locations that store data, the computer operating system, and other critical information.

Suppose that 2/3 of the area of a satellite's processor memory is used for data storage, 1/4 is for the computer's operating system, and the remainder is for program storage.

**Problem 1** - Suppose that the total size of the memory is 1,200 megabytes. How many megabytes are available for program storage?

**Problem 2** - Suppose that for a satellite in space, cosmic rays cause glitches and errors in the computer memory at a rate of 1 glitch per hour for every 1 gigabytes of memory. If the satellite is in operation for 10 hours, how many glitches will this satellite's memory encounter?

**Problem 3** - Given the areas of the different computer memory functions, how many glitches would you expect in the operating system memory?

**Problem 4** - After 10 hours of operation, about how many operating system failures would you expect, and what would be the average time between operating system failures?

**Problem 5** - An engineer decides to re-design the satellite's memory by splitting up the memory for the operating system into 4 separate areas. Why do you think this design might reduce the number of glitches caused by the cosmic rays, or why would this not work?

# Answer Key



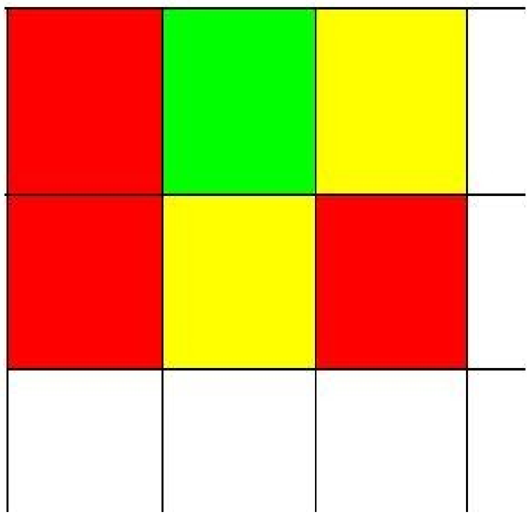
**Problem 1** - Suppose that the total size of the memory is 1,200 megabytes. How many megabytes are available for program storage? Answer: Program storage is 1 - 2/3 - 1/4 = 12/12 - 8/12 - 3/12 = 1/12 of the memory. Since there are 1,200 megabytes of computer memory for all functions, the program memory occupies 1,200 megabytes x 1/12 = **100 megabytes.**

**Problem 2** - Suppose that for a satellite in space, cosmic rays cause glitches and errors in the computer memory at a rate of 1 glitch per hour for every 1 gigabytes of memory. If the satellite is in operation for 10 hours, how many glitches will this satellite's memory encounter? Answer: The computer memory is 1,200 megabytes and since 1,000 megabytes = 1 gigabyte, the memory is 1.2 gigabytes in size. 1 glitch/hour per gigabyte x 1.2 gigabytes = 1.2 glitches/hour. For 10 hours the total number of glitches will be 1.2 x 10 = **12 glitches for the entire memory area.**

**Problem 3** - Given the areas of the different computer memory functions, how many glitches would you expect in the operating system memory? Answer: The operating system occupies 1/4 of the memory, so if the glitches are random, the operating system should experience 12 glitches x 1/4 = **3 glitches.**

**Problem 4** - After 10 hours of operation, about how many operating system failures would you expect, and what would be the average time between operating system failures? Answer: In 10 hours, the average time between operating system glitches would be 10 hours / 3 glitches = **3 1/3 hours between glitches.**

**Problem 5** - An engineer decides to re-design the satellite's memory by splitting up the memory for the operating system into 4 separate areas. Why do you think this design might reduce the number of glitches caused by the cosmic rays, or why would this not work? Answer: **No, because the total area of the operating system memory is exactly the same, so the probability that it will receive a glitch remains the same as before.**



The first few pixels in a large image

Data is sent as a string of '1's and '0's which are then converted into useful numbers by computer programs. A common application is in digital imaging. Each pixel is represented as a 'data word' and the image is recovered by relating the value of the data word to an intensity or a particular color. In the sample image to the left, red is represented by the data word '10110011', green is represented by '11100101' and yellow by the word '00111000', so the first three pixels would be transmitted as the 'three word' string '101100111110010100111000'. But what if one of those 1-s or 0-s was accidentally reversed? You would get a garbled string and an error in the color used in a particular pixel.

Since the beginning of the Computer Era, engineers have anticipated this problem by adding a 'parity bit' to each data word. The bit is '1' if there are an even number of 1's in the word, and' 0' if there is an odd number. In the data word for red '1011001**1**' the last '**1**' to the right is the parity bit.

When data is produced in space, it is protected by parity bits, which alert the scientists that a particular data word may have been corrupted by a cosmic ray accidentally altering one of the data bits in the word. For example, Data Word A '1110001**1**' is valid but Data Word B '1111001**1**' is not. There are five '1's but instead of the parity bit being '0' ('1110001**0**' ), it is '1' which means Data Word B had one extra '1' added somewhere. One way to recover the good data is to simply re-transmit data words several times and fill-in the bad data words with the good words from one of the other transmissions. For example:

| | | | | | |
|---|---|---|---|---|---|
| Corrupted data string: | 1011110**0** | 101101**0** | 1010101**1** | 0011001**1** | 1011101**0** |
| Good data string: | 1011110**0** | 100101**0** | 1010101**1** | 1011001**1** | 1011101**0** |

The second and fourth words have been corrupted, but because the string was re-transmitted twice, we were able to 'flag' the bad word and replace it with a good word with the correct parity bit. Cosmic rays often cause bad data in hundreds of data words in each picture, but because pictures are re-transmitted two or three times, the bad data can be eliminated and a corrected image created.

**Problem:** Below are two data strings that have been corrupted by cosmic ray glitches. Look through the data (a process called parsing) and use the right-most parity bit to identify all the bad data. Create a valid data string that has been 'de-glitched'.

| | | | | | |
|---|---|---|---|---|---|
| String 1: | 10111010 | 11110101 | 10111100 | 11001011 | 00101101 |
| | 01010000 | 01111010 | 10001100 | 00110111 | 00100110 |
| | 01111000 | 11001101 | 10110111 | 11011010 | 11100001 |
| | 10001010 | 10001111 | 01110011 | 10010011 | 11001011 |

| | | | | | |
|---|---|---|---|---|---|
| String 2: | 10111010 | 01110101 | 10111100 | 11011011 | 10101101 |
| | 01011010 | 01111010 | 10001000 | 10110111 | 00100110 |
| | 11011000 | 11001101 | 10110101 | 11011010 | 11110001 |
| | 10001010 | 10011111 | 01110011 | 10010001 | 11001011 |

Answer Key:

**Problem:** Below are two data strings that have been corrupted by cosmic ray glitches. Look through the data (a process called parsing) and use the right-most parity bit to identify all the bad data. Create a valid data string that has been 'de-glitched'.

The highlighted data words are the corrupted ones.

| | | | | | |
|---|---|---|---|---|---|
| String 1: | 10111010 | **11110101** | 10111100 | 11001011 | **00101101** |
| | **01010000** | 01111010 | 10001100 | 00110111 | 00100110 |
| | **01111000** | 11001101 | **10110111** | 11011010 | **11100001** |
| | 10001010 | 10001111 | 01110011 | **10010011** | 11001011 |

| | | | | | |
|---|---|---|---|---|---|
| String 2: | 10111010 | 01110101 | 10111100 | **11011011** | 10101101 |
| | 01011010 | 01111010 | **10001000** | **10110111** | **10100110** |
| | **11011000** | 11001101 | 10110101 | 11011010 | 11110001 |
| | 10001010 | **10011111** | 01110011 | 10010001 | **01001011** |

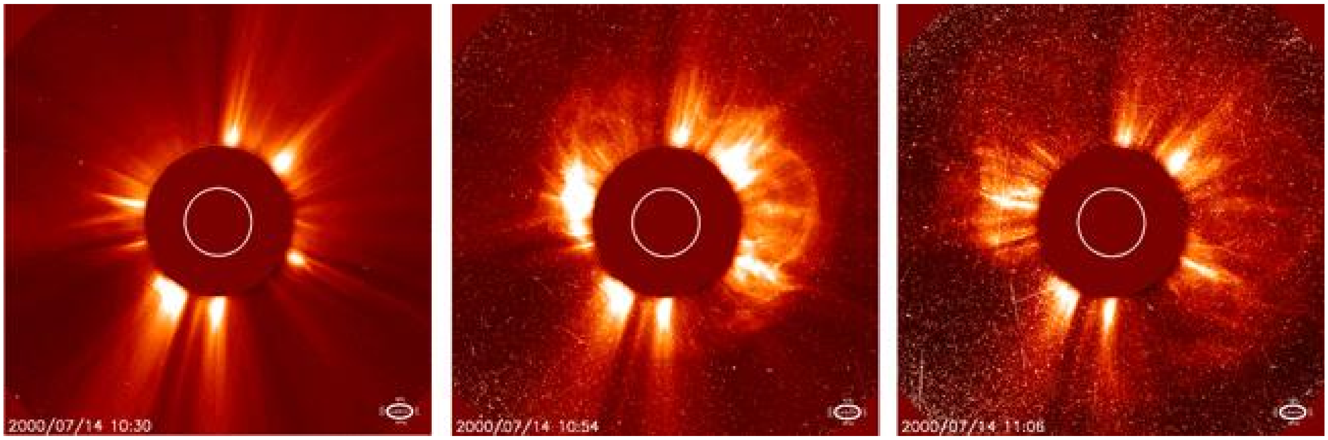
In the first string, 11110101 has a parity bit of '1' but it has an odd number of '1' so its parity should have been '0' if it were a valid word. Looking at the second string, we see that the word that appears at this location in the grid is '01110101' which has the correct parity bit. We can see that a glitch has changed the first '1' in String 2 to a '0' in the incorrect String 1.

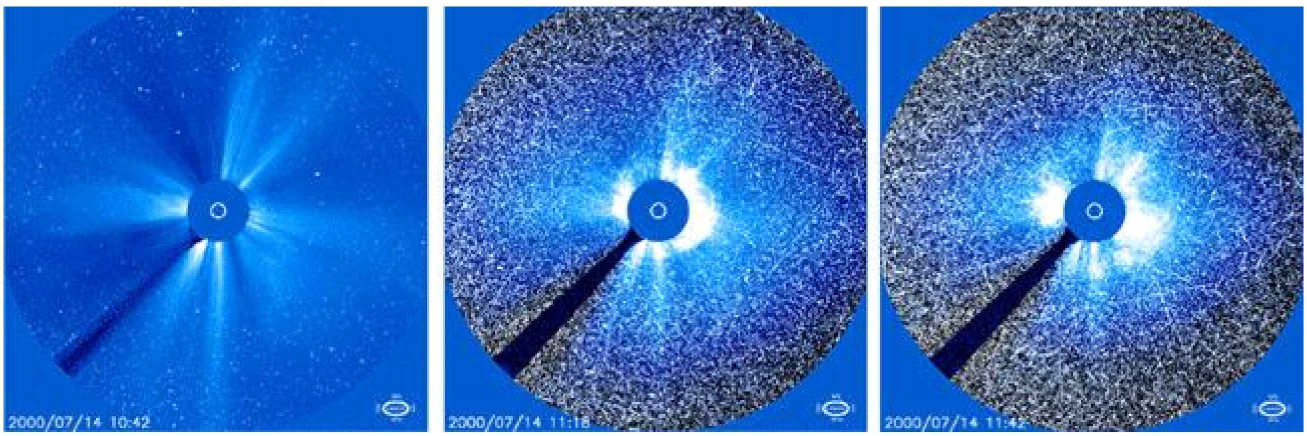
By replacing the highlighted, corrupted data words with the uncorrupted values in the other string, we get the following de-glitched data words:

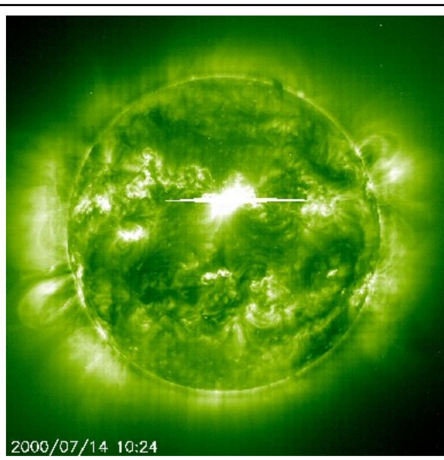
| | | | | | |
|---|---|---|---|---|---|
| Corrected: | 10111010 | 01110101 | 10111100 | 11001011 | 10101101 |
| | 01011010 | 01111010 | 10001100 | 00110111 | 00100110 |
| | **11011000** | 11001101 | 10110101 | 11011010 | 11110001 |
| | 10001010 | 10001111 | 01110011 | 10010001 | 11001011 |

The odd word is the first word in the third row. The first transmission says that it is '0111100**0**' and the second transmission says it is '1101100**0**' Both wrong words have a parity of '1' which means there is an even number of '1' in the first seven places in the data word. But the received parity bit says '**0**' which means there was supposed to be an odd number of '1's in the correct word. Examining these two words, we see that the first three digits are '011' and '110' so it looks like the first and third digits have been altered. Unfortunately, we can't tell what the correct string should have been. Because the rest of the word '11000' has an even parity, all we can tell about the first three digits is that they had an odd number of '1's so that the total parity of the complete word is '0' . This means the correct digits could have been '100', '010', '111', or '111', but we can't tell which of the three is the right one. That means that this data word remains damaged and can't be de-glitched even after the second transmission of the data strings.

# Data Corruption by High-Energy Particles




Solar flares can severely affect sensitive instruments in space and corrupt the data that they produce. On July 14, 2000 the sun produced a powerful X-class flare, which was captured by instruments onboard the Solar and Heliospheric Observatory (SOHO). The EIT imager operating at a wavelength of 195 Angstroms, showed a brilliant flash of light (left image). When these particles arrived at the SOHO satellite some time later, they caused the imaging equipment to develop 'snow' as the individual particles streaked through the sensitive electronic equipment. The above images taken by the SOHO LASCO c2 and c3 imagers show what happened to that instrument when this shower of particles arrived. The date and time information (hr : min) is given in the lower left corner of each image, and give the approximate times of the events.

**Problem 1:** At about what time did the solar flare first erupt on the sun?

**Problem 2:** At about what time did the LASCO imagers begin to show significant signs of the particles having arrived?

**Problem 3**: If the SOHO satellite was located 147 million kilometers from the sun, about what was the speed of the arriving particles?

**Problem 4:** If the speed of light is 300,000 km/sec, what percentage of light-speed were the particles traveling?

## Answer Key:

**Problem 1:** At about what time did the solar flare first erupt on the sun?

**Answer:** The EIT image time says 10:24 or 10 hours and 24 minutes Universal Time
The reason this is not an exact time is because the images were taken at set times, and not at the exact times of the start or end of the events. To within the 24-minute interval between successive EIT images, we will assume that 10:24 UT is the closest time.

**Problem 2:** At about what time did the LASCO imagers begin to show significant signs of the particles having arrived?

**Answer:** The top sequence shows that the 'snow began to fall' at 10:54 UT. The second sequence suggests a later time near 11:18. However, the 11:18 time is later than the 10:54 time. The time interval between exposures is 24 minutes, but the top series started at 10:30 and ended at 10:54 UT, while the lower series started at 10:42 and ended at 11:18. That means, comparing the exposures between the two series, the snow arrived between 10:42 and 10:54 UT. We can split the difference and assume that the snow began around 10:48 UT.

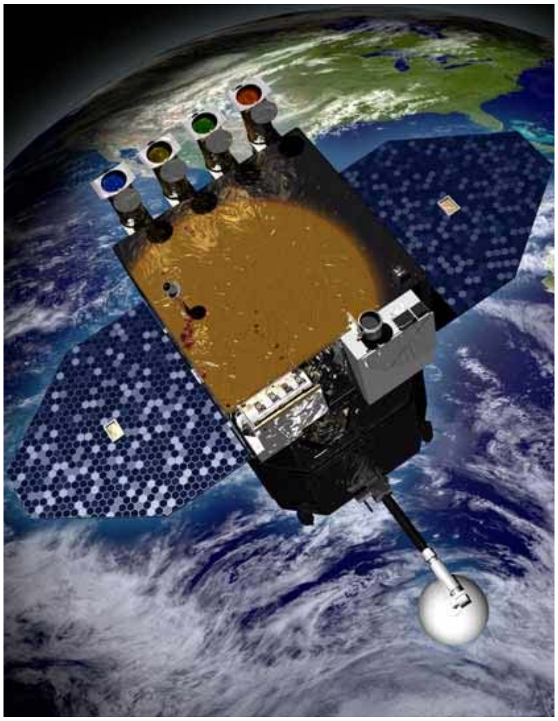
**Problem 3**: If the SOHO satellite was located 147 million kilometers from the sun, about what was the speed of the arriving particles?

**Answer:** The elapsed time between the sighting of the flare by EIT (10:24 UT) and the beginning of the snow seen by LASCO (10:48 UT) is 10:48 UT – 10:24 UT = 24 minutes. The speed of the particles was about 147 million km/24 minutes or 6.1 million km/minute.

**Problem 4:** If the speed of light is 300,000 km/sec, what percentage of light-speed were the particles traveling?

**Answer:** Converting 6.1 million km/minute into km/sec we get 6,100,000 km/sec x (1 min /60 sec) or 102,000 kilometers/sec. Comparing this to the speed of light we see that the particles traveled at (102,000/300,000)x100% = 34% the speed of light!

Note: Because these damaging high-speed particles can arrive only a half-hour after the x-ray flash is first seen on the sun, it can be very difficult to protect sensitive equipment from these storms of particles if you wait for the first sighting of the solar flare flash. In some cases, science research satellites have actually been permanently damaged by these particle storms.



The 15 instruments on NASA's Solar Dynamics Observatory (SDO) will usher in a new era of solar observation by providing scientists with HD-quality viewing of the solar surface in nearly a dozen different wavelength bands.

One of the biggest challenges is how to handle all the data that the satellite will return to Earth 24/7/365! It is no wonder that the design and construction of this data handling network has taken nearly 10 years to put together! To make sense of the rest of this story, here are some units and prefixes you need to recall (1 byte = 8 bits):

Kilo = 1 thousand
Mega = 1 million
Giga = 1 billion
Tera = 1 trillion
Peta = 1,000 trillion
Exa = 1 million trillion

**Problem 1** - In 1982 an IBM PC desktop computer came equipped with a 25 megabyte hard drive (HD) and cost $6,000. In 2010, a $500 desktop comes equipped with a 2.5 gigabyte hard drive. By what factor do current hard drives have more storage space than older models?

**Problem 2** - A 2.5 gigabyte hard drive is used to store music from iTunes. If one typical 4-minute, uncompressed, MPEG-4 song occupies 8 megabytes, about A) How many uncompressed songs can be stored on the HD? B) How many hours of music can be stored on the HD? (Note: music is actually stored in a compressed format so typically several thousand songs can be stored on a large HD)

**Problem 3** - How long will it take to download 2 gigabytes of music from the iTunes store A) Using an old-style 1980's telephone modem with a bit rate of 56,000 bits/sec? B)With a modern fiber-optic cable with a bit rate of 16 megabits/sec?

**Problem 4** - The SDO satellite's AIA cameras will generate 67 megabits/sec of data as they take 4096x4096-pixel images every 3/4 of a second. The other two instruments, the HMI and the EVE, will generate 62 megabits/sec of data. The satellite itself will also generate 20 megabits/sec of 'housekeeping' information to report on the health of the satellite. If a single DVD can store 8 gigabytes of information, how many DVDs-worth of data will be generated by the SDO: A) Each day? B) Each year?

**Problem 5** - How many petabytes of data will SDO generate during its planned 5-year mission?

**Problem 6** - It has been estimated that the total amount of audio, image and video information generated by all humans during the last million years through 2009 is about 50 exabytes including all spoken words (5 exabytes). How many DVDs does this equal?

**Problem 1** - In 1982 an IBM PC desktop computer came equipped with a 25 megabyte hard drive (HD) and cost $6,000. In 2010, a $500 desktop comes equipped with a 2.5 gigabyte hard drive. By what factor do current hard drives have more storage space than older models? Answer: Take the ratio of the modern HD to the one in 1982 to get 2.5 billion / 25 million = 2,500,000,000/25,000,000 = 2,500/25 = **100 times.**

**Problem 2** - A 2.5 gigabyte hard drive is used to store music from iTunes. If one typical 4-minute MPEG-4 song occupies 8 megabytes, about A) how many songs can be stored on the HD? B) How many hours of music can be stored on the HD?
Answer: A) Number of songs = 2.5 gigabytes/8 megabytes = 2,500 megabytes/8 megabytes = 312 B) Time = 312 songs x 4 minutes/song = 1,248 minutes = **20.8 hours.**

**Problem 3** - How long will it take to download 2 gigabytes of music from the iTunes store A) Using an old-style 1980's telephone modem with a bit rate of 56,000 bits/sec? B)With a modern fiber-optic cable with a bit rate of 16 megabits/sec?
Answer; A) 2 gigabytes x 8 bits/1 byte = 16 gibabits. Then 16,000,000,000 bits x ( 1 second/56,000 bits) = 285,714 seconds or **79.4 hours**. B) 16,000,000,000 bits x ( 1 second/16,000,000 bits) = 1,000 seconds or about **17 minutes.**

**Problem 4** - The SDO satellite's AIA cameras will generate 67 megabits/sec of data as they take 4096x4096-pixel images every 3/4 of a second. The other two instruments, the HMI and the EVE, will generate 62 megabits/sec of data. The satellite itself will also generate 20 megabits/sec of 'housekeeping' information to report on the health of the satellite. If a single DVD can store 8 gigabytes of information, how many DVDs-worth of data will be generated by the SDO each A) Day? B) Year? Answer: Adding up the data rates for the three instruments plus the satellite housekeeping we get 67+63+20 = 150 megabits/sec. A) In one day this is 150 megabits/sec x (86,400 sec/1day) = 12.96 terabits or since 1 byte = 8 bits we have 1.6 terabytes. This equals 1,600 gigabytes x (1 DVD/8 gigabytes) = **200 DVDs each day**. B) In one year this equals 365 days x 1.6 terabytes/day = 584 terabytes per year or 365 days/1 year x 200 DVDs/1 year = **73,000 DVDs/year.**

**Problem 5** - How many petabytes of data will SDO generate during its planned 5-year mission? Answer: In 5 years it will generate 5 years x 584 terabytes/year = 2,920 terabytes. Since 1 petabyte = 1,000 terabytes, this becomes **2.9 petabytes**.

**Problem 6** - It has been estimated that the total amount of audio, image and video information generated by all humans during the last million years through 2009 is about 50 exabytes including all spoken words (5 exabytes). How many DVDs does this equal? Answer: 1 exabyte = 1,000 petabytes = 1,000,000 terabytes = 1,000,000,000 gigabytes. So 50 exabytes equals 50 billion gigabytes. One DVD stores 5 gigabytes, so the total human information 'stream' would occupy 50 billion / 5 billon = **10 billion DVDs.**

Modern electronic CCD pixels actually count individual photons of light. This is easily revealed in the dramatic short time exposure photos seen to the left, of a scene taken under very low light levels. The human retina is also an organic light imager that can detect individual photons of visible light.

The images were taken at light levels that produce in each pixel of the image, from top to bottom, 0.1 photons/sec, 1 photon/sec, 10 photons/sec and 100 photons/sec.

The number of photons striking a one-square-meter surface illuminated by the noon-day sun for one second is about 500 microMoles, where 1 mole = $6.02 \times 10^{23}$ photons.

**Problem 1** – In scientific notation, how much is 500 microMoles of photons?

**Problem 2** – Suppose a CCD chip were placed in full sunlight without a camera lens. If the chip measures 2 cm square and consists of 4096x4096 pixels, what is the brightness of the sunlight falling on each pixel in photons/sec?

**Problem 3** – If a pixel will completely saturate (turn 'white') if it accumulates more than 10 million photons on a single exposure, what is the maximum exposure time for an image taken in full sunlight?

**Problem 1** – In scientific notation, how much is 500 microMoles of photons?

Answer: $500 \times 10^{-6}$ moles x ($6.02 \times 10^{23}$ photons/mole) = **$3.0 \times 10^{20}$ photons/$m^2$/sec**.

**Problem 2** – Suppose a CCD chip were placed in full sunlight without a camera lens. If the chip measures 2 cm square and consists of 4096x4096 pixels, what is the brightness of the sunlight falling on each pixel in photons/sec?

Answer: Pixel dimension L = 2 cm/4096 = 0.0005 cm or

L = $5.0 \times 10^{-4}$ cm x (1 meter/100 cm) = $5.0 \times 10^{-6}$ meters.

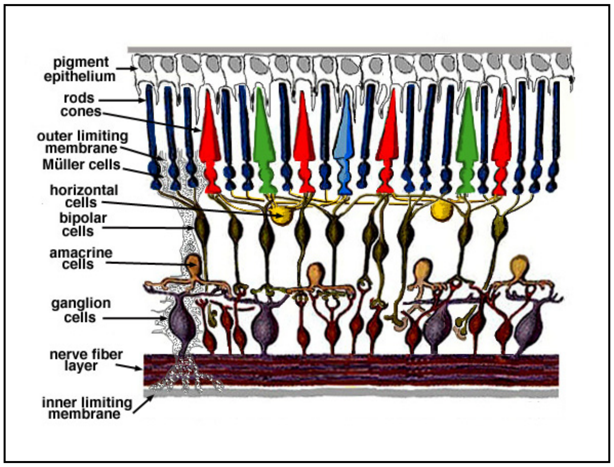
Pixel area A = $(5.0 \times 10^{-6} \text{ meters})^2 = 2.5 \times 10^{-11}$ meters$^2$.

Brightness = $3.0 \times 10^{20}$ photons/$m^2$/sec x $2.5 \times 10^{-11}$ meters$^2$

= **$7.5 \times 10^{9}$ photons/sec/pixel**.

**Problem 3** – If a pixel will completely saturate (turn 'white') if it accumulates more than 10 million photons on a single exposure, what is the maximum exposure time for an image taken in full sunlight?

Answer: The sunlight produces $7.5 \times 10^{9}$ photons/sec, so to accumulate 10 million photons will take T = 10 million / $7.5 \times 10^{9}$ photons/sec so **T = 0.0013 seconds or 1/800 sec.**




Light and other forms of electromagnetic radiation consists of individual particles called photons. We cannot see these particles under most common conditions of lighting. There are just too many of them to be able to perceive all at once. But with a little mathematics we can appreciate the various magnitudes at which they are found.

Photons carry energy. For example, a photon of visible light at a wavelength of 550 nanometers carries about $3.7\times10^{-19}$ Joules of energy.

**Problem 1** – At High Noon on the surface of Earth, the sun delivers about 1000 Joules each second to a surface with an area of one square meter. How many photons pass through a surface with an area of the human pupil for which $A = 2.5\times10^{-5}$ meters$^2$?

**Problem 2** – There are about 5 million cones and rods in the human retina. About how many photons/sec activate a retinal cell during Noon-time illumination?

**Problem 3** – Noon sunlight has an intensity of 130,000 Lux while the Milky Way in the night sky has an intensity of 0.001 Lux. About how may photons strike a rod or cone cell in your retina while looking at the Milky Way at night?

**Problem 4** – Because photons arrive randomly in time, the number that are counted will vary slightly from moment to moment. A statistical rule-of-thumb is that the variation in counts $s = N^{1/2}$ where N is the number of counts in any one time interval. By what percentage will the number of detected photons vary every second if A) N=5?, B) N=100? C) N=10000?

**Problem 5** – From your answer to Problem 4, under which lighting conditions would you probably see a twinkling effect if your retina could detect each individual photon as it arrived?

**Problem 1** – At High Noon on the surface of Earth, the sun delivers about 1000 Joules each second to a surface with an area of one square meter. How many photons pass through a surface with an area of the human pupil for which A = $2.5 \times 10^{-5}$ meters$^2$?

Answer: N = 1000 Joules/sec x (1 photon / $3.7 \times 10^{-19}$ Joules) x ($2.5 \times 10^{-5}$ meters)
**N = $6.7 \times 10^{16}$ photons/second**.

**Problem 2** – There are about 5 million cones and rods in the human retina. About how many photons/sec activate a retinal cell during Noon-time illumination?

Answer: n = ($6.7 \times 10^{16}$ photons/second) / 5 million so **n = 13 billion photons/sec**.

**Problem 3** – Noon sunlight has an intensity of 130,000 Lux while the Milky Way in the night sky has an intensity of 0.001 Lux. About how may photons strike a rod or cone cell in your retina while looking at the Milky Way at night?

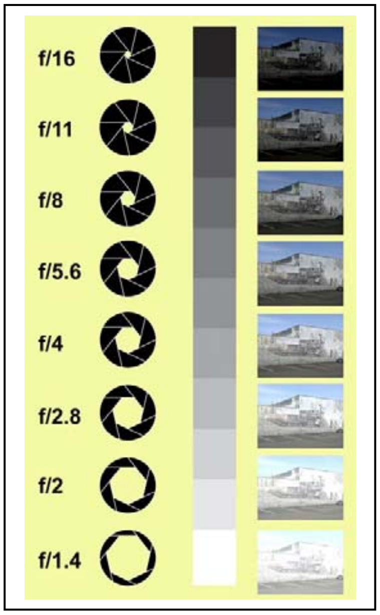
Answer: The Milky Way is about 130,000 Lux / 0.001 Lux = $1.3 \times 10^{8}$ times fainter than sunlight. Since at the retina, sunlight produces 13 billion photons/sec, the Milky Way produces about 13 billion photons/ $1.3 \times 10^{8}$ = **100 photons/sec** in each retinal cell. At these light levels, only rods are sensitive enough to be activated by this little light, so we do not see the Milky Way in color!

**Problem 4** – Because photons arrive randomly in time, the number that are counted will vary slightly from moment to moment. A statistical rule-of-thumb is that the variation in counts $s = N^{1/2}$ where N is the number of counts in any one time interval. By what percentage will the number of detected photons vary every second if A) N=5?, B) N=100? C) N=10000?

Answer: A) 100% x $(5)^{1/2}/5$ = 45%
B) 100% x $(100)^{1/2}/100$ = 10%
C) 100% x $(10000)^{1/2}/10000$ = 1%

**Problem 5** – From your answer to Problem 4, under which lighting conditions would you probably see a twinkling effect if your retina could detect each individual photon as it arrived?

Answer: From **Case A** with 5 photons/sec because the variation from second to second is the highest percentage. This means that sometimes you would se as few as 2 photons/second and other times as high as 7 photons/second which is not a steady rate.




Digital cameras contain lenses that help gather photons into a concentrated beam. This beam comes to a focus on the CCD chip containing the pixels which detect the photons. The larger the area of the lens, the more photons can be collected.

But individual pixels can only accumulate a fixed number of photons, about 40,000, before their capacity is exceeded. This is called 'saturation' and results in the image turning white. By adjusting the aperture size with the camera's F-stop, and the exposure speed, saturation can be avoided and a clear well-balanced picture results!

At High Noon the intensity of photons in 'broad daylight' is about F = 6.7 x $10^{16}$ photons/sec/meter$^2$. A photographer is using a Nikon d3000 camera with a 200 millimeter telephoto lens to take various pictures of flowers, cityscapes and natural settings. The camera has a P = 10 megapixel format.

**Problem 1** – If the relationship between f-number and lens area, A, in meters$^2$ is given in the table below, to 2 significant figures, calculate for all of the possible combinations of f-number and exposure speed, the number of photons in broad daylight that arrive at a single pixel in the CCD if the possible exposures are T = 1/25, 1/60, 1/100, 1/250, 1/500 and 1/1000 second. if the formula is $N = \frac{FTA}{P}$ (Hint: An Excel spreadsheet may make the calculations simpler to perform!)

| | | Exposure | Exposure | Exposure | Exposure | Exposure | Exposure |
|---|---|---|---|---|---|---|---|
| f/number | area (m2) | 1/25 | 1/60 | 1/100 | 1/250 | 1/500 | 1/1000 |
| **4** | 0.001963 | | | | | | |
| **4.5** | 0.001551 | | | | | | |
| **5** | 0.001256 | | | | | | |
| **5.6** | 0.001001 | | | | | | |
| **6.3** | 0.000791 | | | | | | |
| **7** | 0.000641 | | | | | | |
| **8** | 0.000491 | | | | | | |
| **11** | 0.00026 | | | | | | |
| **32** | 0.0000307 | | | | | | |

**Problem 2 -** What combinations of exposure time and f-number will allow pictures to be taken in broad daylight that do not saturate the image, if the maximum number of photons cannot exceed 40,000?

Given: F = $6.7 \times 10^{16}$ photons/sec/meter$^2$. FL = 200 millimeters
CCD has a 10 megapixel format.

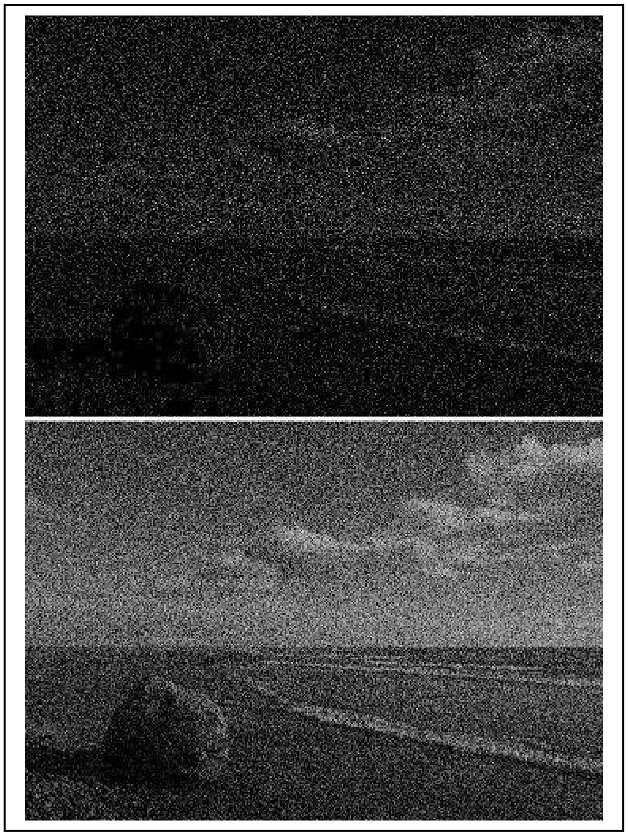
**Problem 1** – If the relationship between f-number and lens area in meters$^2$ is given in the table below, to 2 significant figures, calculate for all of the possible combinations of f-number and exposure speed, the number of photons in broad daylight that arrive at a single pixel in the CCD if the possible exposures are 1/25, 1/60, 1/100, 1/250. 1/500 and 1/1000 second. (Hint: Use an Excel spreadsheet). Answer: See below table.

Example: For F/4 and 1/25 exposure, t = 0.04 seconds. Pixel area = 0.001963, so the total photons striking a pixel is $N = 6.7 \times 10^{16} \times 0.04 \times 0.001963$ /10 million so N = 526,084 which to 2 SF is **N = 530,000 photons per pixel**.

| | | Exposure | | | | | |
|---|---|---|---|---|---|---|---|
| f/number | area (m2) | 1/25 | 1/60 | 1/100 | 1/250 | 1/500 | 1/1000 |
| | | 0.04 | 0.016 | 0.01 | 0.004 | 0.002 | 0.001 |
| 4 | 0.001963 | 530,000 | 220,000 | 130,000 | 53,000 | 26,000 | 13,000 |
| 4.5 | 0.001551 | 420,000 | 170,000 | 100,000 | 42,000 | 21,000 | 10,000 |
| 5 | 0.001256 | 340,000 | 140,000 | 84,000 | 34,000 | 17,000 | 8,400 |
| 5.6 | 0.001001 | 270,000 | 110,000 | 67,000 | 27,000 | 13,000 | 6,700 |
| 6.3 | 0.000791 | 210,000 | 88,000 | 53,000 | 21,000 | 11,000 | 5,300 |
| 7 | 0.000641 | 170,000 | 72,000 | 43,000 | 17,000 | 8,600 | 4,300 |
| 8 | 0.000491 | 130,000 | 55,000 | **33,000** | 13,000 | 6,600 | 3,300 |
| 11 | 0.00026 | 70,000 | **29,000** | **17,000** | 7,000 | 3,500 | 1,700 |
| 32 | 0.0000307 | 8,200 | 3,000 | 2,000 | 820 | 410 | 210 |

**Problem 2 -** What combinations of exposure time and f-number will allow pictures to be taken in broad daylight that do not saturate the image, if the maximum number of photons cannot exceed 40,000?

Answer: The shaded region above indicates combinations that will saturate the image. Unshaded squares are combinations that will lead to usable images.



In many astronomical situations, very faint light levels are being measured, such as the light from distant faint stars and nebulae studied by the Hubble Space Telescope. For other situations, such as the Solar Dynamics Observatory studies of the surface of the Sun, huge amounts of light are available that can easily overwhelm sensitive detectors. Although bright sources can be filtered so that they do not 'saturate' the CCD imagers, faint sources must be amplified to register the meager light. In this problem we look at faint sources first.

The image to the left shows a faint image (top) in which individual photons are being registered. In the bottom image, 10-times more photons were detected.

Photon counting follows many of the same statistical rules as conducting surveys. At the end of an exposure for an image, we want to accurately measure the number of photons that were registered in a pixel, which will determine the exact color to assign to the pixel.

Suppose you asked 16 people in Group A the same true/false question as 16 people in Group B. Statistical counting predicts that the responses in each True and False category will differ by as much as $s = (16)^{1/2} = 4$ people in each response between the two groups. If 8 people in Group A answered True, the number of people in Group B that also answered True could be 8 +/- 4 or the specific values of 4, 5, 6, 7, 8, 9, 10, 11 or 12. The survey would be said to have a sampling error of 100% x 4/16 = +/- 25%. If the survey had 1000 people, $s = (1000)^{1/2} = 32$ so the sampling error is +/- 3%.

**Problem 1** – A CCD camera operates under low light level conditions and in one photograph, one pixel registers N= 25 photons. If a second photograph is taken, what are the possible numbers of photons detected in this pixel in the second image?

**Problem 2** – Three images are compared and the same pixel registers 245, 230 and 252 photons. A) What is the average number of photons registered? B) What is the sampling error of the average number of photons? C) What is the possible of range of values relative to the average value?

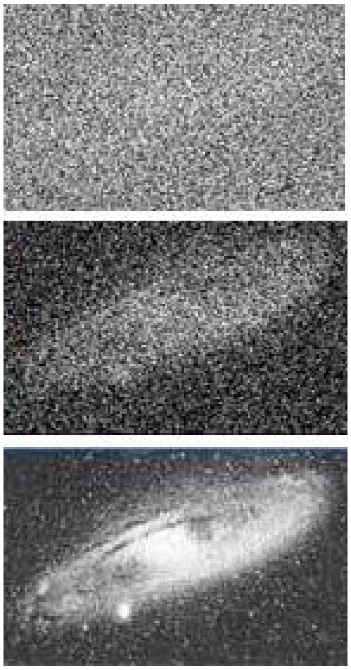
**Problem 1** – A CCD camera operates under low light level conditions and in one photograph, one pixel registers N= 25 photons. If a second photograph is taken, what ae the possible numbers of photons detected in this pixel in the second image?

Answer: $s = (35)^{1/2} = 5$ so
25 +/- 5 becomes N = 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30.

**Problem 2** – Three images are compared and the same pixel registers 245, 230 and 252 photons. A) What is the average number of photons registered? B) What is the sampling error of the average number of photons? C) What is the possible of range of values relative to the average value?

**Answer**: A) Average = (245+230+252)/3 = 242.
B) $s = (242)^{1/2} = 15$
C) N = 242 +/- 15
N = 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 249,
240, 241, **242**, 243, 244, 245, 246 ,247, 248, 249, 250, 251,
252, 253, 254, 255, 256,257

**Note to Teacher:** In statistics, the value of s is called the standard deviation of the distribution. If the distribution is randomly determined (a Gaussian Bell Curve), then repeated sampling of a similar population will result in a mean value which is within 1-standard deviation of the mean value about 68% of the time, within 2-standard deviations 95% of the time, within 3-standard deviations 99% of the time and so on.



Courtesy: Ulmer and Wessels, NorthwesternUniversity: m-ulmer2@northwestern.edu;.

A recognizable image of a surface detail or an astronomical object requires that each pixel's measurement be done with high accuracy. Otherwise, if the measurements are imprecise, the image becomes indistinct. What this means is that we have to make the standard deviation of the final image as small as possible so that we have an image with a small percentage of error in its pixel values. Fortunately, there is a very simple process that guarantees this outcome. By averaging a large number of images together, pixel by pixel, we can greatly increase the accuracy of each pixel measurement. This fundamental process is called image stacking or image coadding. The three images to the left dramatically shows what happens as the number of coadded images is increased from top to bottom.

If the standard deviation of the pixel value in any one image is given by s, then the standard deviation of N coadded images is given by

$$S = \frac{s}{\sqrt{N}}$$

**Problem 1** – A CCD camera is taking images of a dark region of the sky to search for faint stars not visible to the human eye. The camera is set to make one exposure every second, and for each exposure the standard deviation per pixel is +/- 50 photons. A) How many exposures must be combined to lower the standard deviation to +/- 1 photon per pixel so that the faint star can be imaged? B) How long will it take to accumulate these images if it takes one second of exposure for each image, and 0.01 second to store and process each image?

**Problem 2** – The Hubble Space Telescope obtained the Southern Deep Field image (HDF-South) and detected thousands of distant galaxies more than 5 billion light years from Earth. Each WFPC-2 image lasted about 1000 seconds and the total exposure time was about 39 hours. A) How many images were stacked to get the final image? B) By what factor, B, is the uncertainty in the stacked image smaller than a single image of the same field? C) Astronomers measure star brightness in terms of a magnitude scale, m, defined by $B = 10^{-0.4m}$. Example, If Star X is 5 magnitudes fainter than Star Y, its brightness is 1/100 of Star Y. How many magnitudes fainter are the faintest galaxies in the stacked image than in the original image?

**Problem 1** – A CCD camera is taking images of a dark region of the sky to search for faint stars not visible to the human eye. The camera is set to make one exposure every second, and for each exposure the standard deviation per pixel is +/- 50 photons. A) How many exposures must be combined to lower the standard deviation to +/- 1 photon per pixel so that the faint star can be imaged? B) How long will it take to accumulate these images if it takes one second of exposure for each image, and 0.01 second to store and process each image?

Answer: A) The standard deviation of the final image is 1.0 and each frame contributes s = 50 so $N^{1/2}$ = S/s = 50/1 and so N = **2500 images**.

B) the total time consists of two contributions:

Setup time = 2500 images x (0.01 second/image) = 25 seconds.

Exposure time = 2500 images x (1 sec/image) = 2500 seconds

For a total time of **T = 2,525 seconds or 42 minutes**.

**Problem 2** – The Hubble Space Telescope obtained the Southern Deep Field image (HDF-South) and detected thousands of distant galaxies more than 5 billion light years from Earth. Each WFPC-2 image lasted about 1000 seconds and the total exposure time was about 39 hours. A) How many images were stacked to get the final image? B) By what factor, B, is the uncertainty in the stacked image smaller than a single image of the same field? C) Astronomers measure star brightness in terms of a magnitude scale, m, defined by B = $10^{-0.4m}$. Example, If Star X is 5 magnitudes fainter than Star Y, its brightness is 1/100 of Star Y. How many magnitudes fainter are the faintest galaxies in the stacked image than in the original image?

Answer:

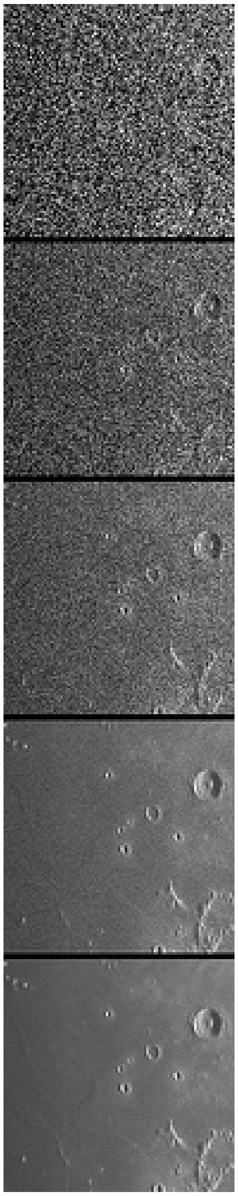
A) T = 39 hours x (3600 seconds/1 hour) = 140,400 seconds. So N = 140,400/1000 **N=140 exposures**.

B) By a factor of B = $(140)^{1/2}$ or **B = 12**

C) We have B = $10^{-0.4m}$ so for B = 12

Log(1/12) = -0.4m

**m = 2.7 so it is 2.7 magnitudes fainter.**


Courtesy Joe Zawodny.

Ordinary digital photography consists of relying on a camera to automatically determine the focus, exposure speed and f/stop in order to produce one 'perfect' image.

In scientific photography, especially astronomical imaging, hundreds or even thousands of images may be taken of the same target. These images are then carefully sorted to eliminate poor quality images, then the remaining images are combined together to produce the final image. Although a single image may have an exposure of only a few seconds, it may be impossible to prolong the exposure for minutes or hours to detect the faintest objects or details. By taking hundreds of short-exposure images, and 'stacking' them together, exposures in the final summed image can exceed hours, or even days.

The images to the left show the dramatic effect of summing or 'coadding' many images together. The top image is the original '1-second' exposure. The second image combines 4 of these 1-second images. The next image combines 16, followed by 256 and 4096 images.

The gaininess in each summed image represents the statistical noise. This noise is caused by a combination of instrumental measuring errors in each pixel, and the quantum aspects of light photons when only a few photons are present. By combining more images, the statistical noise in the final image is greatly reduced, allowing progressively fainter features to be discerned.

The fundamental mathematical formula relating the statistical noise in one image, s, to the final statistical noise in N coadded images, S, is

$$S = \frac{s}{\sqrt{N}}$$

In the 2MASS all-sky infrared survey, astronomers use a ground-based telescope to photograph the sky. The basic digital image lasts 1.6 seconds and the noise is measured to be s = +/- 2.5 DN. The faintest star detectable in these images has a brightness of 0.0004 Jy.

**Problem 1** - In a graph, plot the final noise level in DNs after coadding up to 10,000 images. Use Log(N) as the horizontal axis and S in units of DN as the vertical axis.

**Problem 2** – If the brightness of the faintest star scales linearly with the noise in the image, what is the brightness of the faintest star visible after coadding 10,000 images?

The images were created by Joe Zawodny (http://joe.zawodny.com/index.php) and can be found in his excellent articles on astrophotography.

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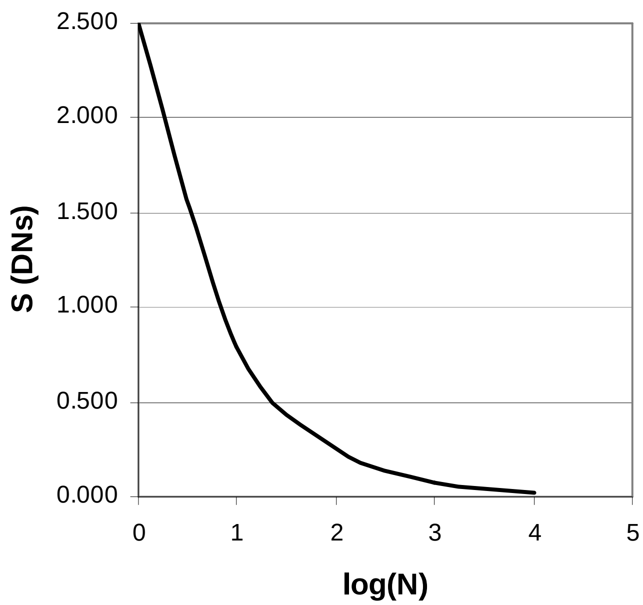
**Problem 1** - In a graph, plot the final noise level in DNs after coadding up to 10,000 images. Use Log(N) as the horizontal axis and S in units of DN as the vertical axis.




**Problem 2** – If the brightness of the faintest star scales linearly with the noise in the image, what is the brightness of the faintest star visible after coadding 10,000 images?

Answer: $S = 2.5/(10000)^{1/2} = 0.025s$
The coadded image can see stars about 1/100 as bright as the original image so F = 0.01 x 0.0004 Janskys and so **F = 0.000004 Janskys**.

Note: In terms of astronomical apparent magnitudes, the original image could see stars as faint as m = +16 but the new image can see stars to m = +16 + 5m or m= +21m.
A factor of 100 in brightness changes the apparent magnitude by exactly +5 magnitudes.


Nikon d3000 image of moon, 1/25-sec, 200mm lens, F/32.

When we take ordinary digital photos for our family album, we are not concerned about the actual quantity of light that fell on the CCD to create the final image. Only the clarity and color balance of the final image matters.

In scientific 'imaging' it is almost always the case that the quantity of light, and its variation from pixel-to-pixel, matter greatly in the study at hand.

The process of working backwards from the digital numbers that code the pixel brightness (DNs) to physical quantities such as watts, watts/meter$^2$ or more complex units, is called calibration.

**Method 1:** If you know exactly how your entire imager (CCD+optical system and filters) responds to light, you can work from the data unit values in DNs to actual brightness units using mathematical steps. This approach is commonly used when you know nothing about the 'unknown' object you are imaging.

**Method 2:** Alternately, you can image a few different types of known objects (such as stars) whose brightness you know exactly, and measure their DNs. You can then establish a relationship that '1 DN equals x number of brightness units'.

**Problem 1** – The bright star Vega has an intensity of I = $3.7 \times 10^{-8}$ watts/meter$^2$/arcsecond$^2$/micron and its light is concentrated into one CCD pixel. The processing of this image indicates a data word for Vega of 150,000 DNs. What is the calibration constant C, that relates the computer DNs to the actual, physical intensity, I?

**Problem 2** - What is the intensity of the faintest star that registers on the CCD as 25 DNs per pixel?

**Problem 1** – The bright star Vega has an intensity of I = $3.7\times10^{-8}$ watts/meter$^{2}$/arcsecond$^{2}$/micron and its light is concentrated into one CCD pixel. The processing of this image indicates a data word for Vega of 150,000 DNs. What is the calibration constant C, that relates the computer DNs to the actual, physical intensity, I?

Answer; We want to create an equation I = C x N where N is the number of DNs, and I is the resulting physical intensity of the light. That means the constant ,C, must have the units appropriate to I/DN. Since we have measured one 'calibration standard' to get I = $3.7\times10^{-8}$ watts/meter$^{2}$/arcsecond$^{2}$/micron which is equivalent to 150,000 DNs ,the calibration constant would be
C = ($3.7\times10^{-8}$ watts/meter$^{2}$/arcsecond$^{2}$/micron) / (150,m000 DNs)
**C = $2.5\times10^{-13}$ watts/meter$^{2}$/arcsecond$^{2}$/micron/ DN**

Note the compound units!

**Problem 2** – What is the intensity of the faintest star that registers on the CCD as 25 DNs per pixel?

Answer:

From our answer to Problem 1,

I = ($2.5\times10^{-13}$ watts/meter$^{2}$/arcsecond$^{2}$/micron/DN) x 25 DN
**I = $6.3 \times 10^{-12}$ watts/meter$^{2}$/arcsecond$^{2}$/micron**

# Calibrating Data

Scientists often invent measuring technology in order to measure some property of an object. A simple thermometer measures temperature, defined in terms of the Celsius scale, as a displacement of a column of mercury (in millimeters).

This example shows a common problem in measurement. It is often the case that the scale of interest (degrees) is very different than the scale upon which the measurement is based (millimeters). To translate between these scales you need to carry out a process called calibration.

An instrument has been created that measures temperature by moving a marker through a specific number of millimeters. The scale was calibrated by measuring the temperature of several bodies of known temperature, measured in Celsius degrees. Suppose that you created a Calibration Table in which you measured how a specific temperature causes a specific displacement. The measurements indicate that at T=10 C, the marker moved to an indicator of x=10 millimeters. At T=30 C, it moved to x=15 mm. At T=50C it moved to x=20mm and at T=70C it moved to x=25 mm.

**Problem 1** - With temperature on the horizontal axis, graph the data to form a Calibration Curve for the instrument.

**Problem 2** - Determine the best-fit linear equation for the data in the form y=mx+b.

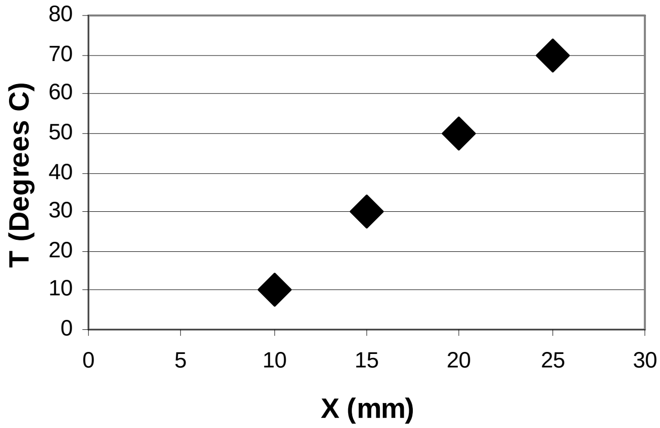
**Problem 3** - What is the Zero-Point (x=0) for the calibration?

**Problem 4** - What is the Calibration Constant (M) for the data, and what are the units for the Calibration Constant?

**Problem 5** - If the instrument indicates x=5mm, what is the temperature being

The Calibration Table indicates that at T=10 C, the marker moved to an indicator of x=10 millimeters. At T=30 C, it moved to x=15 mm. At T=50C it moved to x=20mm and at T=70C it moved to x=25 mm.

**Problem 1** - Graph the data to form a Calibration Curve for the instrument.




**Problem 2** - Determine the best-fit linear equation for the data in the form y=mx+b.

Answer: Use the 2-point formula then simplify to the point-slope form.

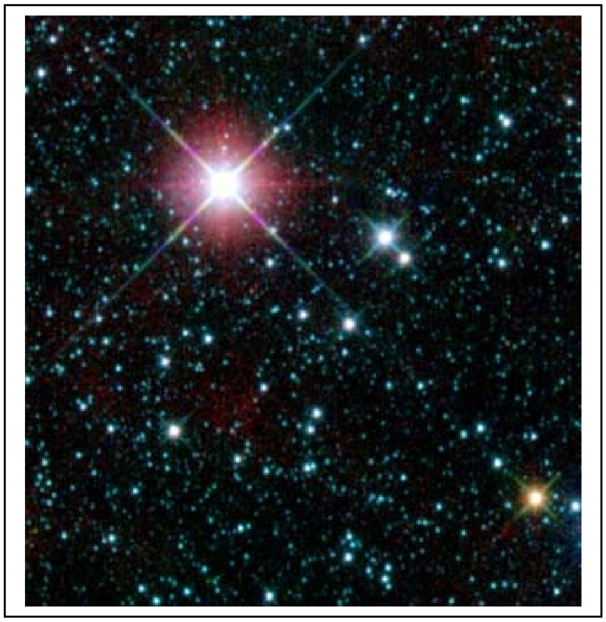
$$T - T1 = \frac{T2 - T1}{x2 - x1}(x - x1)$$ for Point 1 = (15, 30) and Point 2 = (25, 70)

T - 30 = (40/10)(x - 15) so T = 4x - 60 + 30 and so **T(x) = 4x - 30**.

**Problem 3** - What is the Zero-Point (x=0) for the calibration?
Answer: For X = 0, **T(0) = -30 C**

**Problem 4** - What is the Calibration Constant (M) for the data, and what are the units for the Calibration Constant?
Answer: **M = 4.0 degrees C/millimeter**.

**Problem 5** - If the instrument indicates x=5mm, what is the temperature being measured?
Answer: T(5mm) = 4(5) - 30 so **T(5mm) = -10 degrees C**



Astronomers build instruments to photograph distant stars, galaxies and nebulae, such as the NASA WISE satellite image shown to the left. The measuring units that are provided by the instrument do not, by themselves, indicate the proper units required for measuring some aspect of a distant object being imaged.

A process of 'calibration' must be followed to relate the measuring units to the physical units of the distant object being studied.

A CCD camera has been built that consists of 16 million pixels in a square format of 4096 x 4096 pixels. The intensity of the starlight falling upon a pixel as it passes through the optics of the telescope is registered in each pixel as a 20-bit digital word. Each of the $2^{20}$ = 1,048,576 levels indicated by the data word (abbreviated as DN) can be related to a physical brightness level in the distant astronomical body through the calibration process.

The astronomer uses this telescope+camera system to image 5 different stars of known brightness, and records the number of DN units that are measured by an individual pixel centered on the star. The Calibration Data Table is as follows:

| Star | Brightness (picoWatts/$m^2$/nm) | DN value |
|---|---|---|
| Polaris | 4 | 40,000 |
| Regulus | 10 | 100,000 |
| Spica | 16 | 160,000 |
| Arcturus | 40 | 400,000 |
| Sirius | 80 | 800,000 |

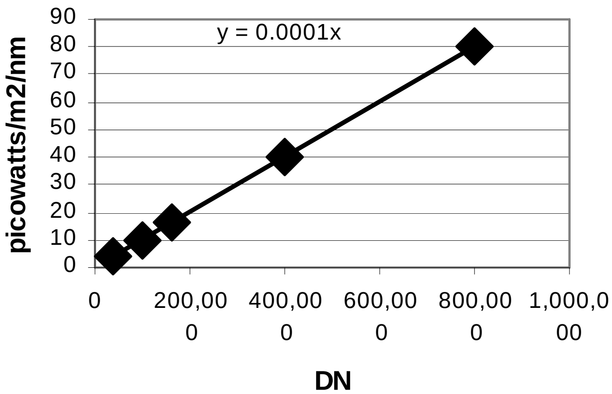
**Problem 1** - With the DN values on the horizontal axis, graph the calibration curve for the data.

**Problem 2** - What is the best-fit linear equation of the form B = mX + b, where B is the brightness and X is the DN value?

**Problem 3** - What is; A) the Zero-Point of the scale (B(0)) in the correct physical units? B) the Calibration Constant (M)

**Problem 4** - The star 36 Ophichi is measured to be x= 12,000 DN on the CCD image. What is the brightness of this star in the correct physical units?

**Problem 1** - Graph the calibration curve for the data.




**Problem 2** - What is the best-fit linear equation of the form B = mX + b?

Answer: Use the two-point formula for Point 1 (50000, 40) and Point 2 (170000, 160)

$$B - 4 = \frac{16 - 4}{160000-40000} (X-40000) \quad \text{so} \quad \mathbf{B(x) = 0.0001X}$$

**Problem 3** - What is; A) the Zero-Point of the scale in the correct physical units? B) the Calibration Constant (M)

Answer: A) **B(x=0) = 0.0 picoWatts/m$^2$/nm**

B) **M = 0.0001 picoWatts/m$^2$/nm per DN**

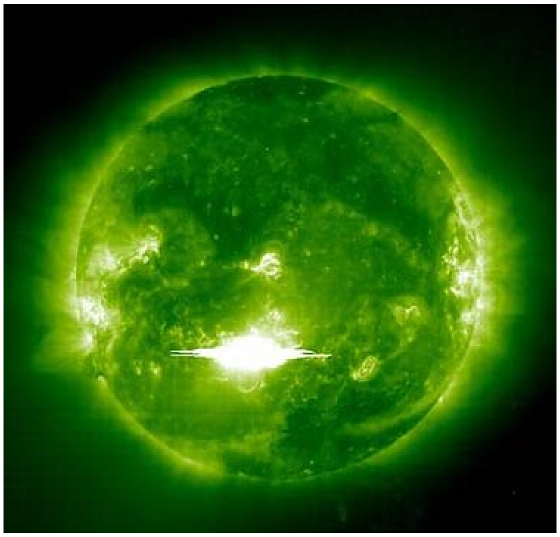
**Problem 4** - The star 36 Ophichi is measured to be x= 12,000 DN on the CCD image. What is the brightness of this star in the correct physical units?

Answer: B(12000) = 0.0001 (12000)

**B(12000) = 1.2 picoWatts/m$^2$/nm**

**Teacher Note:** The physical units of irradiance are picowatts per square meter per nanometer of bandwidth. Irradiance is a measure of the 'density' of power in the spectrum of the object so 'per nanometer' indicates the width of the wavelength band or filter used to measure the star brightness. The faintest star that can be detected with this array has X = 1 DN units which corresponds to a brightness of B(1) = 0.0001 picowatts/m$^2$/nm

# Light Intensity and Pixel 'DNs'



Once the image has been transmitted to Earth and translated back into its original numerical format as an array of numbers, the numbers have to be converted into units that actually describe how much light was detected by the CCD. Without this important step, called calibration, an astronomer will not be able to relate the numbers that make up the image to physically important properties of the object being studied. Here's an example:

At the distance of a satellite from the Sun, the total amount of sunlight power is known to be 1000 watts/meter$^2$. Since this energy is spread out over the sun's entire electromagnetic spectrum, it is convenient to indicate the spectral irradiance of the sun, B. At visible wavelengths (500 nm), this spectral irradiance is about B(500) = 2.0 watts/meter$^2$/nm.

**Problem 1** - A 50 nm, narrow-band filter is used on the satellite's imager that only passes radiation between wavelengths of 500 nm and 550 nm. What is the sunlight power delivered to the surface of the CCD if P = B x bandwidth?

**Problem 2** - The surface area of the CCD chip is 1 cm$^2$. It is a 16 megapixel array. What is the surface area of one pixel in meters$^2$?

**Problem 3** - How much power is falling on one CCD pixel based on your answers to Problem 1 and 2?

**Problem 4** - The measured value of a pixel in the CCD is based on a 16-bit data word which has a maximum value of DN=65,536. After flat-fielding, a single pixel centered on the Sun registers a value of DN=63,000. In terms of actual solar power, what is the conversion constant that converts DN values into solar power values of watts for this CCD array?

**Problem 5** - A pixel measures a faint detail on the sun with a pixel value of 50,000 DN units. What is the corresponding solar power incident on the pixel?

**Problem 1** - A 50 nm, narrow-band filter is used on the satellite's imager that only passes radiation between wavelengths of 500 nm and 550 nm. What is the sunlight power delivered to the surface of the CCD if P = B x bandwidth?

Answer: P = 2.0 watts/meter$^2$/nm x (50 nm) so **B = 100 watts/meter$^2$**.

**Problem 2** - The surface area of the CCD chip is 1 cm$^2$. It is a 16 megapixel array. What is the surface area of one pixel in meters$^2$?

Answer: Accd = 1 cm$^2$ x (1 meter/100 cm)$^2$ = 1.0x10$^{-4}$ meter$^2$.
Apixel = 1.0x10$^{-4}$ meters$^2$ / 16 million so Apixel = **6.3x10$^{-12}$ meters$^2$**.

**Problem 3** - How much power is falling on one CCD pixel based on your answers to Problem 1 and 2?

Answer: P = B x Area so
P = 100 watts/meter$^2$ x (6.3x10$^{-12}$ meters$^2$)
**P = 6.3x10$^{-10}$ watts**.

**Problem 4** - The measured value of a pixel in the CCD is based on a 16-bit data word which has a maximum value of DN=65,536. After flat-fielding, a single pixel centered on the Sun registers a value of DN=63,000. In terms of actual solar power, what is the conversion constant that converts DN values into solar power values of watts for this CCD array?

**Answer:** From Problem 3 we know that the power of the Sun falling on a single pixel is 6.3 x 10$^{-10}$ watts. The pixel value that corresponds to this amount of power is DN=63,000. That means that 1 DN will equal 6.3x10$^{-10}$ watts / 63000 so 1 DN = 1.0x10$^{-14}$ Watts. The conversion constant is then **C = 1.0x10$^{-14}$ Watts/DN**.

**Problem 5** - A pixel measures a faint detail on the sun with a pixel value of 50,000 DN units. What is the corresponding solar power incident on the pixel?

Answer: P = 50,000 DN x (1.0x10$^{-14}$ watts/DN) so **P = 5.0x10$^{-10}$ watts.**


You can turn your camera into a scientific instrument by calibrating it against an object of known brightness! The simplest calibration involves working with grayscale images. The goal is to convert the grayscale units (256 levels) into increments in terms of the physical brightness unit of Lux.

| Object | Lux |
|---|---|
| Sun (-26.7m) | 130,000 |
| Full Moon (-12.5m) | 0.267 |
| Venus at Brightest (-4.3) | 0.000139 |
| Sirius (-1.4m) | 0.0000098 |
| Faint Star (+6m) | 0.0000000105 |

Take a photograph of one of the calibration objects (not the sun!), such as the image of the moon to the left. We now have to determine what the exact values are for each of the image pixels that covers the moon disk. To do this, we have to read the gif or jpeg image file of the moon to extract these numbers. This can easily be done by visiting the Harvard/Smithsonian Center for Astrophysics MicroObservatory (http://mo-www.harvard.edu/MicroObservatory/) and downloading their Image 2.2 software.

**Step 1** – Follow the directions to install the program on your PC,
**Step 2** - Run the program by clicking the ‘run’ icon.
**Step 3** - Following a setup screen, it will open a second ‘work area’ screen.
**Step 4** - Click on the top menu bar and open your moon image file.
**Step 5** - Scrolling your mouse cursor over your moon image, the pixel values will appear in the top-right information box.

**Correcting for background light:**
**Problem 1** – A) Select 10 pixels in the ‘black’ portion of the picture and compute the average pixel value, B. B) Select 10 pixels across the moon disk and compute the average pixel value,M. C) what is the background-corrected brightness of the moon D = M-B?

**Computing angular resolution of image pixels:**
**Problem 2** – The diameter of the moon is 30 arcseconds. What is A) the diameter of the moon in pixels? B) The resolution of the image in arcseconds per pixel? C) The area of the moon, X, in arcseconds$^2$? D)The area of the moon, N, in pixels?

Refer to the sample image above, taken with a Nikon 3000D camera, using a 200 mm lens, with an exposure speed of 1/25 sec at F/32, and a film speed setting of ISO 100

**Correcting for background light:**
**Problem 1** – A) Select 10 pixels in the 'black' portion of the picture and compute the average pixel value, B. B) Select 10 pixels across the moon disk and compute the average pixel value,M. C) what is the background-corrected brightness of the moon D = M-B?

Answer:
A) 40, 40, 40, 40, 40, 40, 40, 40, 40, 40 so **B = 40 DNs**.
B) 212, 212, 169, 169, 170, 255, 170, 169, 212, 212 so **M = 178 DNs**
C) D = 178 – 40 so **C = 138 DNs**.

**Computing angular resolution of image pixels:**
**Problem 2** – The diameter of the moon is 30 arcseconds. What is A) the diameter of the moon in pixels? B) The resolution of the image in arcseconds per pixel? C) The area of the moon, A, in arcseconds$^2$? D)The area of the moon, N, in pixels?

Answer:
A) Horizontal = 149-82 , H=67 pixels, Vertical=116-47 so V = 69 pixels so average diameter is **68 pixels**.
B) R = 30 arcsec/68 pixels , **R = 0.4 arcsec/pixel**.
C) A = 3.14 x $(30/2)^2$ so **A = 707 arcsec$^2$**.
D) N = 3.14 x $(68/2)^2$ so **N = 3,630 pixels**.

The goal is to convert the grayscale units (256 levels) into increments in terms of the physical brightness unit of Lux.

1 Lux = 1 Lumen/meter$^2$

= 0.0015 watts/meter$^2$ at 550 nm

| Object | Lux |
|---|---|
| Sun (-26.7m) | 130,000 |
| Full Moon (-12.5m) | 0.267 |
| Venus at Brightest (-4.3) | 0.000139 |
| Sirius (-1.4m) | 0.0000098 |
| Faint Star (+6m) | 0.0000000105 |

We have determined the diameter of the moon, Dp, in pixels, the total area, N, in pixels, the total area, A, in arcseconds$^2$, and the average moon brightness in DNs/pixel, Bp, in DNs, Example from above image:

Dp = 34 pixels N = 3,630 pixels, A = 707 arcseconds$^2$ Bp = 138 DNs/pixel

We move to the next step which is to relate the DN values to a physical light brightness unit.

**Problem 1** – From the average DN per pixel and the moon area in pixels, what is T, the total number of DNs for all the moon pixels combined?

**Problem 2** – From the above table, what is the total flux, F, of light from the full moon in A) Lux and B) watts/meter$^2$?

**Determing the three calibration constants:**

**Problem 3** – What is the calibration factor for this camera A) Ca = F/(AT) in watts/meter$^2$/arcsec$^2$? ; B) Cp=F/(NT) in watts/meter$^2$/pixel? and C) Cl=Fl/N in Lux/Dn/Pixel?

**Problem 4** – You measure a star image as a total of 1500 DN in a total of 6 pixels. What is its brightness in A) watts/meter$^2$? B) Lux?

**Problem 1** – From the average DN per pixel and the moon area in pixels, what is T, the total number of DNs for all the moon pixels combined?

Answer: There are N=3,630 pixels covering the moon, with an average brightness of Bp = 138 DN/pixel, so T = 3,630 pixels x (138 DN/pixel) and so **T = 500,940 DN**.

**Problem 2** – From the above table, what is the total flux, F, of light from the full moon in A) Lux and B) watts/meter$^2$?

Answer: A) **Fl = 0.267 Lux**

B) Fw = 0.267 Lux x (0.0015 watts meter$^2$/ 1 lux)

**Fw = 0.0004 watts/m$^2$**

**Problem 3** – What is the calibration factor for this camera A) Ca = F/AT in watts/DN/meter$^2$/arcsec$^2$? , B) Cp=F/NT in watts/DN/meter$^2$/pixel? C) Cl=Fl/NT in Lux/Dn/Pixel?

Answer:

A) Ca = 0.0004/(707x500940) so **Ca = 1.1x10$^{-12}$ watts/DN/meter$^2$/arcsec$^2$**

B) Cp = 0.0004/(3630x500940) so **Cp = 2.2x10$^{-13}$ watts/DN/meter$^2$/pixel**.

C) Cl = 0.267 / (3630x500940) so **Cl = 1.5x10$^{-10}$ Lux/Dn/pixel**

**Problem 4** – You measure a star image as a total of 1500 DN in a total of 6 pixels. What is its brightness in A) watts/meter$^2$? B) Lux?

Answer: A) Use the calibration constant Cp then

B = 1500 DN x 6 pixels x (2.2x10$^{-13}$ watts/DN/meter$^2$/pixel**)**

**B = 2.0x10$^{-9}$ watts/meter$^2$**

B) Use the calibration constant Cl, then

B = 1500 DN x 6 pixels x (1.5x10$^{-10}$ Lux/DN/pixel**)**

**B = 1.4x10$^{-7}$ Lux**

Common digital cameras use CCD chips whose pixels count the arriving photons. The counts are converted into a 256-level grayscale representation that is used in the storage of the image as a gif or jpeg file. We can use the calibration and photography information to determine how many photons correspond to a change by 1 DN in the 256-DN grayscale number.

This moon image was taken by a Nikon d3000 camera with a focal length of 200 mm, at F/32, with ISO=100 and an exposure speed of 1/25 sec.

We have determined the diameter of the moon, Dp, in pixels, the total area, N, in pixels, the total area, A, in arcseconds$^2$, and the average moon brightness in DNs/pixel, Bp:

**Dp = 34 pixels** **A = 707 arcseconds$^2$**
**N = 3,630 pixels**, **Bp = 138 DNs/pixel**

We have also determined for this lunar image the various calibration constants

**Ca = $1.1 \times 10^{-12}$ watts/DN/meter$^2$/arcsec$^2$**
**Cp = $2.2 \times 10^{-13}$ watts/DN/meter$^2$/pixel**.
**Cl = $1.5 \times 10^{-10}$ Lux/Dn/pixel**

**Problem 1** – The lens diameter is determined from its focal length L, and f-stop f, according to D = L/f. What is the area of the lens, in square meters, used to create the above photo of the Moon?

**Problem 2** – The energy of a single photon of light at visible wavelengths (550 nm) is about Ep= $3.7 \times 10^{-19}$ Joules. If 1 watt = 1 Joule/ 1 second, to two significant figures, what was the rate, F, at which photons were falling onto one pixel on the disk of the full moon through a lens with an area of A?

**Problem 3** – During the time that the exposure was being made, how many photons entered a single pixel?

**Problem 4** – For this lunar image, how many photons correspond to 1 DN to one significant figure?

**Problem 1** – **Problem 1** – The lens diameter is determined from its focal length L, and f-stop f, according to D = L/f. What is the area of the lens, in square meters, used to create the above photo of the Moon?

Answer: D = 200 mm/32 = 6.3 mm or 0.0063 meters. Then for a circular area, A = $\pi$ $(0.0063/2)^2$ so **A = $3.1\times10^{-5}$ meters$^2$**.

**Problem 2** – The energy of a single photon of light at visible wavelengths (550 nm) is about Ep= $3.7\times10^{-19}$ Joules. If 1 watt = 1 Joule/ 1 second, what was the rate, F, at which photons were falling onto one pixel on the disk of the full moon through a lens with an area of A?

Answer: The brightness of an average pixel in the full moon image is Bp = 138 DNs. Using the constant, Cp, this corresponds to a brightness of
B = 138 x $2.2\times10^{-13}$ watts/DN/meter$^2$ so B = $3.0\times10^{-11}$ watts/meter$^2$.

The lens collecting area is A = $3.1\times10^{-5}$ meters$^2$

F = (BxA) / Ep so
F = $(3.0\times10^{-11})\times(3.1\times10^{-5})/(3.7\times10^{-19})$
F = 2513
**F = 2500 photons/sec to 2 SF**

**Problem 3** – During the time that the exposure was being made, how many photons entered a single pixel?

Answer: The exposure time was 1/25 sec, so the number of photons producing the average pixel illumination on the lunar disk (138 DN) was n = 2,500 photons/sec x 1/25 sec so **n = 100 photons**.

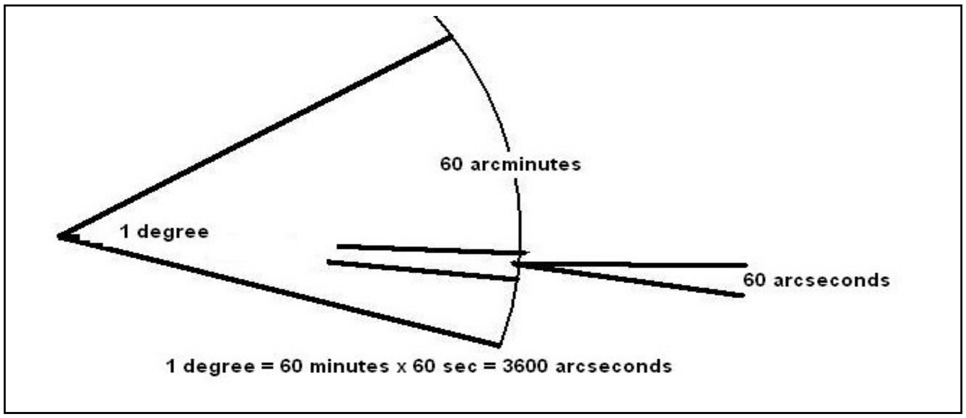
**Problem 4** – For this lunar image, how many photons correspond to 1 DN?

Answer: 100 photons/138 DN so 1 DN = 0.72 photons,

and to 1 SF this is just **1 photon**

# Angular Measure in Astronomy

The easiest, and most basic, unit of measure in astronomy is the angular degree. Because the distances to objects in the sky are not directly measurable, a photograph of the sky will only indicate how large, or far apart, objects are in terms of degrees, or fractions of degrees. It is a basic fact in angle mensuration in geometry, that 1 angular degree (or arc-degree) can be split into 60 arc-minutes of angle, and that 1 arc-minute equals 60 arc-seconds. A full degree is then equal to 60 x 60 = 3,600 'arcseconds'. High-precision astronomy also uses the unit of milliarcsecond to represent angles as small as 0.001 arcseconds and microarcseconds to equal 0.000001 arcseconds.




**Problem 1** – The moon has a diameter of 0.5 degrees (a physical size of 3,474 km) A telescope sees a crater 1 arcsecond across. What is its diameter in meters?

**Problem 2** – A photograph has an image scale of 10 arcseconds/pixel. If the image has a size of 512 x 512 pixels, what is the image field-of-view in degrees?

**Problem 3** – An astronomer wants to photograph the Orion Nebula (M-42) with an electronic camera with a CCD format of 4096x4096 pixels. If the nebula has a diameter of 85 arcminutes, then what is the resolution of the camera in arcseconds/pixel when the nebula fills the entire field-of-view?

**Problem 4** – An electronic camera is used to photograph the Whirlpool Galaxy, M-51, which has a diameter of 11.2 arcminutes. The image will have 1024x1024 pixels. What is the resolution of the camera, in arcseconds/pixel, when the galaxy fills the entire field-of-view?

**Problem 5** – The angular diameter of Mars from Earth is about 25 arcseconds. This corresponds to a linear size of 6,800 km. The Mars Reconnaissance Orbiter's HiRISE camera, in orbit around Mars, can see details as small as 1 meter. What is the angular resolution of the camera in microarcseconds as viewed from Earth?

**Problem 6** – The Hubble Space Telescope can resolve details as small as 46 milliarcseconds. At the distance of the Moon, how large a crater could it resolve, in meters?

**Problem 1** – Answer: 0.5 degrees x 3600 arcsec/degree = 1800 arcseconds. Using proportions 1/1800 = x/3474 so X = 3474/1800 = **1.9 kilometers**.

**Problem 2** –Answer: 512 pixels x 10 arcsec/pixel x 1 degree/3600 arcseconds = 5120 arcseconds /3600 = 1.4 degrees, so the image is **1.4 x 1.4 degrees**.

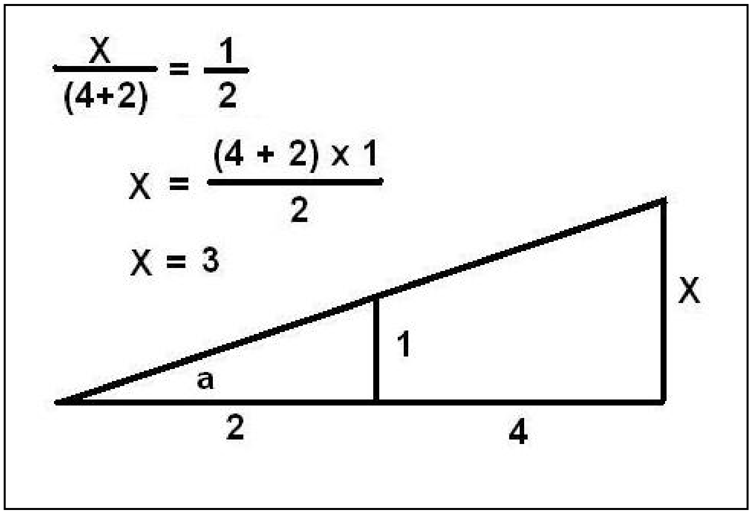
**Problem 3** –Answer: 85 arcminutes x 60 arcsec/arcmin = 5,100 arcseconds. This corresponds to 4096 pixels so the scale is 5,100 arcsec/4096 pixels = **1.2 arcsec/pixel.**

**Problem 4** –Answer: 11.2 arcminutes x 60 arcsec/arcmin = 672 arcsec. This equals 1024 opixels so the scale is 672/1024 = **0.656 arcsec/pixel**.

**Problem 5** –Answer: 25 arcsec = 6800 km so 1 arcsec = 6800 km/25 = 272 km from Earth. For 1-meter resolution at Earth, the angular scale would have to be 1 sec x 1m/272000m = 0.0000037 arcseconds or **3.7 microarcseconds**.

**Problem 6** – Answer: From Problem-1, 1 arcsecond = 1.9 kilometers. By proportions, 0.046 arcsec/1 arcsec = x/1.9 km so X = 0.046 x 1.9 km = **0.087 kilometers or 87 meters.**

The picture below was taken by the Cassini spacecraft orbiting Saturn. It is of the satellite Phoebe, which from Earth subtends an angular size of about 32 milliarcsec. The smallest crater, about 1 km across, would subtend about 160 microarcseconds as seen from Earth.

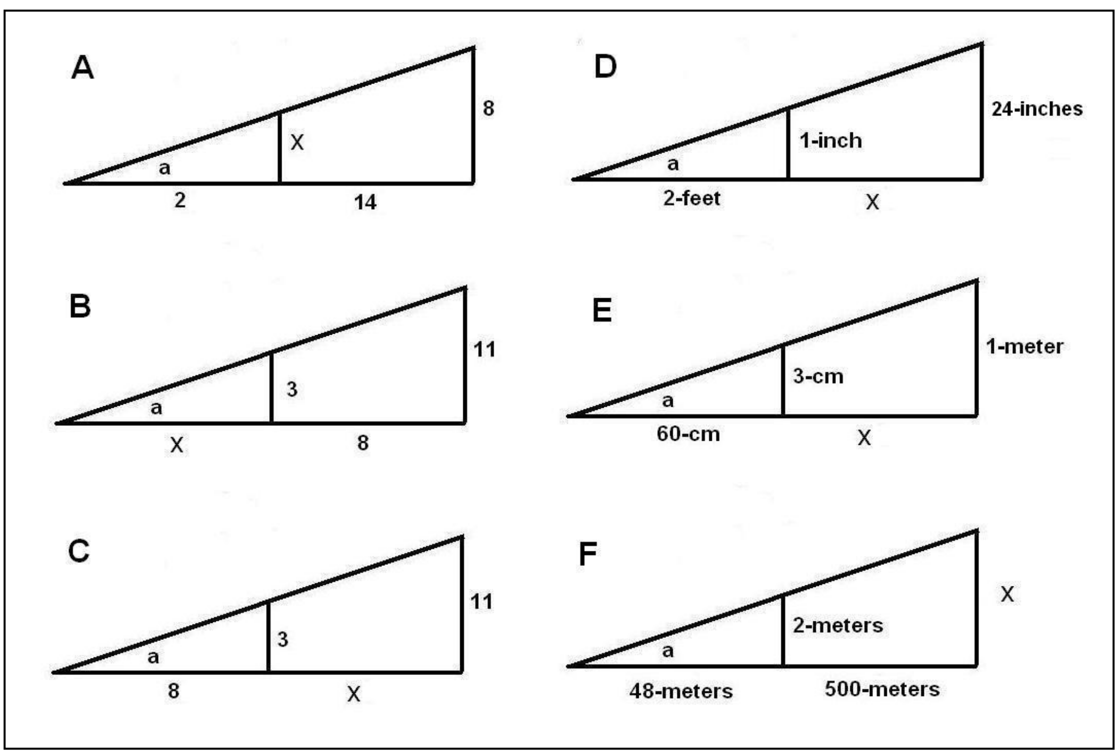



The corresponding sides of similar triangles are proportional to one another as the illustration to the left shows. Because the vertex angle of the triangles are identical in measure, two objects at different distances from the vertex will subtend the same angle, **a** . The corresponding side to 'X' is '1' and the corresponding side to '2' is the combined length of '2+4'.

**Problem 1:** Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

**Problem 2:** Which triangles must have the same measure for the indicated angle **a** ?

**Problem 3:** The sun is 400 times the diameter of the moon. Explain why they appear to have about the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?



# Answer Key

**Problem 1:** Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

A) X / 2 = 8 / 16 so **X = 1**

B) 3 / X = 11 / (X+8) so 3 (X + 8) = 11 X ; 3X + 24 = 11 X ; 24 = 8X and so **X = 3.**

C) 3 / 8 = 11 / (x + 8) so 3 ( x + 8) = 88 ; 3X + 24 = 88 ; 3X = 64 and so **X = 21 1/3**

D) 1-inch / 2-feet = 24 inches / ( D + 2 feet); First convert all units to inches;
1 / 24 = 24 / ( D + 24); then solve (D + 24) = 24x24 so D = 576 - 24;
**D = 552 inches or 46 feet**.

E) 3 cm / 60 cm = 1 meter / ( X + 60 cm). 3/60 = 1 meter / ( X + 0.6 m) then
3( X + 0.60) = 60 ; 3X + 1.8 = 60 ; 3X = 58.2 meters so **X = 19.4 meters.**

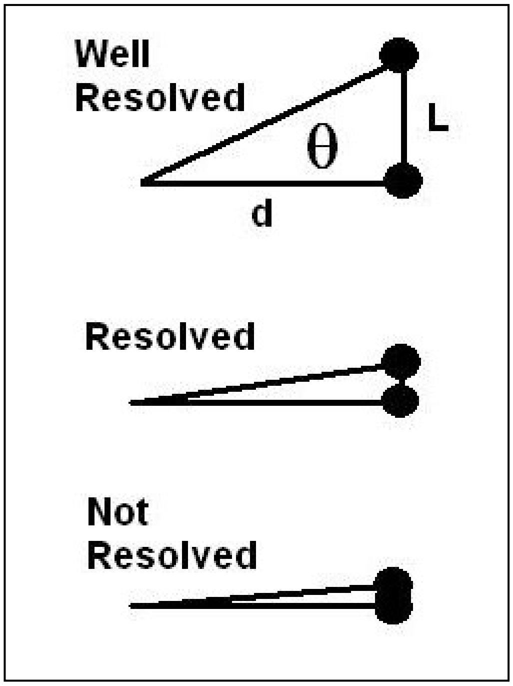
F) 2 meters / 48 meters = X / 548 meters ; 1/24 = X/548 ; X = 548 / 24 ; so **X = 22.8.**

**Problem 2:** Which triangles must have the same measure for the indicated angle **a** ?

Answer: Because the triangle (D) has the side proportion 1-inch /24-inches = 1/24 and triangle (F) has the side proportion 2 meters / 48 meters = 1/24 these two triangles, **D and F, have the same angle measurement for angle a**

**Problem 3:** The Sun is 400 times the diameter of the Moon. Explain why they appear to have the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?

Answer: From one of our similar triangles, the long vertical side would represent the diameter of the sun; the short vertical side would represent the diameter of the moon; the angle **a** is the same for both the sun and moon if the distance to the sun from Earth were 400x farther than the distance of the moon from Earth. Since the lunar distance is 384,000 kilometers, the sun must be at a distance of 154 million kilometers, which is close to the number given.




Satellites are often designed to photograph or 'image' the surface of Earth, the Moon or other celestial objects. One of the most basic properties of imaging systems is how well they can resolve details.

The most elementary way to define resolution is in terms of the angle between two closely-spaced objects that can just be distinguished by the imaging system. The figure shows how the angle, $\theta$, changes as the objects are considered well-resolved, resolved or not-resolved. Imaging systems are designed to be resolved for objects separated by a length **L** viewed from a distance of **d**.

From trigonometry, the angle separating two objects is simply $\tan\theta = L/d$. However, for angles much smaller than $1^{o}$, which is a common resolution angle for modern imaging systems, the trigonometric relationship becomes $\theta = L/d$ when $\theta$ is measured in radians. One radian = $57.296^{o}$, and since $1^{o}$ = 3600 arcseconds, therefore, we have $\theta = 206265\ L/d$, where the apparent angular size $\theta$ is now in units of arcseconds when L andd are measured in the same units (meters, kilometers, light years). This is the fundamental formula for determining angular scales in astronomy and remote sensing.

**Problem 1** – The altitude of the imaging satellite is designed to be 350 kilometers. If a biologist wants to study deforestation in plots of land 10-meters across, what will be the minimum angular resolution of the CCD camera system used on the satellite?

**Problem 2** – The Lunar Reconnaissance Orbiter (LRO) operates from a lunar altitude of 60 kilometers. What is the resolution of the CCD imager which can resolve details at a level of 1-meter per pixel?

**Problem 3** – The Solar Dynamics Observatory (SDO) has an imaging system with 1 arcsecond per pixel resolution. At a distance of 150 million kilometers, what is the resolution of this system in kilometers per pixel?

**Problem 1** – The altitude of the imaging satellite is designed to be 350 kilometers. If a biologist wants to study deforestation in plots of land 10-meters across, what will be the minimum angular resolution of the CCD camera system used on the satellite?

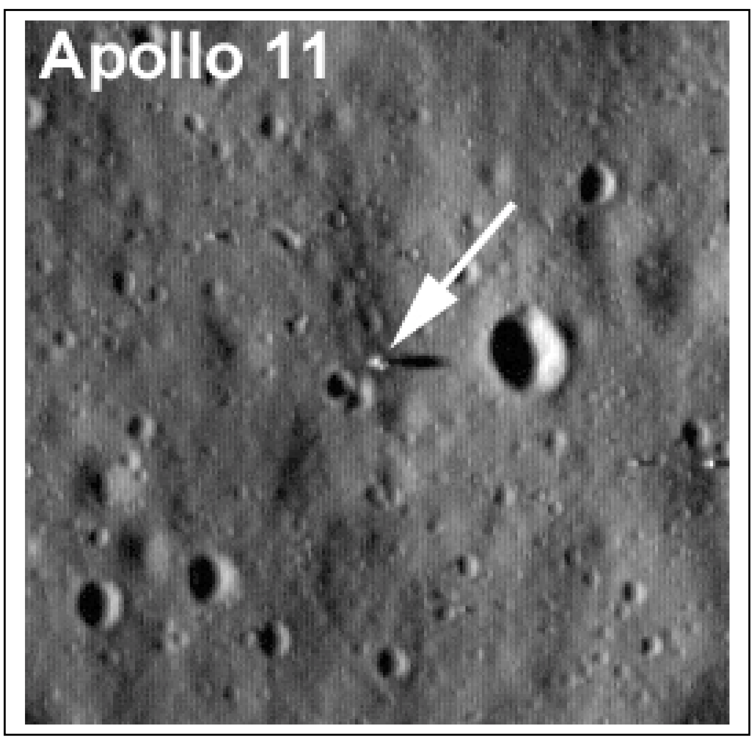
Answer: $\theta$ = 206265 x (10 meters/350000 meters) so $\theta$ **= 6 arcseconds**.

**Problem 2** – The Lunar Reconnissance Orbiter operates from a lunar altitude of 60 kilometers. What is the resolution of the CCD imager which can resolve details at a level of 1-meter per pixel?

Answer: $\theta$ = 206265 x (1 meter/60000 meters) so $\theta$ **= 3 arcseconds**

**Problem 3** – The Solar Dynamics Observatory (SDO) has an imaging system with 1 arcsecond per pixel resolution. At a distance of 150 million kilometers, what is the resolution of this system in kilometers per pixel?

Answer: $\theta$ = 206265 x (L/d) so
L = 150 million kilometers x (1 arcsecond/206265 arcseconds)
**L = 727 kilometers per pixel**



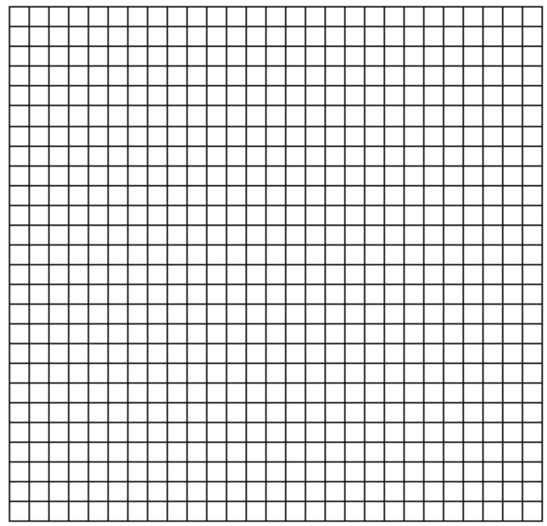

This LRO image was obtained in 2009 from an altitude of 60 kilometers above the Apollo-11 landing area. The **1-meter** resolution clearly shows the Apollo Landing Module that served as the launch pad for the returning Lunar Excursion Module (LEM) carrying Astronauts Armstrong and Aldrin.

# Resolving the Moon

Although a pair of binoculars or a telescope can see amazing details on the Moon, the human eye is not so gifted!

The lens of the eye is so small, only 2 to 5 millimeters across, that the sky is 'pixelized' into cells that are about one arcminute across. We call this the resolution limit of the eye, or the eye's visual acuity.

One degree of angle measure can be divided into 60 minutes of arc. For an object like the full moon, which is 1/2-degree in diameter, it also measures 30 arcminutes in diameter. This means that, compared to the human eye, the moon can be divided into an image that is 30-pixels in diameter.



**Problem 1** - Convert the following degree measures into their equivalent measure in arcminutes (amin); A) 5 degrees; B) 2/3 degree; C) 15.5 degrees; D) 0.25 degrees

**Problem 2** - Convert the following arcminute measures into their equivalent measure in degrees: A) 15 amin; B) 1/2 amin; C) 120.5 amin; D) 3600 amin.

**Problem 3** - Convert the following area measures in square-degrees into their equivalent measures in square arcminutes ($amin^2$): A) 1.0 $deg^2$; B) 0.25 $deg^2$

**Problem 4** - The figure to the above-left is a telescopic photo of the full moon showing its many details including craters and dark mare. Construct a simulated image of the moon in the grid to the right to represent what the moon would look like at the resolution of the human eye. First sketch the moon on the grid. Then use the three shades; black, light-gray and dark-gray, and fill-in each square with one of the three shades using your sketch as a guide.

**Problem 5** - Why can't the human eye see any craters on the Moon?

# Answer Key

**Problem 1** - Convert the following degree measures into their equivalent measure in arcminutes (amin); A) 5 degrees; B) 2/3 degree; C) 15.5 degrees; D) 0.25 degrees

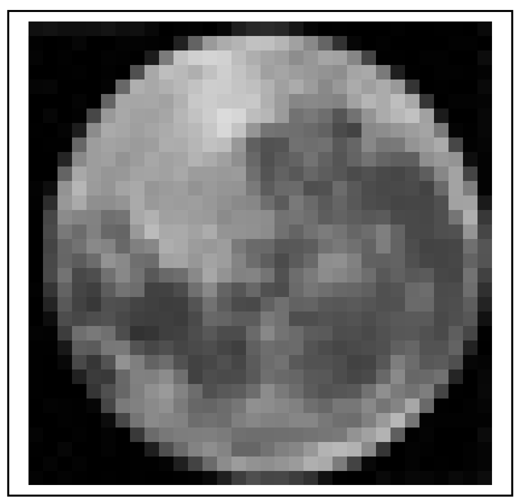
Answer: A) 5 degrees x (60 amin/1 deg) = **300 amin.** B) 2/3 degree x (60 amin/1 deg) = 120/3 = **40 amin**. C) 15.5 degrees x (60 amin/1deg) = **930 amin**; D) 0.25 deg x (60 amin/1 deg) = **15 amin.**

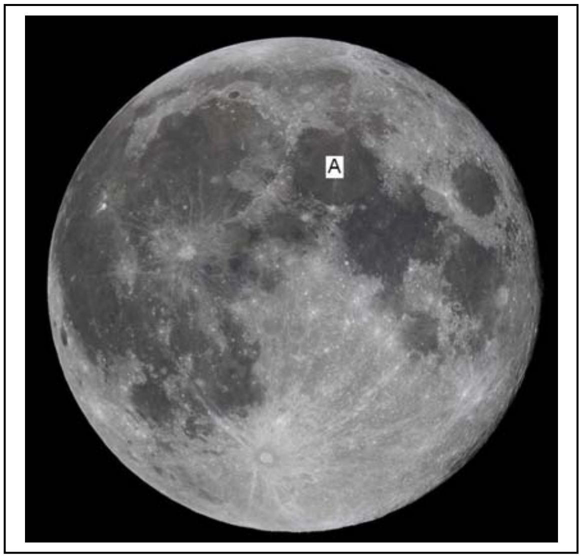
**Problem 2** - Convert the following arcminute measures into their equivalent measure in degrees: A) 15 amin; B) 1/2 amin; C) 120.5 amin; D) 3600 amin.
Answer: A) 15 amin x (1 deg/60 amin) = **0.25 deg**. B) 1/2 amin x (1 deg / 60 amin) = **1/120 deg**. C) 120.5 amin x (1 deg/60 amin) = **2.0 deg**. D) 360 amin x (1 deg/60 amin) = **60 deg**.

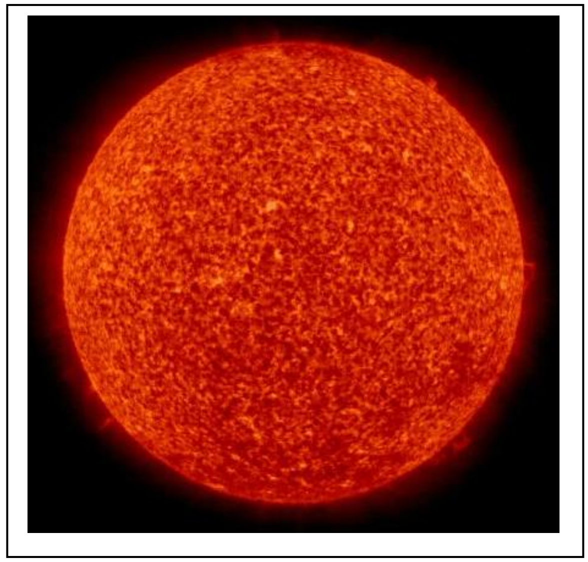
**Problem 3** - Convert the following area measures in square-degrees into their equivalent measures in square arcminutes ($amin^2$): A) 1.0 $deg^2$; B) 0.25 $deg^2$
Answer; A) 1.0 $deg^2$ x ( 60 amin/1 deg) x (60 amin/1 deg) = **3600 $amin^2$**. B) 0.25 $deg^2$ x (60 amin/1 deg) x (60 amin/1 deg) = 0.25 x 3600 = **900 $amin^2$**.

**Problem 4** - See the image below which has been pixelized to the grid resolution. How well did your version match the image on the right?

**Problem 5** - Why can't the human eye see any craters on the Moon? Answer: The human eye can only see details 1 arcminute across and this is too low a resolution to see even the largest craters.





The Sun (Diameter = 1,400,000 km) and Moon (Diameter = 3,476 km) have very different physical diameters in kilometers, but in the sky they can appear to be nearly the same size. Astronomers use the angular measure of arcseconds (asec) to measure the apparent sizes of most astronomical objects. (1 degree equals 60 arcminutes, and 1 arcminute equals 60 arcseconds). The photos above show the Sun and Moon at a time when their angular diameters were both about 1,865 arcseconds.

**Problem 1** - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter?

**Problem 2** - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon?

**Problem 3** - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon?

**Problem 4** - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle?

**Problem 5** - What is the area of Mare Serenitatis in square kilometers?

**Problem 6** - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun?

# Answer Key

**Problem 1** - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter? Answer: Moon diameter = 65 mm and sun diameter = 61 mm so the lunar image scale is 1,865 asec/65mm = **28.7 asec/mm** and the solar scale is 1865 asec/61 mm = **30.6 asec/mm.**

**Problem 2** - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon? Answer: the smallest feature is about 0.5 mm or 0.5 x 28.7 asec/mm = **14.4 asec for the Moon** and 0.5 x 30.6 asec/mm = **15.3 asec for the Sun**.

**Problem 3** - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon? Answer: The diameter of the mare is 1 centimeter, so the radius is 5 mm or 5 mm x 28.7 asec/mm = 143.5 asec. Assuming a circle, the area is $A = \pi \times (143.5 \text{ asec})^2$ = **64,700 $asec^2$.**

Note that a 1 millimeter measurement uncertainty (4 mm = 115 asec vs 6 mm = 172 asec) corresponds to a range of areas from 41, 539 to 92,923 $asec^2$ !

**Problem 4** - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle? Answer: The angular scale at the sun would correspond to 400 x 1.9 km = **760 kilometers** per arcsecond.

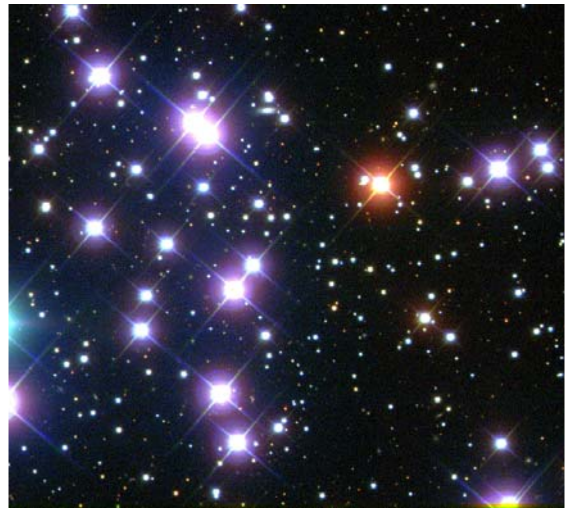
**Problem 5** - What is the area of Mare Serenitatis in square kilometers? Answer: We have to convert from square arcseconds to square kilometers using a two-step unit conversion 'ladder'.

64,700 $asec^2$ x ( 1.9 km/asec) x (1.9 km/asec) = **233,600 $km^2$.**

**Problem 6** - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun? Answer: The angular area is 400-times further away, so we have to use the scaling of 760 kilometers/asec deduced in Problem 4. The unit conversion for the solar area becomes:

64,700 $asec^2$ x (760 km/asec) x (760 km/asec) = **37,400,000,000 $km^2$**.

# Angular Size : The Moon and Stars



Although many astronomical objects may have the same angular size, most are at vastly different distance from Earth, so their actual sizes are very different. If your friends were standing 200 meters away from you, they would appear very small, even though they are as big as you are!

The pictures show the Moon (d = 384,000 km) and the star cluster Messier-34 (d = 1,400 light years). The star cluster photo was taken by the Sloan Digital Sky Survey, and although the cluster appears the same size as the Moon in the sky, its stars are vastly further apart than the diameter of the Moon!

**In the problems below, round all answers to one significant figure.**

**Problem 1** - The images are copied to the same scale. Use a metric ruler to measure the diameter of the Moon in millimeters. If the diameter of the moon is 1,900 arcseconds, what is the scale of the images in arcseconds per millimeter?

**Problem 2** - The relationship between angular size, **Θ**, and actual size, **L**, and distance, **D**, is given by the formula:

$$L = \frac{\Theta}{206{,}265} D$$

Where Θ is measured in arcseconds, and L and D are both given in the same units of length or distance (e.g. meters, kilometers, light years). A) In the image of the Moon, what does 1 arcsecond correspond to in kilometers? B) In the image of M-34, what does 1 arcsecond correspond to in light years?

**Problem 3** - What is the smallest detail you can see in the Moon image in A) arcseconds? B) kilometers?

**Problem 4** - What is the smallest star separation you can measure in Messier-34 in among the brightest stars in A) arcseconds? B) Light years?

# Answer Key

**Problem 1** - The images are copied to the same scale. Use a metric ruler to measure the diameter of the Moon in millimeters. If the diameter of the moon is 1,900 arcseconds, what is the scale of the images in arcseconds per millimeter? Answer: The diameter of the Moon is about 64 millimeters, and since this corresponds to 1,900 arcseconds, the scale is 1,900 asec/64 mm = 29.68 or **30 asec/mm**.

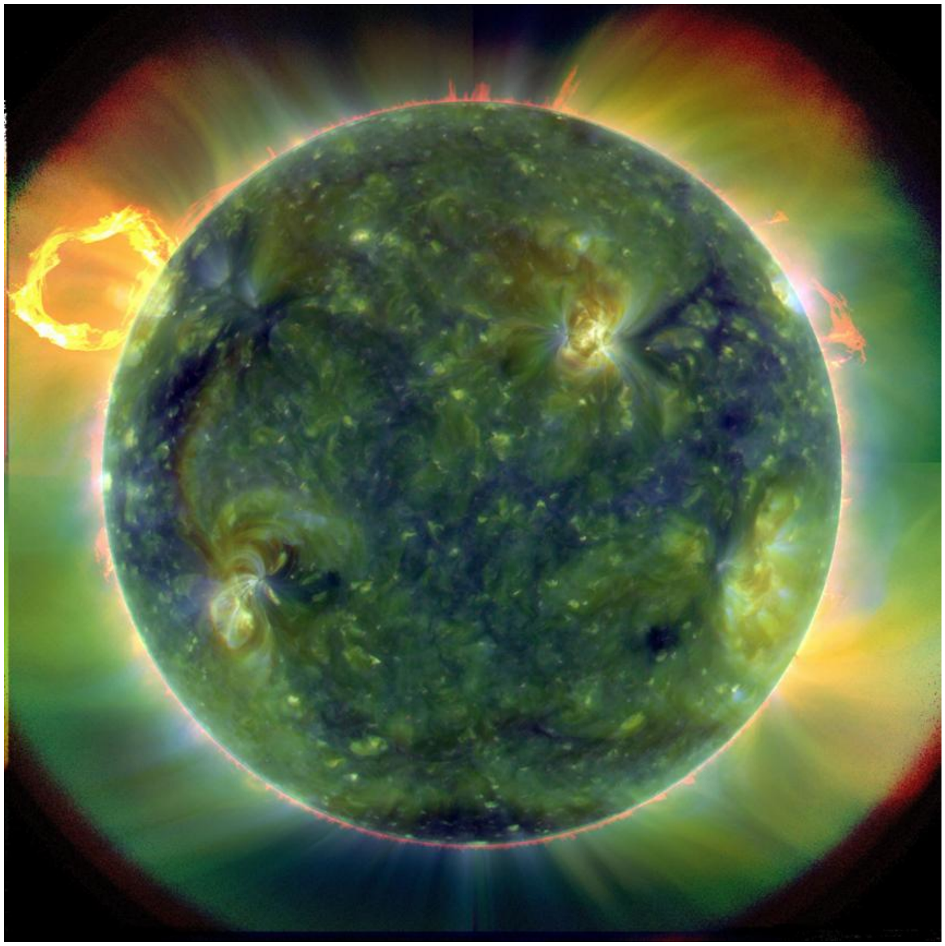
**Problem 2** - The relationship between angular size, **Θ**, and actual size, **L**, and distance, **D**, is given by the formula:

$$L = \frac{\Theta}{206,265} D$$

Where Θ is measured in arcseconds, and L and D are both given in the same units of length or distance ( e.g. meters, kilometers, light years). A) In the image of the Moon, what does 1 arcsecond correspond to in kilometers? B) In the image of M-34, what does 1 arcsecond correspond to in light years? Answer: A) For the Moon: L = 1 arcsec/206265 x (384,000 km) = 1.86 or **2.0 kilometers**. B) For the cluster, L = 1 arcsec/206265 x (1,400 light years) = **0.007 light years**.

**Problem 3** - What is the smallest detail you can see in the Moon image in A) arcseconds? B) kilometers? Answer: A) About 1 millimeter, which corresponds to **1.0 arcsec**. B) One arcsec corresponds to **2.0 kilometers**.

**Problem 4** - What is the smallest star separation you can measure in Messier-34 among the brightest stars in A) arcseconds? B) Light years? Answer: A) Students may find that some of the bright stars are about 3 millimeters apart, which corresponds to 3 mm x 30 asec/mm = **90 arcseconds**. B) At the distance of the cluster, 1 asec = 0.007 light years, so 90 asec corresponds to 90 x (0.007 light years/asec) = 0.63 or **0.6 light years** to 1 significant figure.



On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the Sun. The image above shows a full-disk, multi-wavelength, extreme ultraviolet image of the sun taken by SDO on March 30, 2010. False colors trace different gas temperatures. Black indicates very low temperatures near 10,000 Kelvin close to the solar surface (photosphere). Reds are relatively cool plasma heated to 60,000 Kelvin (100,000 F); blues, greens and white are hotter plasma with temperatures greater than 1 million Kelvin (2,000,000 F) located in the sun's outer layer (atmosphere) called the corona.

**Problem 1** – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

**Problem 2** – What are the smallest features you can find on this image, and how large are they in kilometers? In comparison to Earth, how big are these features if the radius of Earth is 6378 kilometers?

**Problem 3** – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

**Problem 1** – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Answer: The diameter of the Sun is 98 millimeters, so the scale is 1,380,000 km/98 mm = **14,000 km/mm**.

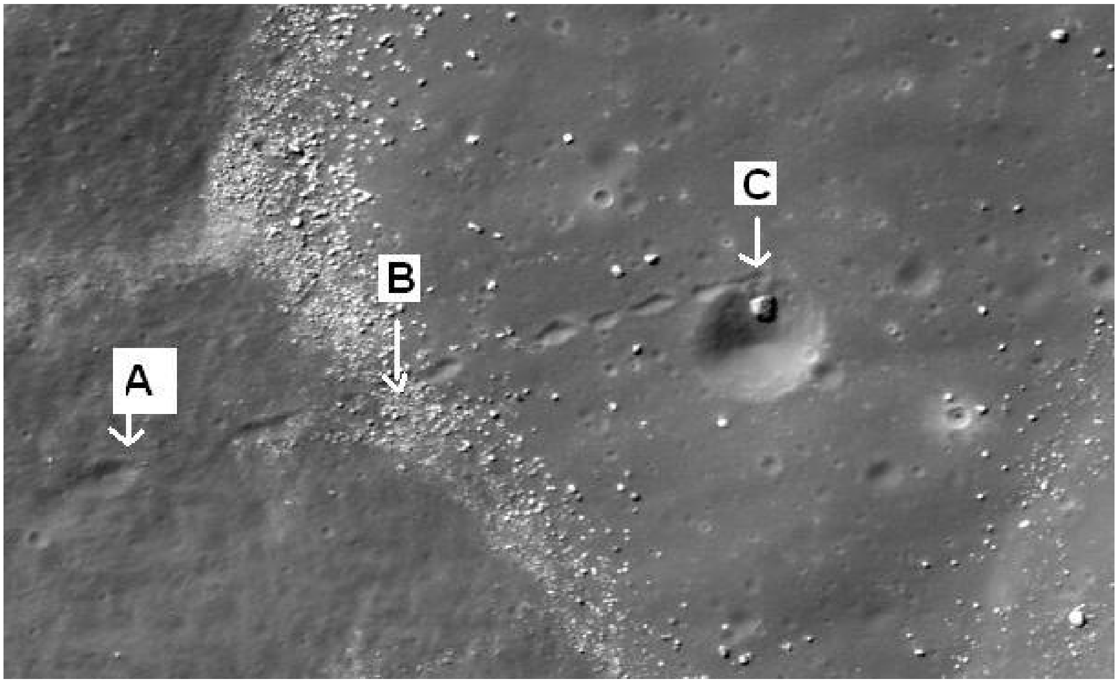
**Problem 2** – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

Answer: **Students should see numerous bright points freckling the surface, the smallest of these are about 0.5 mm across or 7,000 km. This is slightly larger than ½ the diameter of Earth.**

**Problem 3** – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

Answer: **There are large irregular blotches all across the disk of the sun that are dark blue-black. These are regions where thee is little of the hot coronal gas and only the 'cold' photosphere can be seen. The hottest gas seems to reside in the corona, and in the very small point-like 'microflare' regions that are generally no larger than the size of Earth**.

Note: Microflares were first observed, clearly, by the Hinode satellite between 2007-2009. Some solar physicists believe that these microflares, which erupt violently, are ejecting hot plasma that eventually ends up in the corona to replenish it. Because the corona never disappears, these microflares happen all the time no matter what part of the sunspot cycle is occurring.




This image (LROC MAC M122597190L), taken by the Lunar Reconnaissance Orbiter shows a boulder that has rolled and skipped down hill from the left-hand edge of the image to a ‘hole-in-one’ location in a small crater. The width of the image is 510 meters. To two significant figure accuracy in your answers:

**Problem 1** – Mark those portions of the path where the boulder must have A) rolled and B) skipped, in order to cover the distance.

**Problem 2** – What is the scale of this image in meters/millimeter?

**Problem 3** – Assuming that the boulder is roughly spherical in shape with a density of D=3000 kg/$m^3$, what is A) The diameter of the boulder? B) The mass of the boulder in tons?

**Problem 4** – How far did the boulder skip and roll from A) Point A to B? B) From Point B to C?

**Problem 1** – Mark those portions of the path where the boulder must have A) rolled and B) skipped, in order to cover the distance.

Answer: Wherever you see a track is where the boulder was rolling in contact with the lunar regolith. Wherever you see gaps in the track is where the boulder was skipping through the air.

**Problem 2** – What is the scale of this image in meters/mm?

Answer: The width of the image is about 142 mm, so the scale is 510 meters/142mm = **3.6 meters/mm**.

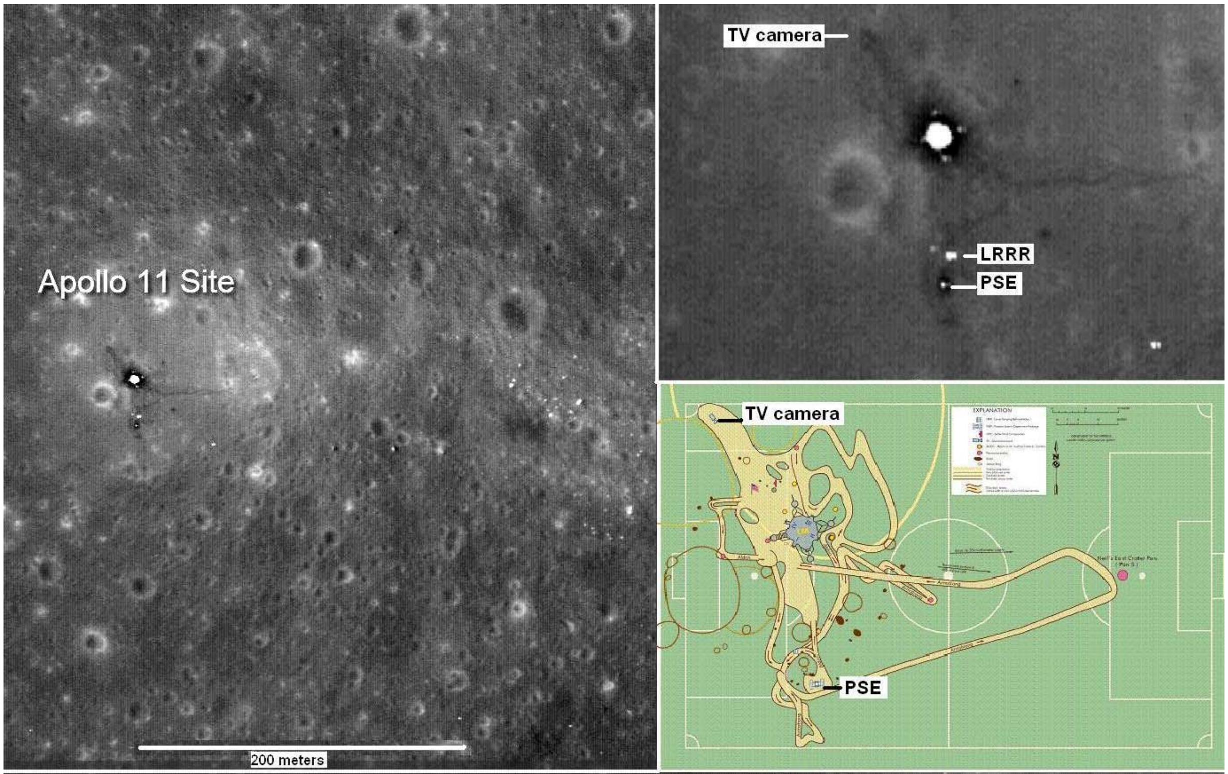
**Problem 3** – Assuming that the boulder is roughly spherical in shape, with a density of D=3000 $kg/m^3$, what is A) The diameter of the boulder? B) The mass of the boulder in tons?

Answer; A) The measured diameter is 3mm so the physical size is 3 mm x 3.6 meters/mm = **11 meters**.

B) The volume is $V= 4/3 (3.14)(5.5)^3 = 700\ meter^3$. The mass is then M = V x D so M = 700 x 3000 ; M = 2,100,000 kg ; M = **2,100 tons**.

**Problem 4** – How far did the boulder skip and roll from A) Point A to B? B) From Point B to C?

Answer: A) L(AB) = 35 mm x (3.6m/mm) = **130 meters**.
B) L(BC) = 50 mm x (3.6 m/mm) = **180 meters**.




The Lunar Reconnaissance Orbiter (LRO) recently imaged the Apollo-11 landing area at high-resolution and obtained the image above (Top left). An enlargement of the area is shown in the inset (Top right) and a rough map of the area is also shown (bottom right). The landing pad with three of its four foot-pads is clearly seen, together with the Lunar Ranging Retro Reflector experiment (LRRR), the Passive Seismic Experiment (PSE) and the TV camera area. The additional white spots seen in the left image are boulders from the West Crater located just off the right edge of the image.

**Problem 1** - Using a millimeter ruler and the '200 meter' metric bar, what is the scale of each of the two images and the map?

**Problem 2** - About what is the distance between the TV camera and the PSE?

**Problem 3** - From the left-hand image; A) What is the height and width of the field? B) What is the area of the field in square-kilometers?

**Problem 4** - In the left-hand image, what is the diameter, in meters, of A) the largest crater, and B) the smallest crater?

**Problem 5** - By counting craters in the left-hand image, what is the surface density of cratering in this region of the moon in units of craters per square kilometer?

**Problem 1** - Using a millimeter ruler and the '200 meter' metric bar, what is the scale of each of the two images and the map?
Answer: On the main image, the length is 43 millimeters so the scale is 200 meters/43 mm = **4.7 meters/mm for the left-hand image.** The distance between the landing pad and the LRRR on this image is 5 millimeters or 5 x 4.7 = 24 meters. In the upper right image, the landing pad and the LRRR are 16 mm apart, so the scale of this image is 24 meters/16mm = **1.5 meters/mm**. The PSE and landing pad are clearly indicated in the map, which the top image says are 20 mm or 20x1.5 = 30 meters apart. On the map, these points are also 20 mm apart, so the scale is also 1.5 meters/mm on the map.

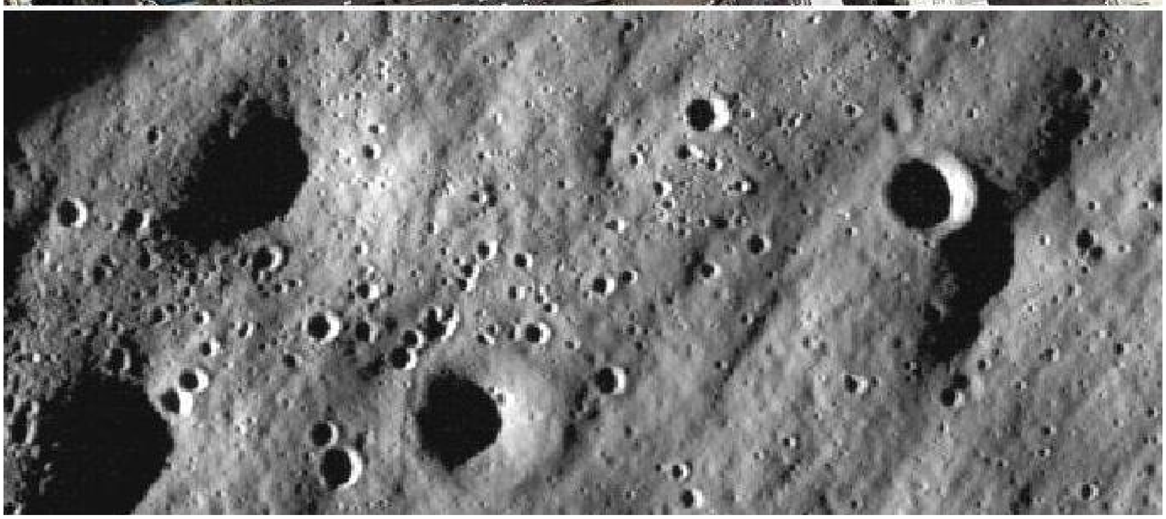
**Problem 2** - About what is the distance between the TV camera and the PSE?
Answer: According to the map, the distance is 38 millimeters or 38 mm x (1.5 m/mm) = **57 meters apart**.

**Problem 3** - From the left-hand image; A) What is the height and width of the field? B) What is the area of the field in square-kilometers?
Answer; A) Height x Width = 101mm x 86mm and for a scale of 4.7 meters/mm this equals **475m x 404m**. B) The are in square kilometers is 0.475 m x 0.404 m= **0.19 km$^2$.**

**Problem 4** - In the left-hand image, what is the diameter, in meters, of A) the largest crater, and B) the smallest crater? Answer: A) The largest circular feature is about 8mm in diameter or **38 meters across**. B) The smallest feature is about 1 millimeter across or **4.7 meters**.

**Problem 5** - By counting craters in the left-hand image, what is the surface density of cratering in this region of the moon in units of craters per square kilometer?
Answer: Depending on the quality of the printed copy, students may count between 20 and 100 craters. Assuming the lower value, the crater density is 20 craters/0.19 km$^2$ = 105 craters/km$^2$. If the PDF file is displayed on the computer screen, a much better contrast is obtained and students should be able to count about 225 craters for a density of 1,200 craters/km$^2$. **Values between 100 and 1000 craters/km$^2$ are acceptable**.

# Mare Nubium and Las Vegas

The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above 700-meter wide image shows Downtown Las Vegas, Nevada (Top - Courtesy of Digital Globe, Inc.), and Mare Nubium (bottom - LRO) at this same resolution.

**Problem 1** - About how big, in meters, are the large, medium and small-sized craters in the LRO image?

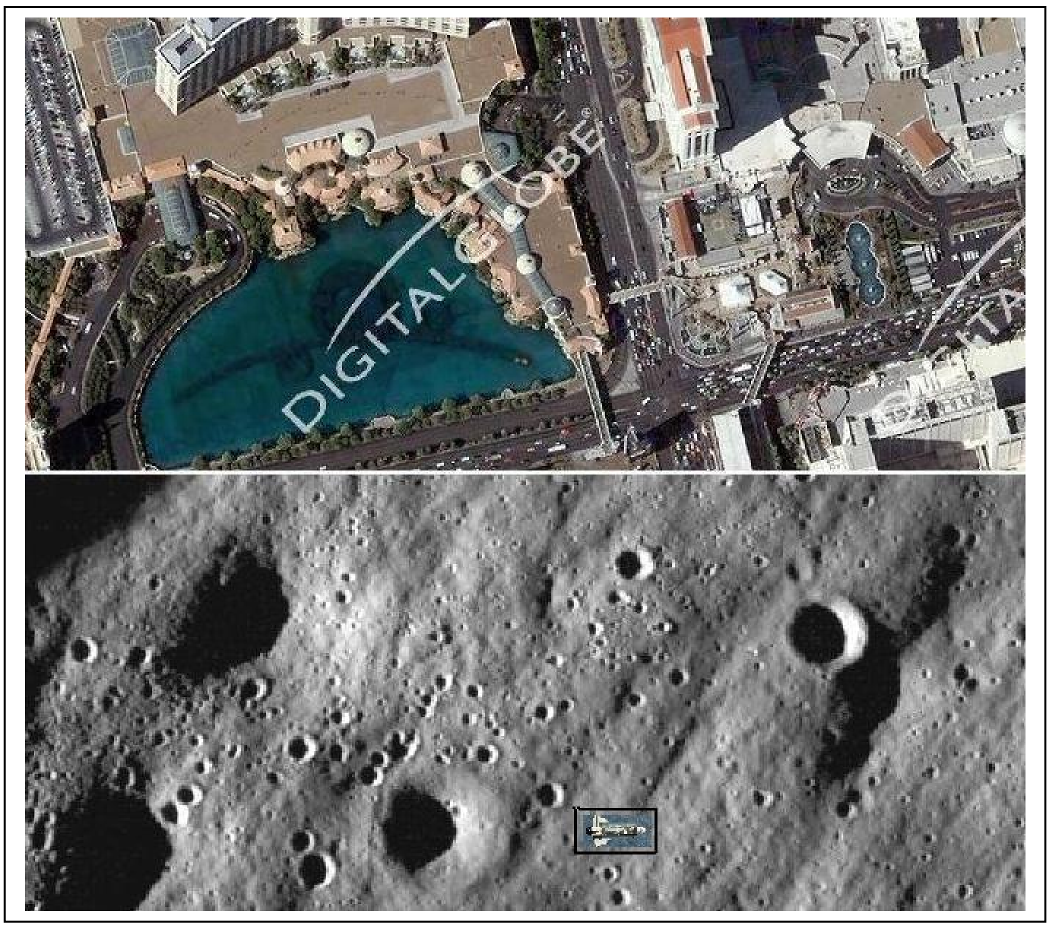
**Problem 2** - How do the large, medium and small-sized craters compare to familiar objects in Downtown Las Vegas, or in your neighborhood?

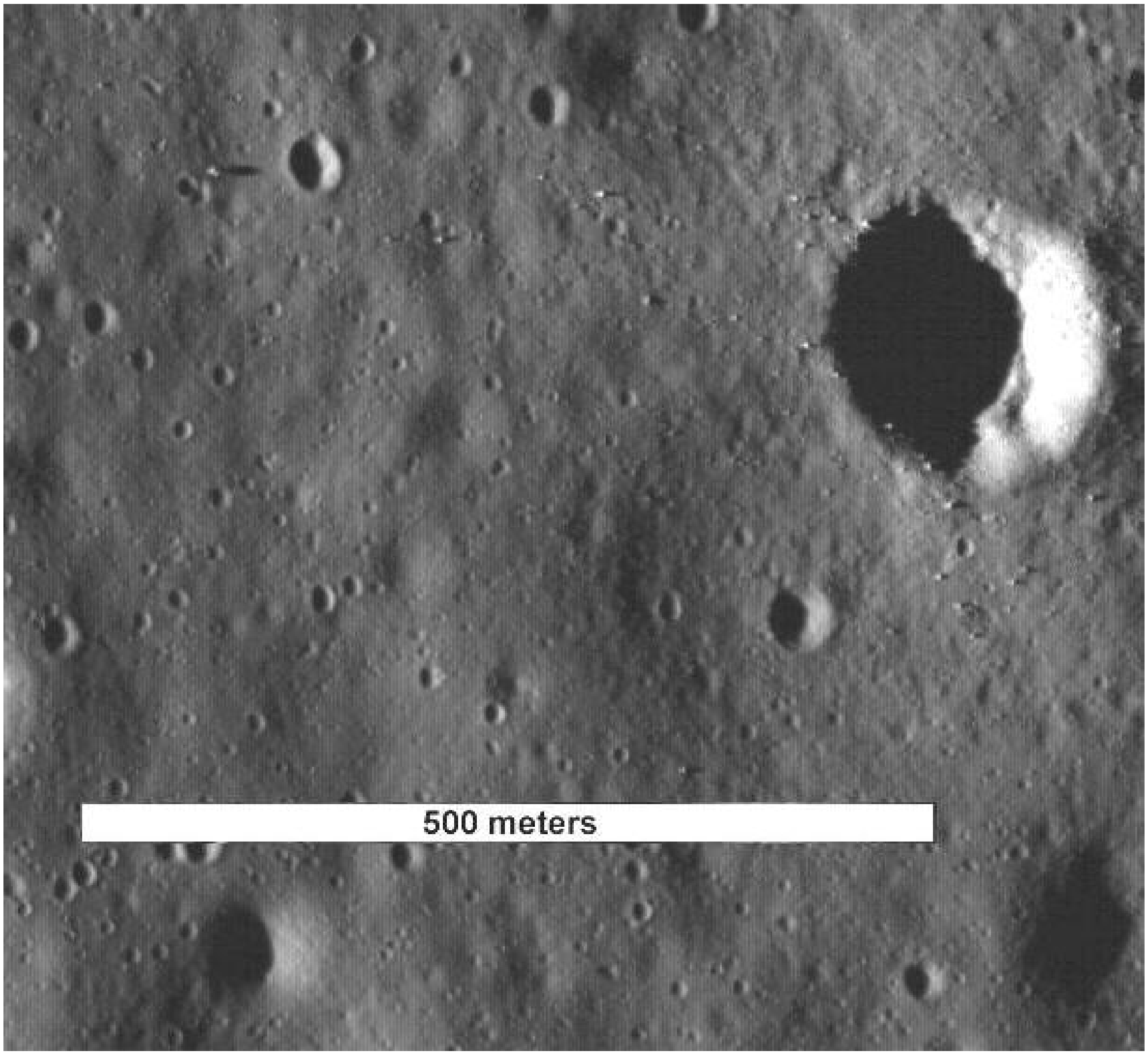
**Problem 3** - The Space Shuttle measures 37 meters long and has a wingspan of 24 meters. Draw a sketch of the Shuttle in the LRO image. Would you be able to see the Space Shuttle on the moon's surface at this resolution scale? (Note that the Space Shuttle is not equipped to travel to the moon and land!).

**Problem 1** - About how big, in meters, are the large, medium and small-sized craters in the LRO image? Answer: the image is 153 millimeters wide so the scale is 700 meters/153 mm = **4.6 meters/mm.** Small craters are about 4-5 meters across; medium craters are about 10 to15 meters across, and the few large craters are about 30 to 100 meters across.

**Problem 2** - How do the large, medium and small-sized craters compare to familiar objects in Downtown Las Vegas, or in your neighborhood? Answer: The small craters are about as wide as your car, mini-van or street. The medium craters are about as wide as large as your house. The big craters are as big as your entire yard or a large Boulevard.

**Problem 3** - The Space Shuttle measures 37 meters long and has a wingspan of 24 meters. Draw a sketch of the Shuttle in the LRO image. Would you be able to see the Space Shuttle on the moon's surface at this resolution scale? (Note that the Space Shuttle is not equipped to travel to the moon and land!). Answer: **The shuttle would be 37 meters/4.6 M/mm = 8 millimeters long by 24/4.6 = 5.2 millimeters wide.** The figure below shows the Shuttle to the same scale as the LRO image. It would occupy about 26 x 17 pixels and be easy to see at the LRO image scale.






The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above image shows a region near the Apollo-11 landing site. The Lunar Module (LM) can be seen from its very long shadow near the large crater in the upper left corner of the image.

**Problem 1** - With a millimeter ruler, determine the scale of this image in meters/mm. What is the total area of this image in square-kilometers?

**Problem 2** - Measure all of the craters larger than or equal to 9 meters and create a histogram of the numbers of the craters. Divide the number of craters in each bin, by the total area of the field, to get $\mathbf{A_c}$ : the Areal Crater Density (craters/km$^2$).

**Problem 3** - The average distance between craters of a given size is found by taking the square-root of the reciprocal of $\mathbf{A_c}$. About what is the average distance between craters with a diameter close to 5 meters?

Problem 1 - With a millimeter ruler, determine the scale of this image in meters/mm. What is the total area of this image in square-kilometers?

Answer: The 500-meter bar is 111 millimeters long so the scale is 500 M/111mm = **4.5 meters/mm.** The image has the dimensions of 149 mm x 136 mm or 670m x 612 m for an area of **0.41 kilometers$^2$.**

**Problem 2** - Measure all of the craters larger than 9 meters and create a histogram of the numbers of the craters. Divide the number of craters in each bin, by the total area of the field, to get $\mathbf{A_C}$ : the Areal Crater Density (craters/km$^2$). Answer: The following table shows an example. Students bin intervals may differ.

| Crater diameter (mm) | Crater diameter (meters) | Number of craters close to this size | Areal Density | **Problem 3** Average distance in kilometers |
|---|---|---|---|---|
| 2 mm | 9 | 70 | 70/0.41 = 171 | 0.08 |
| 4 mm | 18 | 6 | 15 | 0.25 |
| 6 mm | 27 | 3 | 7 | 0.38 |
| 8 mm | 36 | 2 | 5 | 0.44 |
| 10 mm | 45 | 1 | 2 | 0.71 |

Students may extend this table to include craters of 1-mm diameter and also the single, very large crater that is 35mm in diameter. The number of counted craters, especially in the smallest bins, will vary. Student data may be averaged together to improve accuracy in each bin.

**Problem 3** - The average distance between craters of a given size is found by taking the square-root of the reciprocal of $\mathbf{A_C}$. About what is the average distance between craters with a diameter close to 5 meters?
Answer: See above table for tabulated values. Students may also convert the answers to meters. For example, '0.08 km' = 80 meters. Students will need to estimate the Areal Crater Density for craters just below the tabulated threshold of 9 meters. This can be done by estimating the shape of the plotted curve through the points, and extrapolating it to 5 meters. It is also possible to use Excel Spreadsheets by entering the data and plotting the 'scatter plot' with a trendline added. Reasonable values for the Areal Creater Density would range from 171 craters/km$^2$ to 1000 craters/km$^2$ ,which lead to distances **between 80 meters and 30 meters, but probably closer to 30 meters given the rapidly decreasing trend of the curve based on the data in the bins for 18-meter and 9-meter crater diameters**

# Landsat - Exploring Washington D.C.


**Problem 1** - This Landsat-7 image of Washington D.C. was taken on May 9, 2005. Using a metric ruler, and the conversion 1 kilometer = 0.62 miles, what is the scale of the image in meters per millimeter, and how large is the field in kilometers?

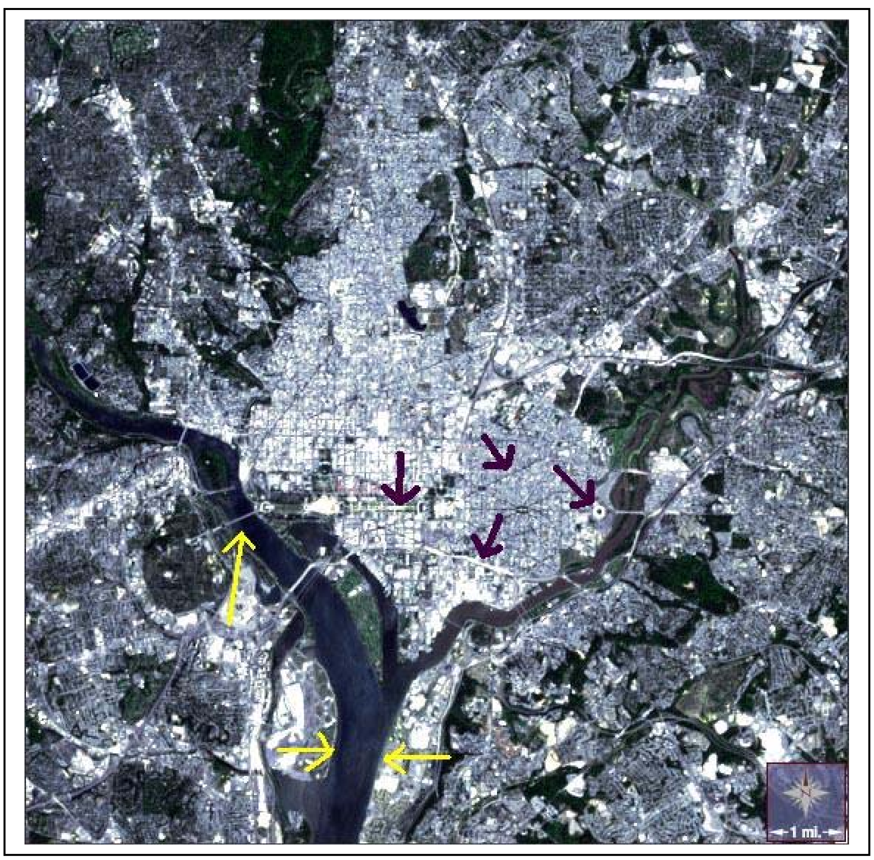
**Problem 2** - Use a map of the area (e.g.GOOGLE maps) to find the following features and determine their width: A) Potomac River; B) Arlington Memorial Bridge; C) National Mall; D) Highway 395; E) RFK Stadium; F) A large Boulevard.

**Problem 3** - Comparing the park lands (dark areas), the rivers and the man-made developments (light areas); about what would you estimate as the percentage of this metropolitan area that is developed?

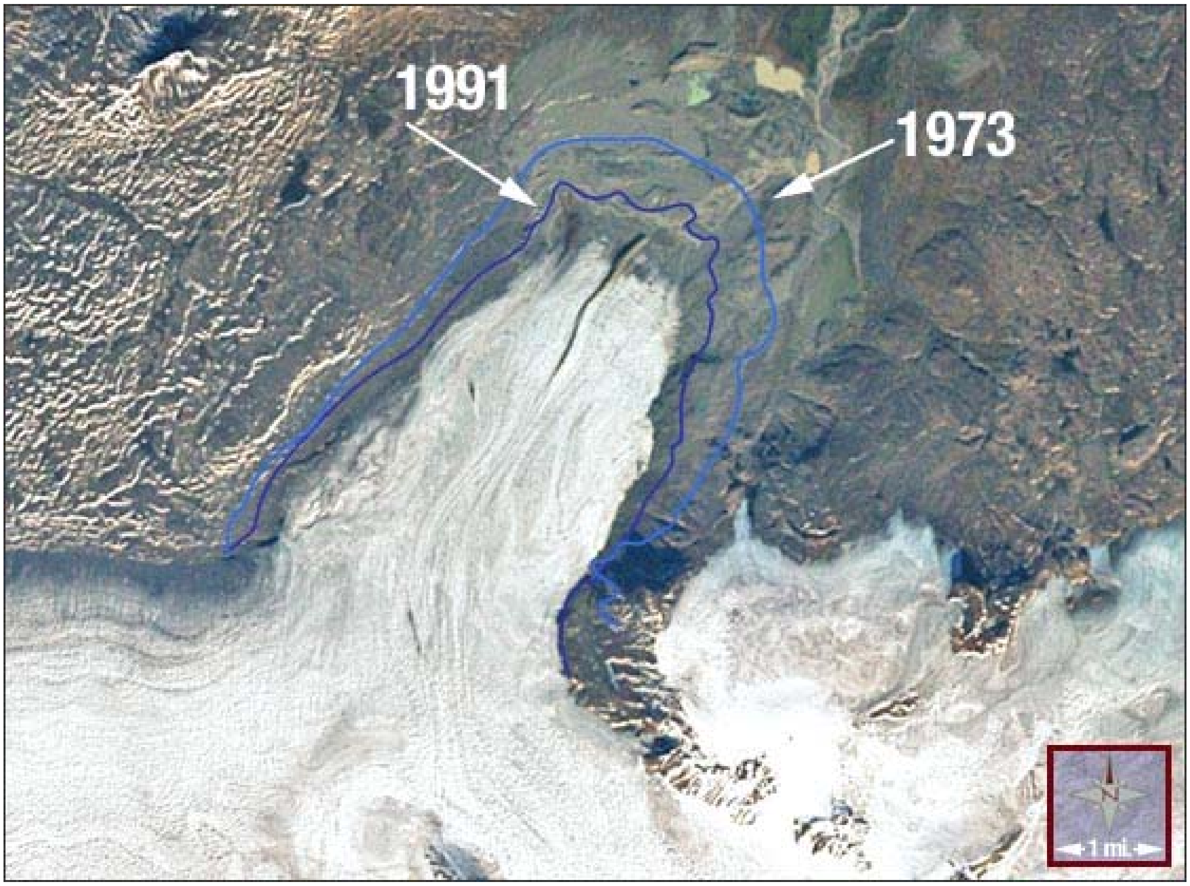
**Problem 1** - Answer: The legend in the lower right indicates that 1 mile = 14 millimeters. This means 1.61 kilometers = 14 millimeters or that the scale is 115 meters/mm. The field measures 156mm x 156mm or **17.9 km on a side**.

**Problem 2** - Use a map of the area (e.g.GOOGLE maps) to find the following features and determine their width (See labels in below image):
**A) Potomac River; About 9 millimeters wide before split with Anacostia River between two arrows, or 9mm x 115 m/mm = 1.0 kilometers.**
**B) Arlington Memorial Bridge; The second bridge to the north that crosses the Potomac River. Width is about 0.5 mm x 115 m/mm = 58 meters.**
**C) National Mall; About 1 mm or 155 meters.**
**D) Highway 395; White roadway, about 1 mm wide or 115 meters.**
**E) RFK Stadium; Round building, about 2mm in diameter or 230 meters.**
**F) A large Boulevard. Black streak about 0.3mm wide or 34 meters.**

**Problem 3** - Answer: The full area is 17.9 km x 17.9 km = 320 $km^2$. The developed areas are the ones in grey which represent concrete and asphalt surfaces or buildings. The areas that include the park lands and river can be cut out and fitted together into a square, or can be determined more accurately by dividing the full area into 10x10 squares and adding up the number of squares that mostly cover the dark areas. **Students estimates will vary depending on the method used for estimating the irregular areas, but should amount to about 25% of the full area. The fraction of the full area that is developed in this particular view is about 3/4 or 75%.**



# Landsat - Glacier Retreat




The Eyjabakkajökull Glacier is an outlet glacier of the Vatnajökull ice cap in Iceland. It has been retreating since a major surge occurred in 1973. This true-color Landsat-7 image shows the glacier terminus in September 2000. The light- and dark-blue outlines show the terminus extent in 1973 and 1991, respectively.

**Problem 1** - Using a metric ruler, and the conversion 1 kilometer = 0.62 miles, what is the scale of the image in meters per millimeter?

**Problem 2** - How many kilometers did the glacier retreat between A) 1973 and 1991? B) 1991 and 2000?

**Problem 3** - From your answers to Problem 2, what is the average rate of retreat in kilometers per year between A) 1973-1991, and B) 1991 to 2000? C) Is the retreat of the glacier speeding up or slowing down?

**Problem 4** - Assume that the height of the glacier is 1000 meters. About what volume of ice has been lost between 1973 and 1991 in cubic kilometers, assuming that the missing ice is shaped like a wall?

# Answer Key

**Problem 1** - Using a metric ruler, and the conversion 1 kilometer = 0.62 miles, what is the scale of the image in meters per millimeter? Answer; The 1-mile legend on the lower right measures 14mm wide, and since 1 mile = 1.61 km, the scale is 1610 meters/14mm = **115 meters/mm**.

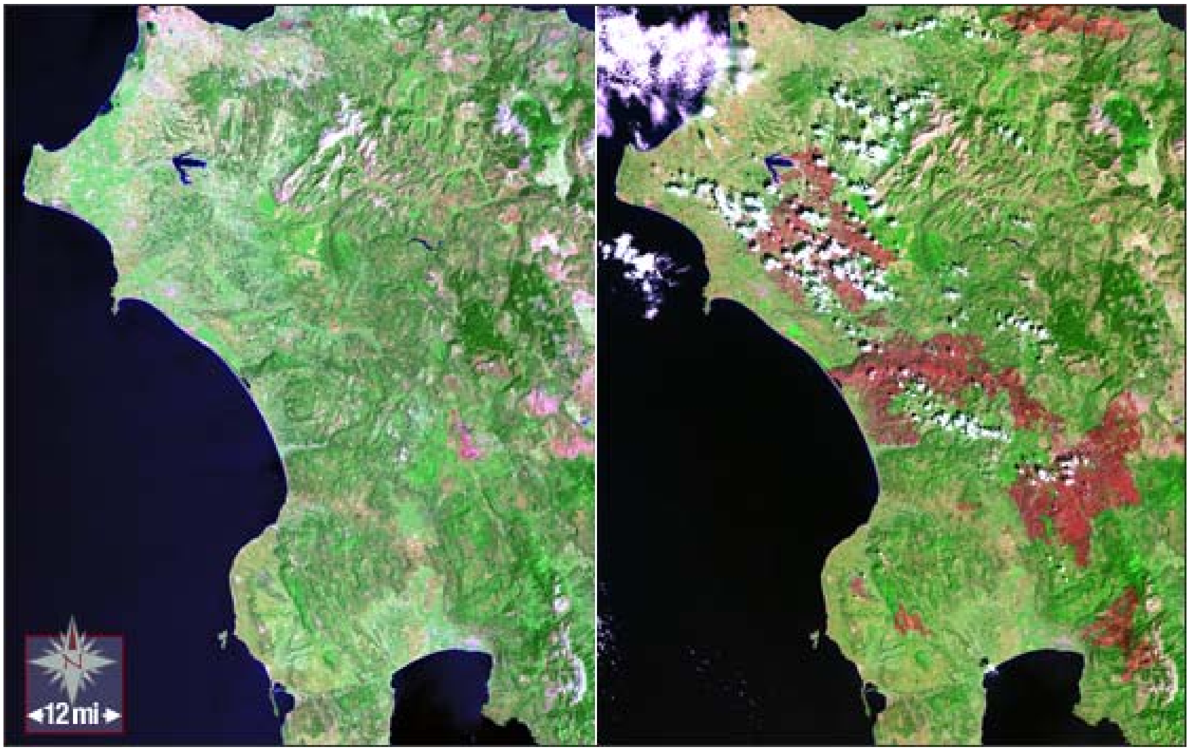
**Problem 2** - How many kilometers did the glacier retreat between
A) 1973 and 1991? Answer; At the head of the glacier (top end) the distance is 8 mm or 8 x 115 = **920 meters**.
B) 1991 and 2000? Answer: the distance traveled is about 10mm or 10 x 115m = **1,150 meters.**

**Problem 3** - From your answers to Problem 2, what is the average rate of retreat in kilometers per year between A) 1973-1991: 920 meters/18 years = **51 meters/year**. and B) 1991 to 2000? 1150 meters/9years = **128 meters/year**.
C) Is the retreat accelerating (speeding up or slowing down?) Answer: the retreat is definitely **speeding up** ( 51 m/yr compared to 128 m/yr).

**Problem 4** - Assume that the height of the glacier is 1000 meters. About what volume of ice has been lost between 1973 and 1991 in cubic kilometers?

Answer: The height of the wall is 1000 meters. The width of the wall is estimated by using the average width of the retreated ice between 1973 and 2000, which from the photo is about (5mm + 8mm + 20 mm)/3 = 11 mm or 11 x 115 m/mm = 1,300 meters. The length of the wall is the perimeter of the retreating ice which is about 140 mm x 115 m/mm = 16,000 meters. The volume in cubic kilometers is then 1.0km x 1.3km x 16 km = **21 cubic kilometers**!

# Landsat - Estimating Biomass Loss from a Large Fire




The fires in Greece during the summer of 2007 devastated large areas of forest and ground cover in this Mediterranean region. These before (left) and after (right) images were taken on July 18 and September 4 by Landsat-7. The red areas show the extent of the biomass loss from the fires.

**Problem 1** - Using a metric ruler, and the conversion 1 kilometer = 0.62 miles, what is the scale of the image in meters per millimeter?

**Problem 2** - About what is the total area, in square-kilometers, of this photo of Greece and its surroundings?

**Problem 3** - About what was the land area, in square-kilometers, that was burned?

**Problem 4** - What percentage of the total area was lost to the fires?

**Problem 5** - Suppose that a typical forest in this region contains about 5.0 kilograms of biomass per square meter. How many metric tons of biomass were lost during the fires?

# Answer Key

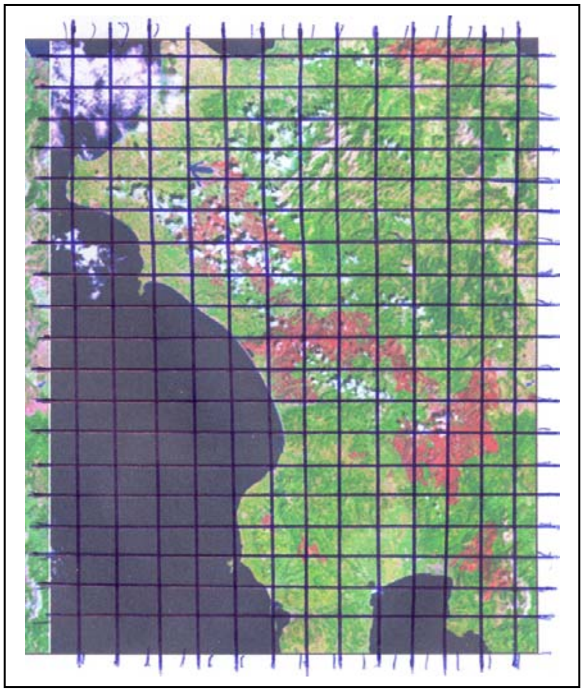
**Problem 1** - Answer: The legend on the lower right indicates that 12 miles = 12 millimeters, so in kilometers this becomes 19.4 km /12mm = **1.6 km/mm**

**Problem 2** - Answer: The field on the right measures 78mm x 98mm = 125 km x 157 km = **19,700 km$^2$**.

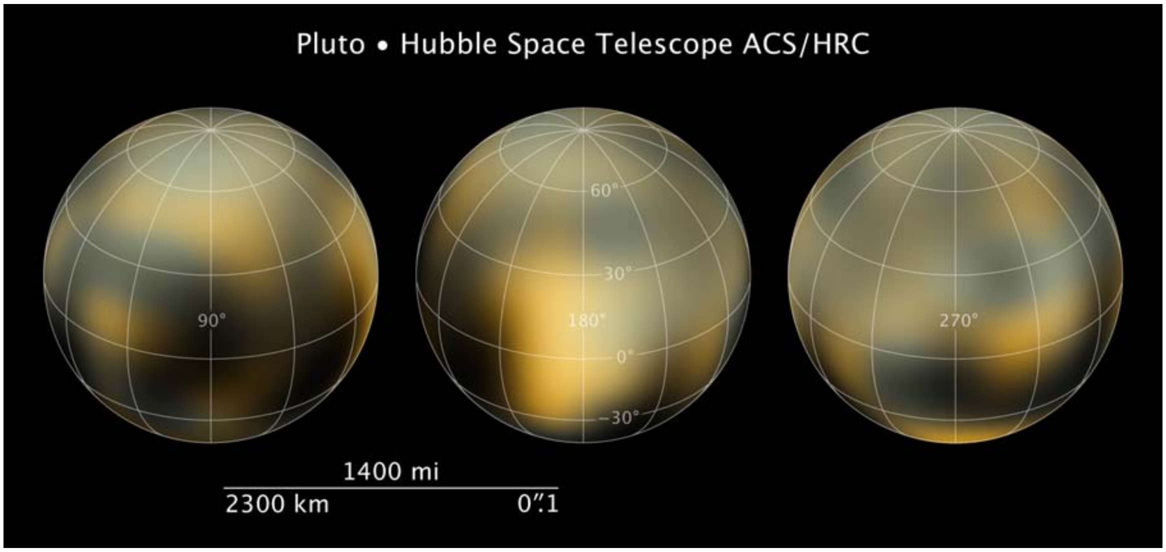
**Problem 3** - Answer. To estimate the area of irregular regions, divide the image into a suitable number of smaller squares, for example, 5mm on a side ( = 8 km on a side or an area of 64 km$^2$) as shown in the figure below. The full area has 13 squares across and 19 squares vertically, for a total of 247 cells and a total area of 16,000 km$^2$. Because the drawn cells are slightly irregular, we can re-calculate their average area as 19,700 km$^2$/247 cells = 80 km$^2$. The land area is covered by 173 cells for a total area of 173 x 80 = 13,800 km$^2$. The red areas that were burned total about 30 cells or 2,400 km$^2$. Students answers will vary depending on how they counted the cells. Students may combine their counts and average them to get a more accurate estimate.

**Problem 4** - Answer: 100% x 2400/13800 = **17%**

**Problem 5** - Answer: 5.0 kg/m$^2$ x (1,000,000 m$^2$/km$^2$) x 2,400 km$^2$ = **12 billion kg or 12 million metric tons.**



# The Changing Atmosphere of Pluto




Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. Let's see how this is possible!

**Problem 1** - The equation for the orbit of Pluto can be approximated by the formula $2433600=1521x^2+1600y^2$. Determine from this equation, expressed in Standard Form, A) the semi-major axis, a; B) the semi-minor axis, b; C) the ellipticity of the orbit, e; D) the longest distance from a focus called the aphelion; E) the shortest distance from a focus, called the perihelion. (Note: All units will be in terms of Astronomical Units. 1 AU = distance from the Earth to the Sun = $1.5 \times 10^{11}$ meters).

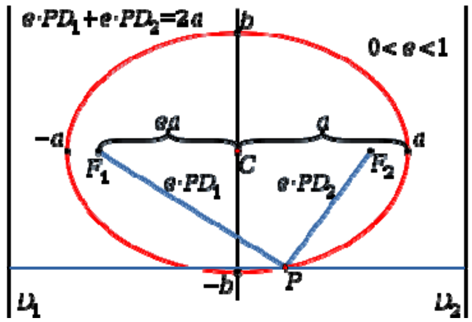
**Problem 2** - The temperature of the methane atmosphere of Pluto is given by the formula

$$T(R) = \left(\frac{L(1-A)}{16\pi\sigma R^2}\right)^{\frac{1}{4}} \qquad \text{degrees Kelvin (K)}$$

where L is the luminosity of the sun ($L=4 \times 10^{26}$ watts); $\sigma$ is a constant with a value of $5.67 \times 10^{-8}$, R is the distance from the sun to Pluto in meters; and A is the albedo of Pluto. The albedo of Pluto, the ability of its surface to reflect light, is about A = 0.6. From this information, what is the predicted temperature of Pluto at A) perihelion? B) aphelion?

**Problem 3** - If the thickness, H, of the atmosphere in kilometers is given by H(T) = 1.2 T with T being the average temperature in degrees K, can you describe what happens to the atmosphere of Pluto between aphelion and perihelion?

**Problem 1** - Answer:




In Standard Form $2433600=1521x^2+1600y^2$ becomes

$$1 = \frac{x^2}{1600} + \frac{y^2}{1521} \quad = \quad \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Then A) **a = 40 AU** and B) **b=39 AU**. C) The ellipticity **$e = (a^2 - b^2)^{1/2}/a = 0.22$**. D) The longest distance from a focus is just a(1 + e) = 40(1+0.22) = **49 AU**. E) The shortest distance is just a(1-e) = (1-0.22)(40) = **31 AU**. Written out in meters we have a= $6x10^{12}$ meters; b= $5.8x10^{12}$ meters; aphelion = $7.35x10^{12}$ meters and perihelion = $4.6x10^{12}$ meters.

**Problem 2** - Answer: For R in terms of AU, the formula simplifies to

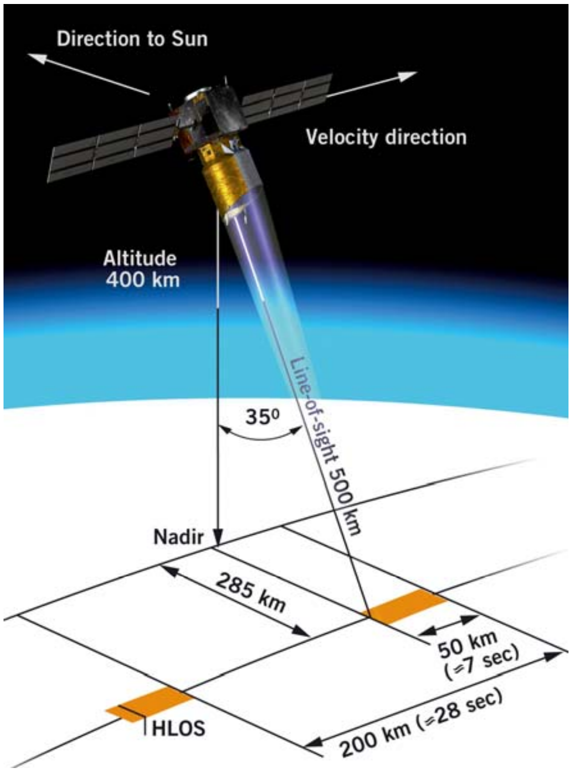
$$T(R) = \left( \frac{4x10^{26}(1-0.6)}{16(3.14)(5.67x10^{-8})(1.5x10^{11})^2 R^2} \right)^{\frac{1}{4}}$$

$$T(R) = \frac{223}{\sqrt{R}}$$

A) For a perihelion distance of 31 AU we have T = $223/(31)^{1/2}$ = 40 K; B) At an aphelion distance of 49 AU we have T = $223/(49)^{1/2}$ = 32 K. Note: The actual temperatures are about higher than this and average about 50K.

**Problem 3** - Answer: At aphelion, the height of the atmosphere is about H=1.2x(32) = 38 kilometers, and at perihelion it is about H=1.2x(40) = 48 kilometers, so as Pluto orbits the sun its atmosphere increases and decreases in thickness.

Note: In fact, because the freezing point of methane is 91K, at aphelion most of the atmosphere freezes onto the surface of the dwarf planet, and at aphelion it returns to a mostly gaseous state. This indicates that the simple physical model used to derive H(T) was incomplete and did not account for the freezing-out of an atmospheric constituent.



*Credits: ESA - AOES Medialab*

Most satellite imaging systems do not remain fixed over a target because they are in orbit around Earth or the Moon. For ordinary digital photos we do not want our Subject to move and cause blurring of the image. For satellite photography, it is unavoidable that the satellite in its orbit, or the Target are in motion.

Once we have determined the resolution that our satellite camera needs to study a Target, we also have to keep track of image and Target motion which can also blur the image.

To avoid blurring, we do not want the scene being photographed to move by more than one pixel during the exposure time.

**Problem 1** – The satellite travels at a ground speed of 10 kilometers/sec. The CCD camera will not be designed to mechanically track the Target as it passes-by. What will be the angular speed, W, in pixels/sec, of the ground Target traveling across the CCD image if the satellite is in an orbit 350 km above the ground and has a resolution of 6 arcseconds/pixel?

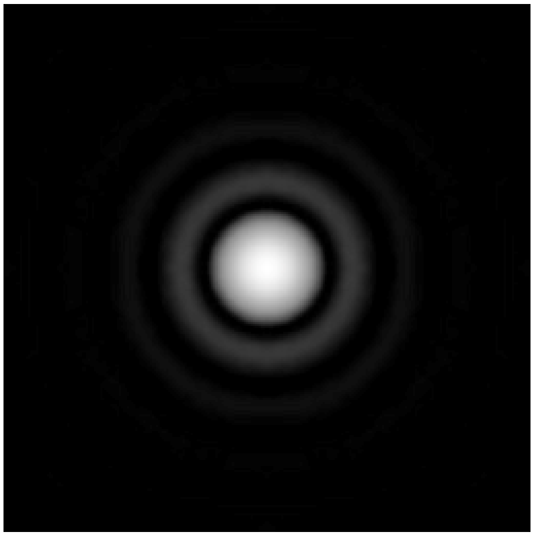
**Problem 2** – What must be the maximum exposure time of the CCD image in order to avoid image blurring?

**Problem 1** – The satellite travels at a ground speed of 10 kilometers/sec. The CCD camera will not be designed to mechanically track the Target as it passes-by. What will be the angular speed, W, in pixels/sec, of the ground Target traveling across the CCD image if the satellite is in an orbit 350 km above the ground and has a resolution of 6 arcseconds/pixel?

Answer: Since $\theta$ = 206265 L/d, and L = 10km, d = 350 km, the angular speed W = 206265 x (10/350) = 5893 arcseconds/sec. The resolution is 6 arcseconds/pixel, so the speed is
w = 5893/6
= **982 pixels/sec**.

**Problem 2** – What must be the maximum exposure time of the CCD image in order to avoid image blurring?

Answer: T = 1 Frame x 1 second /982 Frames
so **T = 0.001 seconds**



The bright spot is where most of the light energy falls, but it is surrounded by a large number of rings of light, called the diffraction pattern. The angle between the center of the main spot, and the first ring is given by the Airy Disk formula for θ.

Because like is a wave-like phenomenon, it causes interference when it is reflected and concentrated in an optical system. This pattern of interference makes it impossible to clearly see details that are smaller than this interference pattern.

There is a geometric relationship between the resolution of an imaging system and the wavelength at which it operates given by

$$\theta = 1.22\frac{\lambda}{D}$$

where θ is the resolution in units of radians, λ is the wavelength of the radiation in meters, and **D** is the diameter of the camera or telescope lens or mirror in meters.

**Problem 1** - If 1 radian = 206265 arcseconds, what is the resolution formula in terms of arcseconds?

**Problem 2** - A biologist wants to study deforestation with a satellite camera that has a pixel resolution of 10-meters/pixel, which at the orbit of the satellite corresponds to an angular resolution of 6 arcseconds. To measure the loss of plant matter, she detects the reflection by the ground of chlorophyll, which is the most intense at a wavelength of 700 nanometers (1 nanometer = $10^{-9}$ meters). What is the diameter of the camera lens that will insure this resolution at the orbit of the satellite?

**Problem 3** – Construct a graph that shows the diameter of lens or mirror that is needed to obtain a resolution of 1 arcsecond from far-ultraviolet wavelengths of 200 nanometers to infrared wavelengths of 10 micrometers. From orbit, a human subtends an angle of 1 arcseconds, and emits infrared energy at a wavelength of 10 microns. How large would the camera have to be to resolve a human by his heat emission?

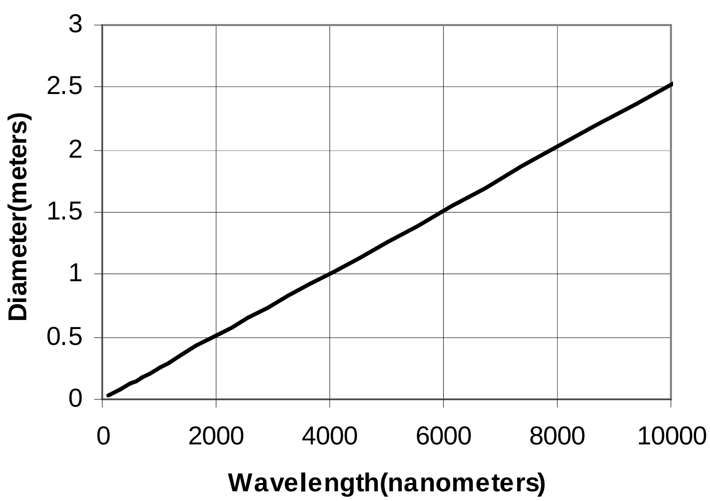
**Problem 1** - If 1 radian = 206265 arcseconds, what is the resolution formula in terms of arcseconds?

Answer: $\theta$ = (1.22 x 206265) $\lambda$/D so $\theta$ = 251,643 $\lambda$/D

**Problem 2** - A biologist wants to study deforestation with a satellite camera that has a pixel resolution of 10-meters/pixel, which at the orbit of the satellite corresponds to an angular resolution of 6 arcseconds. To measure the loss of plant matter, she detects the reflection by the ground of chlorophyll, which is the most intense at a wavelength of 700 nanometers (1 nanometer = $10^{-9}$ meters). What is the diameter of the camera lens that will insure this resolution at the orbit of the satellite?

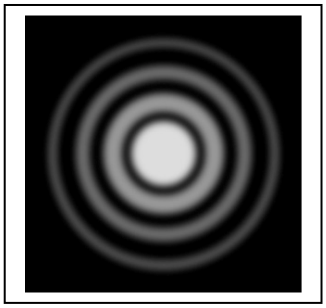
Answer: We want q = 6 arcseconds. Then for l = $7.0 \times 10^{-7}$ meters we have
D = 251,643 x $7.0 \times 10^{-7}$/6.0
**D = 0.03 meters or 3 centimeters**.

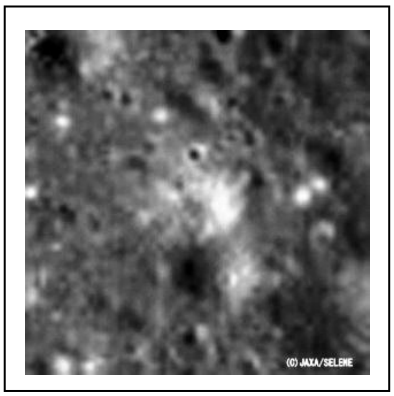
**Problem 3** – Construct a graph that shows the diameter of lens or mirror that is needed to obtain a resolution of 1 arcsecond from far-ultraviolet wavelengths of 100 nanometers to infrared wavelengths of 10 micrometers (10000 nanometers). From orbit, a human subtends an angle of 1 arcseconds, and emits infrared energy at a wavelength of 10 microns. How large would the camera have to be to resolve a human by his heat emission?

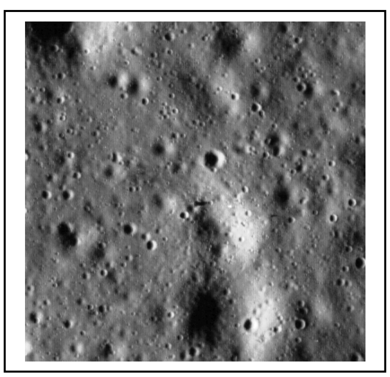



Answer: **The graph suggests a mirror diameter of 2.5 meters!**

$$R = 1.22 \frac{L}{D}$$





There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

In optics, the best focused spot of light that a perfect lens with a circular aperture can make is limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, L, and the size of the circular aperture (mirror, lens), given by D. When L and D are expressed in the same units (e.g. centimeters, meters), R will be in units of angular measure called radians ( 1 radian = 57.3 degrees).

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm ) with the lower image taken by the LRO satellite (0.5 meters/pixel at a 50km orbit elevation: aperture = ). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

**Problem 1** - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

**Problem 2** - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

**Problem 3** - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

**Problem 1** - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Answer: First convert all numbers to centimeters, then use the formula to calculate the resolution in radian units: L = 21 centimeters, D = 100 meters = 10,000 centimeters, then R = 1.22 x 21 cm /10000 cm so R = 0.0026 radians. There are 57.3 degrees to 1 radian, so A) 0.0026 radians x (57.3 degrees/ 1 radian) = **0.14 degrees.** And B) There are 60 arc minutes to 1 degrees, so 0.14 degrees x (60 minutes/1 degrees) = **8.4 arcminutes.**

**Problem 2** - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Answer: R = 1.22 x (0.00006 cm/10400 cm) = 0.000000069 radians. A) Since 1 microradian = 0.000001 radians, the resolution of this telescope is **0.069 microradians.** B) Since 1 radian = 57.3 degrees, and 1 degree = 3600 arcseconds, the resolution is 0.000000069 radians x (57.3 degrees/radian) x (3600 arcseconds/1 degree) = 0.014 arcseconds. One thousand milliarcsecond = 1 arcseconds, so the resolution is 0.014 arcsecond x (1000 milliarcsecond / arcsecond) = **14 milliarcseconds**.

**Problem 3** - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

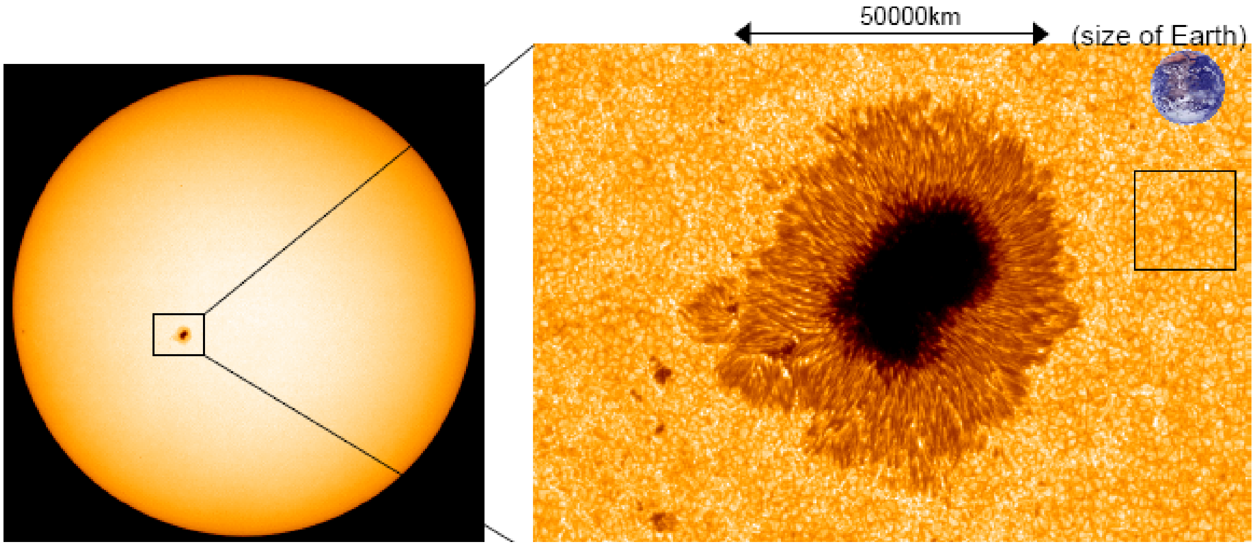
Answer: From R = 1.22 L/D we have R = 1 arcsecond and L = 20 micrometers and need to calculate D, so with algebra we re-write the equation as D = 1.22 L/R.
Convert R to radians:

R = 1 arcsecond x (1 degree/3600 arcsecond) x (1 radian / 57.3 degrees) = 0.0000048 radians.

L = 20 micrometers x (1 meter/1,000,000 micrometers) = 0.00002 meters.

Then D = 1.22 (0.00003 meters)/( 0.0000048 radians) = **5.1 meters.**

# Hinode - Close-up of a Sunspot




After a successful launch on September 22, 2006 the Hinode solar observatory caught a glimpse of a large sunspot on November 4, 2006. An instrument called the Solar Optical Telescope (SOT) captured this image, showing sunspot details on the solar surface.

**Problem 1** - From the clues in this image, what is the scale of the image on the right in units of kilometers per millimeter?

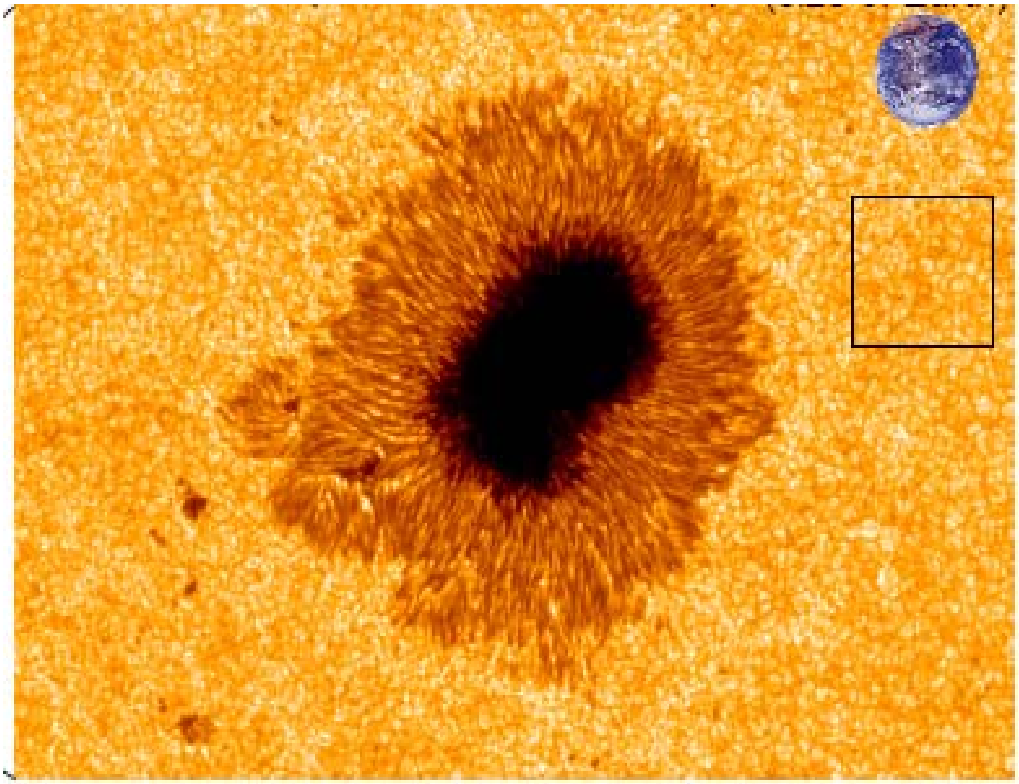
**Problem 2** - What is the size of the smallest detail you can see in the image?

**Problem 3** - Compared to familiar things on the surface of Earth, how big would the smallest feature in the solar image be?

**Problem 4** - The gold-colored textured surface is the photosphere of the sun. The texturing is produced by heated gas that is convecting from the hot interior to the cooler outer layers of the sun. The convecting gases form cells, called granulations, at the surface, with upwelling gas flowing from the center of each cell, outwards to the cell boundary, where it cools and flows back down to deeper layers. What is the average size of a granulation cell within the square?

**Problem 5** - Measure several granulation cells at different distances from the sunspot, and plot the average size you get versus distance from the spot center. Do granulation cells have about the same size near the sunspot, or do they tend to become larger or smaller as you approach the sunspot?

Answer Key:



**Problem 1** - From the 40 millimeter length of the 50,000 km arrow marker, the scale of the image is 50,000 km/40 mm = **1250 kilometers per millimeter**

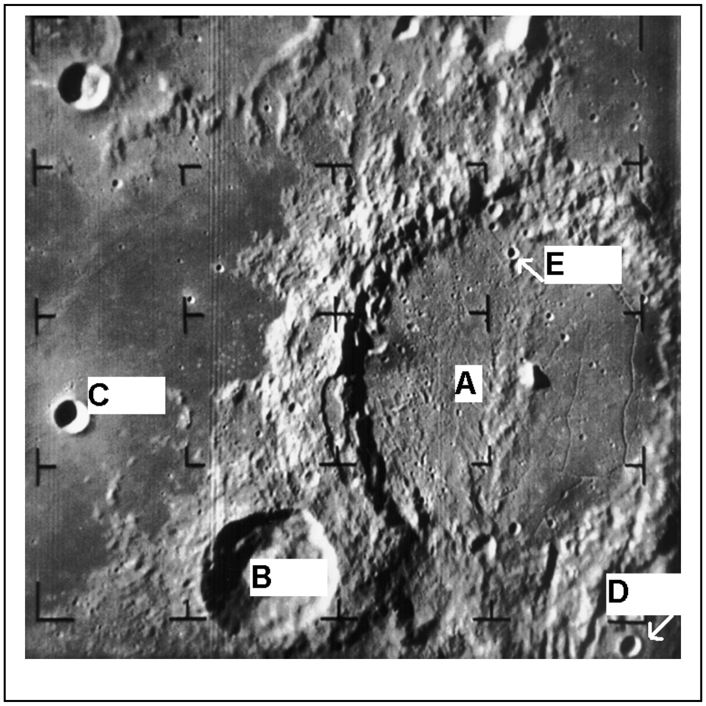
**Problem 2** - The smallest detail is about 0.5 millimeters or 0.5 x 1250 = **625 kilometers across.**

**Problem 3** - Similar features on Earth would be continents like Greenland (1,800 km) or England (700 km).

**Problem 4** - Measure about 5 cells to get: 1.5 mm, 1.0 mm, 0.8mm, 1.2mm and 1.4 mm. The average is about 1.2 mm, so the average size is (1.2) x 1250km = **1,500 km**.

**Problem 5** - Students should measure about 5 granulation cells in three groups; Group 1 should be far from the center of the spot. Group 3 should be as close to the outer, tan-colored, 'penumbra' of the spot as possible, and Group 2 should be about half-way in between Group 1 and 3. The average granulation sizes do not change significantly.

# Craters are a Blast!




Have you ever wondered how much energy it takes to create a crater on the Moon. Physicists have worked on this problem for many years using simulations, and even measuring craters created during early hydrogen bomb tests in the 1950's and 1960's. One approximate result is a formula that looks like this:

$$\mathbf{E = 4.0 \times 10^{15} D^3 \text{ Joules.}}$$

where D is the crater diameter in kilometers.

As a reference point, a nuclear bomb with a yield of one-megaton of TNT produces 4.0 x $10^{15}$ Joules of energy!

**Problem 1** - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

**Problem 2** - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of the labeled craters in the picture.

**Problem 3** - Use the formula from Problem 1 to determine the energy needed to create the labeled craters.

Note: To get a better sense of scale, the table below gives some equivalent energies for famous historical events:

| Event | Equivalent Energy (TNT) |
|---|---|
| | |
| Cretaceous Impactor | 100,000,000,000 megatons |
| Valdiva Volcano, Chile 1960 | 178,000 megatons |
| San Francisco Earthquake 1909 | 600 megatons |
| Hurricane Katrina 2005 | 300 megatons |
| Krakatoa Volcano 1883 | 200 megatons |
| Tsunami 2004 | 100 megatons |
| Mount St. Helens Volcano 1980 | 25 megatons |

# Answer Key

**Problem 1** - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

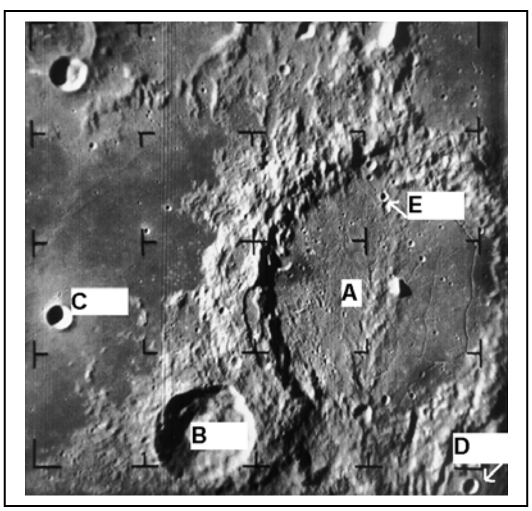
Answer: $E = 4.0 \times 10^{15} D^3$ Joules x (1 megaton TNT/$4.0 \times 10^{15}$ Joules)

**$E = 1.0 D^3$ megatons of TNT**

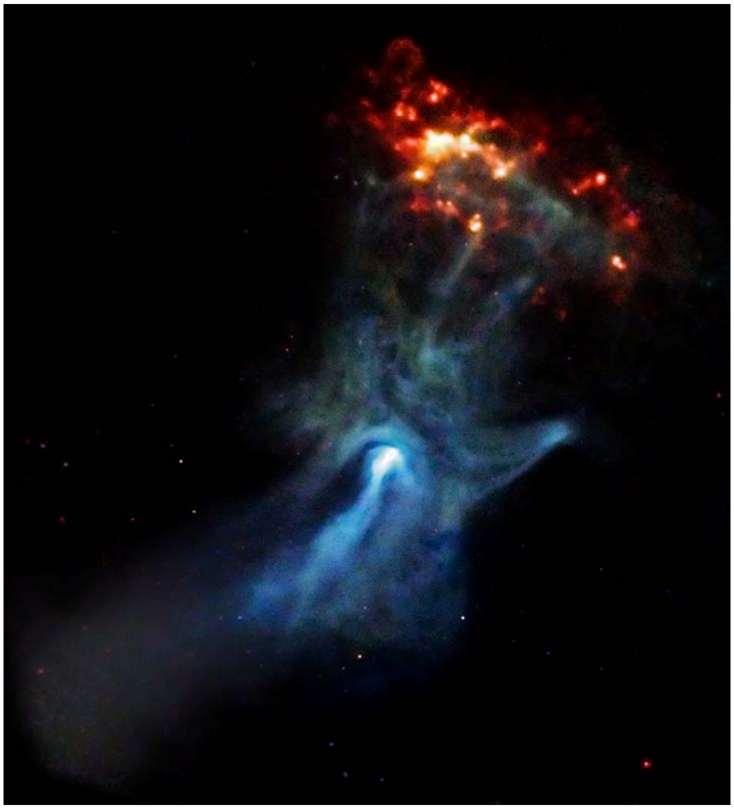
**Problem 2** - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of a range of craters in the picture. Answer: The width of the image is 92 mm, so the scale is 183/92 = 2.0 km/mm. See figure below for some typical examples: See column 3 in the table below for actual crater diameters.

**Problem 3** - Use the formula from Problem 1 to determine the energy needed to create the craters you identified. Answer: See the table below, column 4. Crater A is called Alphonsis. Note: No single formula works for all possible scales and conditions. The impact energy formula only provides an estimate for lunar impact energy because it was originally designed to work for terrestrial impact craters created under Earth's gravity and bedrock conditions. Lunar gravity and bedrock conditions are somewhat different and lead to different energy estimates. The formula will not work for laboratory experiments such as dropping pebbles onto sand or flour. The formula is also likely to be inaccurate for very small craters less than 10 meters, or very large craters greatly exceeding the sizes created by nuclear weapons. (e.g. 1 kilometer).

| Crater | Size (mm) | Diameter (km) | Energy (Megatons) |
|---|---|---|---|
| A | 50 | **100** | **1,000,000** |
| B | 20 | **40** | **64,000** |
| C | 5 | **10** | **1,000** |
| D | 3 | **6** | **216** |
| E | 1 | **2** | **8** |



# The Hand of Chandra!



A small, dense object only twelve miles in diameter is responsible for this beautiful X-ray nebula that spans 150 light years and resembles a human hand!

At the center of this image made by NASA's Chandra X-ray Observatory is a very young and powerful pulsar known as PSR B1509-58.

The pulsar is a rapidly spinning neutron star which is spewing energy out into the space around it to create complex and intriguing structures, including one that resembles a large cosmic hand.

Astronomers think that the pulsar and its nebula is about 1,700 years old, and is located about 17,000 light years away (e.g. 5,200 parsecs). Finger-like structures extend to the north, apparently energizing knots of material in a neighboring gas cloud known as RCW 89. The transfer of energy from the wind to these knots makes them glow brightly in X-rays (orange and red features to the upper right).

**Problem 1** - This field of view is 19 arcminutes across. Using similar triangles and proportions, if 1 arcminute at a distance of 1,000 parsecs equals a length of 0.3 parsecs how wide is the image in parsecs?

**Problem 2** - Measure the width of this image with a millimeter ruler. What is the scale of this image in parsecs per millimeter? How far, in light years, is the bright spot in the 'palm' where the pulsar is located, from the center of the ring-like knots in RCW 89? (1 parsec = 3.26 light years). Round your answer to the nearest light year.

**Problem 3** - If the speed of the plasma is 10,000 km/sec, how many years did it take for the plasma to reach RCW-89 if 1 light year = $9.5 \times 10^{12}$ kilometers, and there are $3.1 \times 10^{7}$ seconds in 1 year?

**Problem 1** - This field of view is 19 arcminutes across. Using similar triangles and proportions, if 1 arcminute at a distance of 1,000 parsecs equals a length of 0.3 parsecs how wide is the image in parsecs?

Answer: If 0.3 parsecs seen from a distance of 1,000 parsecs subtends an angle of 1 arcminute, then 1 arcminute at a distance of 5,200 parsecs will subtend

$$\frac{X}{0.3\ \text{parsecs}} = \frac{5{,}200\ \text{parsecs}}{1{,}000\ \text{parsecs}}$$

so that X = 0.3 x (5200/1000) = 1.6 parsecs. Then 19 arcminutes will subtend

$$\frac{X}{1.6\ \text{parsecs}} = \frac{19\ \text{arcminutes}}{1\ \text{arcminute}}$$

so that X = 1.6 x (19/1) = **30.4 parsecs**.

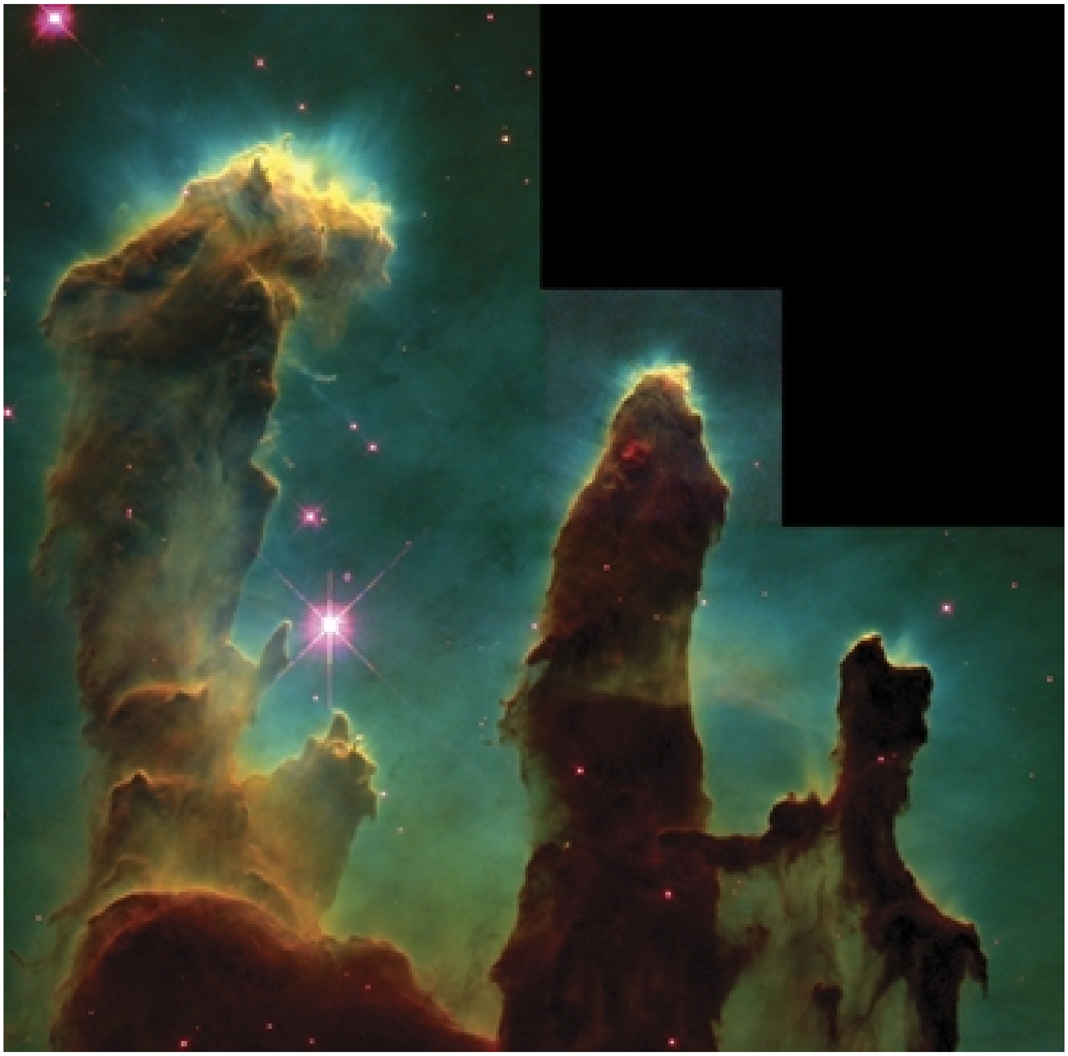
**Problem 2** - Measure the width of this image with a millimeter ruler. What is the scale of this image in parsecs per millimeter? How far, in light years, is the center of the 'palm' where the pulsar is located, from the ring-like knots in RCW 89? (1 parsec = 3.26 light years).

Answer: The width is about 96 millimeters. The scale is then 30.4 parsecs/96 millimeters or **0.32 parsecs/millimeter**. The distance from the bright spot to the ring of knots is about 40 millimeters. At the scale of the image, this equals 40 mm x 0.32 pc/mm = 12.8 parsecs. Converting this to light years we get 12.8 parsecs x (3.26 light years/parsec) or 12.8 x 3.26 = 41.728 light years, which is rounded to **42 light years**.

**Problem 3** - If the speed of the plasma is 10,000 km/sec, how many years did it take for the plasma to reach RCW-89?

Answer: Time = distance/speed. First convert light years to kilometers: 42 light years x (9.5 x $10^{12}$ kilometers/light year) = 4.0 x $10^{14}$ kilometers. Then divide this by the speed to get Time = 4.0 x $10^{14}$ kilometers / (10,000 km/s) = 4.0 x $10^{10}$ seconds. Converting this to years: 4.0 x $10^{10}$ seconds x ( 1 year/3.1 x $10^{7}$ seconds) = **1,300 years.**

# The Eagle Nebula Close-up



The Hubble Space Telescope took this image of the Eagle Nebula (M-16). This star-forming region is in the constellation Serpens, and located 6,500 light years from Earth. It is only about 6 million years old, and the dense clouds of interstellar gas are still collapsing to form new stars. This image is 2.5 arcminutes across.

**Problem 1** – If an angular size of 200 arcseconds corresponds to 1 light year at a distance of 1000 light years, then what is the size of this field at the distance of the nebula?

**Problem 2** – What is the scale of this image in light years/millimeter?

**Problem 3** – Our Solar System is about 1/400 of a light year across. How big is it, in millimeters, at the scale of this photo?

**Problem 4** - How many times the size of our solar system is the smallest nebula feature you can see in the photo?

**Problem 1** – If an angular size of 200 arcseconds corresponds to 1 light years at a distance of 1000 light years. What is the size of this field at the distance of the nebula?

Answer: First we have to find out the scale of the image at the distance of the nebula. The field is stated to be 2.5 arcminutes across. At the distance of the nebula, 6,500 light years, the scale would be 200 arcseconds = 6,500/1000 x 1 light year = **6.5 light years.**

Since 1 arcminute = 60 arcseconds, by converting units we have 2.5 arcminutes x (60 arcseconds/arcminutes) x (6.5 ly/200 arcseconds) = 4.9 light years. **The field is 4.9 x 4.9 light years in size.**

**Problem 2** – What is the scale of this image in light years/millimeter? Answer: The Hubble image is 140 millimeters across. Since this equals 4.9 light years, the scale is 4.9 ly/140mm = **0.035 light years/millimeter**.

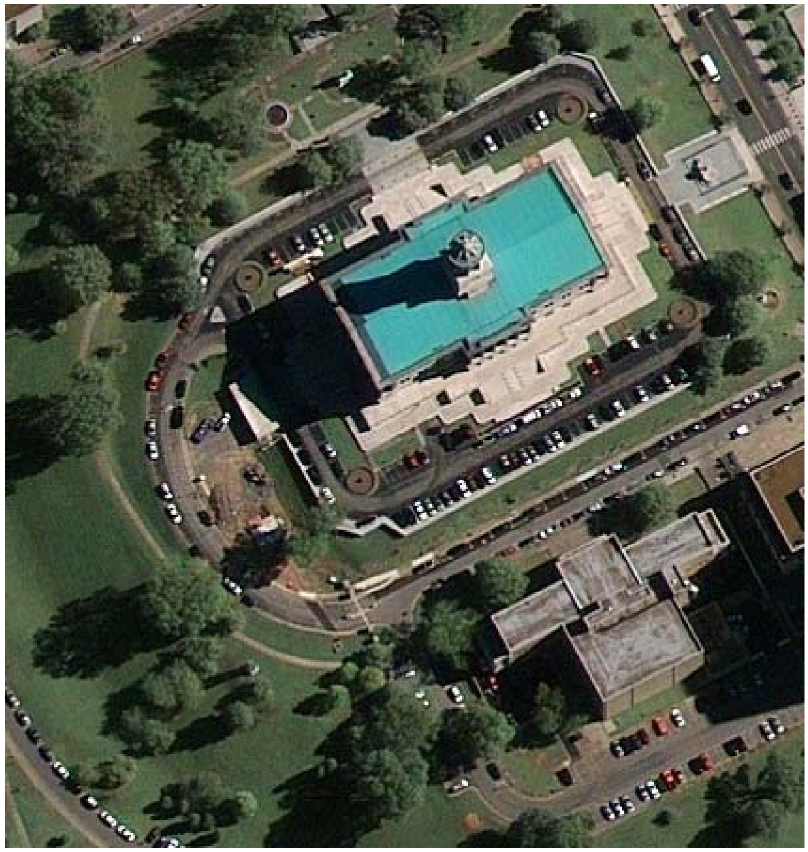
**Problem 3** – Our Solar System is about 1/400 of a light year across. How big is it, in millimeters, at the scale of this photo? Answer: At the scale of the photo ,1/400 of a light years = 0.0025 light years = 0.0025/.035 = **0.07 millimeters**. This is about the same size as the 'period' at the end of this sentence .

**Problem 4** - How many times the size of our solar system is the smallest nebula feature you can see in the photo? Answer: Some features are about 0.2 millimeters in size, which equals 0.2 mm/0.07 **= 2.8 times the diameter of our solar system**!

# A High-Resolution Satellite Photo



The Lunar Reconnaissance Orbiter (LRO) will take photographs of the lunar surface at a resolution of 0.5 meters per pixel. The 425x425 pixel image below (Copyright © 2009 GeoEye) was taken of the Tennessee Court House from the GeoEye-1 satellite with a width of about 212 meters.



**Problem 1** - What is the scale of the image in: A) meters per millimeter? B) meters per pixel?

**Problem 2** - How does the resolution of the expected LRO images compare with the resolution of the above satellite photo?

**Problem 3** - What are the smallest features you can easily identify in the above photo?

**Problem 4** - From the length of the shadows, what would you estimate as the elevation of the sun above the horizon?

# Answer Key

**Problem 1** - What is the scale of the image in: A) meters per millimeter? B) meters per pixel? Answer: A millimeter ruler would indicate a width of 105 millimeters, so the scale is A) 212 meters/105 mm = **2 meters/millimeter** and B) 212 meters/425 pixels = **0.5 meters/pixel**.

**Problem 2** - How does the resolution of the expected LRO images compare with the resolution of the above satellite photo? Answer: **The resolutions are identical** and both equal to 0.5 meters/pixel, so we should be able to see about the same kinds of details at the lunar surface with LRO.

**Problem 3** - What are the smallest features you can easily identify in the above photo?
Answer: With a millimeter ruler you can determine that the smallest features are comfortably about 0.5 millimeters across or 1 meter. Examples would include the parked cars, the widths of the various footpaths, and the lane and street markings stripes. Some of the smaller spots may be the shadows of people!

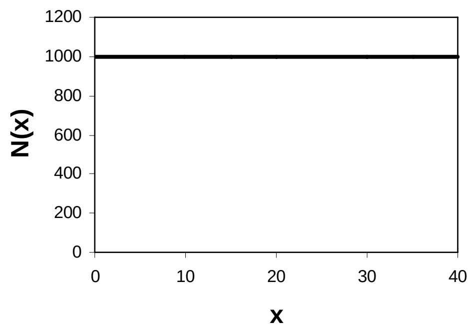
**Problem 4** - From the length of the shadows, what would you estimate as the elevation of the sun above the horizon?

Answer: This is a challenging problem for students because they first need to estimate what the height of the object is that is casting the shadow. From this they can construct a triangle and determine the sun angle, or use trigonometry.

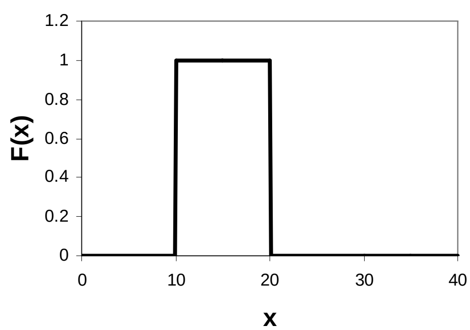
For example, suppose that the large tree in the lower left corner of the picture has a height of 50-feet (17 meters). We can measure the shadow length with a millimeter ruler to get 20 millimeters or 20 mm x 2 meters/mm = 40 meters. Then tan(theta) = 17 meters/40 meters = 0.45, and so **theta = 24 degrees** above the horizon.

This problem shows students one of the problems encountered when studying photos like this. We can easily measure the widths of objects, but measuring their heights can be challenging, especially when the objects in question are unfamiliar.

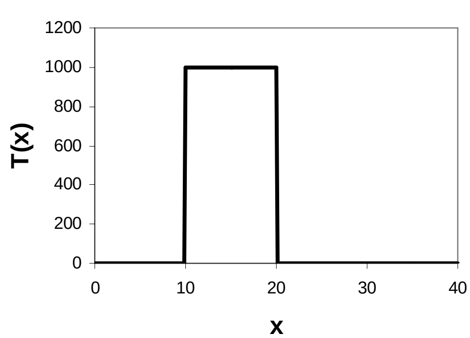
Graph of photon emission N(x)




Graph of filter transmission F(x)




Graph of photon transmission T(x)




Sunglasses are one of the most common, every-day filters that we use. They work in much the same way as the far more sophisticated filters used in professional and scientific photography and digital imaging.

A light source creates huge numbers of photons all across the electromagnetic spectrum. A filter blocks out all of the photons and passes only a narrow range of photons with the desired wavelengths. This process can be described mathematically.

Suppose a light source emits N photons according to the function N(x)=1000 photons shown in the graph to the left (Top). Suppose a filter can be defined according to the piecewise function F(x) = 1.0 for $10 < x < 20$, and F(x)=0 for all other values of x (middle graph). The number of photons passed by this filter is given by T(x) = N(x)F(x). It is easy to see in the bottom graph for T(x) that only photons between $10 < x < 20$ will be passed. The number of photons passed is just P = T(x)x(dx) where the base length is defined by the dx=20-10 = 10-unit width of the filter between x=10 and x=20, and the height is just 1000, so P = 1000x10 = 10,000 photons.

**Problem 1** – Suppose that N(x)=1000 and the filter is designed to match the table below:

| x | F(x) |
|---|---|
| 0 to 20 | 0 |
| 21 to 25 | 0.5 |
| 26 to 30 | 1.0 |
| 31 to 40 | 0.5 |
| 41 to infinity | 0 |

A) Graph N(x) and F(x). B) What is the total number of photons passed? (Hint: create a table for each wavelength interval) and list N(x), F(x),T(x) and P)

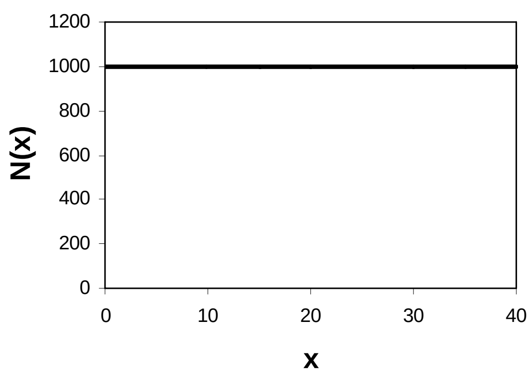
**Problem 1** – Suppose that N(x)=1000 and the filter is designed to match the table below:

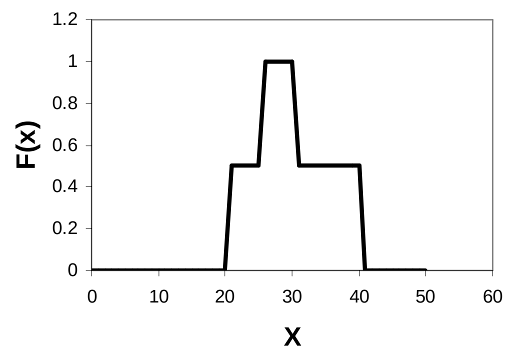
| x | F(x) |
|---|---|
| 0 to 20 | 0 |
| 21 to 25 | 0.5 |
| 26 to 30 | 1.0 |
| 31 to 40 | 0.5 |
| 41 to infinity | 0 |

A) Graph N(x) and F(x). B) What is the total number of photons passed?

Answer:

A) See below:

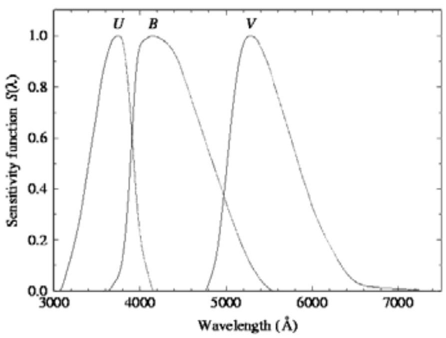



B) For each wavelength interval defined by the filter function, compute the product of N(x)F(x) and the wavelength interval, dx, and then sum the results as shown in the table below:

| x | dx | N(x) | F(x) | P=N(x)F(x)(dx) |
|---|---|---|---|---|
| 0 to 20 | 20 | 1000 | 0 | 0 |
| 21 to 25 | 4 | 1000 | 0.5 | 2000 |
| 26 to 30 | 4 | 1000 | 1.0 | 4000 |
| 31 to 40 | 9 | 1000 | 0.5 | 4500 |
| 41 to infinity | infinity | 1000 | 0 | 0 |

The total number of photons passed is
P = 2000+4000+4500 so
**P=10,500 photons**




The 'UBV' set of filters are used in astronomy to classify the light from distant stars and galaxies. The U, B and V brightness of a star can be used to determine the star's temperature. For example, a hot star is brighter in the U-band than in the B or V bands. A cold star is brighter in the V-band than in the U or B bands.

To handle more complex filters with realistic functions, one needs to use calculus to compute the total number of photons passed. The general formula is

$$P = \int_0^{+\infty} S(\lambda)F(\lambda)d\lambda$$

where $S(\lambda)$ is the Source Function that defines how the source emits radiation at each wavelength, and $F(\lambda)$ is the filter function which defines the transmission of the filter over the wavelength range.

$S(\lambda)$ is a physical function whose units are photons per square meter wavelength interval (example, photons/meter$^2$/nanometer). $F(\lambda)$ is a function that gives the filter transmission at each wavelength as a number from 1.0 to 0.0.

An astronomer is studying the distant quasar 3C273 using the Very Large Array radio telescope in Socorro, New Mexico. The quasar has an emission spectrum represented by the power function

$S(\lambda) = 100\lambda^{-3/4}$ Jansky/cm

where the wavelength, $\lambda$, is given in centimeters. Suppose that the radio telescope uses a filter at a wavelength of $\lambda = 3.0$ cm that has a parabolic shape defined by the piecewise function:

$F(\lambda) = -4(\lambda-2.5)(\lambda-3.5)$ for $2.5 < \lambda < 3.5$ and
$F(\lambda) = 0$ for all other $\lambda$

**Problem 1** – Graph the functions $F(\lambda)$ and $S(\lambda)$.

**Problem 2** – Over what domain will you need to perform the integration?

**Problem 3** - How bright, in Janskys, will quasar 3C273 appear at the wavelength being studied?

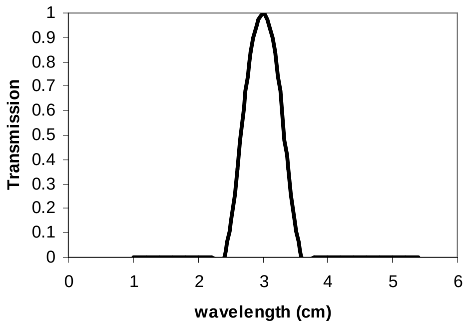
Given:

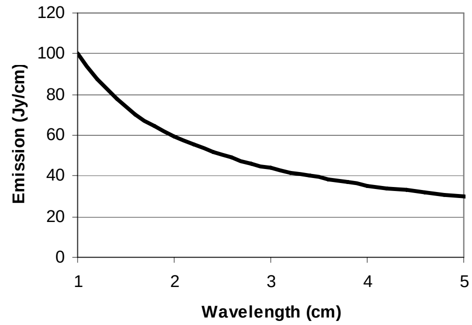
$S(\lambda) = 100\lambda^{-3/4}$ Jansky/cm

$F(\lambda) = -4(\lambda-2.5)(\lambda-3.5)$ for $2.5 < \lambda < 3.5$ and

$F(\lambda) = 0$ for all other $\lambda$

**Problem 1** – Graph the functions $F(\lambda)$ and $S(\lambda)$.




**Problem 2** – Over what domain will you need to perform the integration?

Answer: **Only over 2.5 < $\lambda$ < 3.5 because F($\lambda$) = 0 for all other values of $\lambda$.**

**Problem 3** - How bright, in Janskys, will quasar 3C273 appear at the wavelength being studied?

**Answer:**

$$P = \int_{2.5}^{3.5} 100\lambda^{-\frac{3}{4}}(-4)(\lambda-3.5)(\lambda-2.5)d\lambda$$ Substitute functions into integrand

$$P = 400\int_{3.5}^{2.5} \lambda^{-\frac{3}{4}}(\lambda^2-6\lambda+8.75)d\lambda$$ Reverse limits since integrand negative

$$P = 400\int_{3.5}^{2.5} (\lambda^{\frac{5}{4}}-6\lambda^{\frac{1}{4}}+8.75\lambda^{-\frac{3}{4}})d\lambda$$ Simplify integrand

$$P = 400\left[\frac{4}{9}\lambda^{\frac{9}{4}} - 6\left(\frac{4}{5}\right)\lambda^{\frac{5}{4}} + 8.75(4)\lambda^{\frac{1}{4}}\right]_{3.5}^{2.5}$$ Evaluate definite integral

$$P = 400\left[\frac{4}{9}(2.5)^{\frac{9}{4}} - 6\left(\frac{4}{5}\right)(2.5)^{\frac{5}{4}} + 8.75(4)(2.5)^{\frac{1}{4}}\right] - 400\left[\frac{4}{9}(3.5)^{\frac{9}{4}} - 6\left(\frac{4}{5}\right)(3.5)^{\frac{5}{4}} + 8.75(4)(3.5)^{\frac{1}{4}}\right]$$

P = 1397.1 – 6035.7 + 17604.1 – 2978.7 + 9191.5 – 19149.0

**P = 29.3 Janskys**

# Identifying Materials by their Reflectivity 54

| | |
|---|---|
| 50% | 78% |
| 3% | 30% |

| Material | Reflectivity |
|---|---|
| Snow | 80% |
| White Concrete | 78% |
| Bare Aluminum | 74% |
| Vegetation | 50% |
| Bare Soil | 30% |
| Wood Shingle | 17% |
| Water | 5% |
| Black Asphalt | 3% |

When light falls on a material, some of the light energy is absorbed while the rest is reflected. The absorbed energy usually contributes to heating the body. The reflected energy is what we use to actually see the material! Scientists measure reflectivity and absorption in terms of the percentage of energy that falls on the body. The combination must add up to 100%.

The table above shows the reflectivity of various common materials. For example, snow reflects 80% of the light that falls on it, which means that it absorbs 20% and so 80% + 20% = 100%. This also means that if I have 100 watts of light energy falling on the snow, 80 watts will be reflected and 20 watts will be absorbed.

**Problem 1** - If 1000 watts falls on a body, and you measure 300 watts reflected, what is the reflectivity of the body, and from the Table, what might be its composition?

**Problem 2** - You are given the reflectivity map at the top of this page. What are the likely compositions of the areas in the map?

**Problem 3** - What is the average reflectivity of these four equal-area regions combined?

**Problem 4** - Solar radiation delivers 1300 watts per square meter to the surface of Earth. If the area in the map is 20 meters on a side; A) how much solar radiation, in watts, is reflected by each of the four materials covering this area? B) What is the total solar energy, in watts, reflected by this mapped area? C) What is the total solar energy, in watts, absorbed by this area?

**Problem 1** - If 1000 watts falls on a body, and you measure 300 watts reflected, what is the reflectivity of the body, and from the Table, what might be its composition?

Answer: The reflectivity is 100% x (300 watts/1000 watts) = 30%. From the table, Bare Soil has this same reflectivity and so is a likely composition.

**Problem 2** - You are given the reflectivity map at the top of this page. What are the likely compositions of the areas in the map?

Answer: 50% = Vegetation
78% = White Concrete
30% = Bare Soil
3% = Black Asphalt

**Problem 3** - What is the average reflectivity of these four equal-area regions combined? Answer: Because each of the four materials cover the same area, we just add up their reflectivities and divide by 4 to get (50% + 78% + 30% + 3%)/4 = **40%.**

**Problem 4** - Solar radiation delivers 1300 watts per square meter to the surface of Earth. If the area in the map is 20 meters on a side; A) how much solar radiation, in watts, is reflected by each of the four materials covering this area? B) What is the total solar energy, in watts, reflected by this mapped area? C) What is the total solar energy, in watts, absorbed by this area?

Answer: Each material covers 10 meters x 10 meters = 100 square meters:

A) Vegetation: 0.5 x 1300 watts/$m^2$ x 100 $m^2$ = **65,000 watts.**

Concrete: 0.78 x 1300 watts/$m^2$ x 100 $m^2$ = **101,400 watts.**

Bare Soil: 0.30 x 1300 watts/$m^2$ x 100 $m^2$ = **39,000 watts.**

Black Asphalt: 0.03 x 1300 watts/$m^2$ x 100 $m^2$ = **3,900 watts.**

B) 65,000 + 100,000 + 39,000 + 3,900 = **209,300 watts.**

C) The total wattage entering this area is 1,300 watts/$m^2$ x 100 $m^2$ x 4 = 520,000 watts. Since 209,300 watts are reflected, that means that 520,000 watts - 209,300 watts = **310,700 watts are being absorbed.**

# Reflectivity Fingerprints!

| Material | R(UV) | R(Vis) | R(NIR) |
|---|---|---|---|
| Snow | 90% | 80% | 70% |
| White Concrete | 22% | 80% | 73% |
| Aluminum Roof | 75% | 74% | 68% |
| Vegetation | 15% | 50% | 40% |
| Bare Soil | 15% | 30% | 50% |
| Wood Shingle | 7% | 17% | 28% |
| Water | 2% | 5% | 1% |
| Black Asphalt | 4% | 3% | 3% |

The amount of light a body reflects isn't the same for all of the different light wavelengths that fall on its surface. Because of this, each substance can have a unique fingerprint of reflectivity at different wavelengths that lets you identify it. The table above shows the reflectivity of various common materials. For example, snow reflects 80% of the light that falls on it at visible light wavelengths (400 - 600 nm), but reflects quite a bit more at ultraviolet wavelengths (200 - 300 nm), and quite a bit less at infrared wavelengths (700 - 1500 nm).

**Problem 1** - If 1000 watts falls on a body in the ultraviolet band, and you measure 150 watts reflected, what is the reflectivity of the body, and from the Table, A) what might be its composition? B) What other reflectivity measurements can you make to tell the difference between your choices?

**Problem 2** - You are given the reflectivity maps in each of the three wavelength bands, UV, VIS and NIR at the bottom of this page. What are the likely compositions of the areas in the map?

**UV**

| | | |
|---|---|---|
| 15 | 15 | 15 |
| 15 | 15 | 22 |
| 22 | 90 | 75 |

**VIS**

| | | |
|---|---|---|
| 50 | 50 | 30 |
| 50 | 30 | 80 |
| 80 | 80 | 74 |

**NIR**

| | | |
|---|---|---|
| 40 | 40 | 50 |
| 40 | 50 | 73 |
| 73 | 70 | 68 |

# Answer Key

**Problem 1** - If 1000 watts falls on a body in the ultraviolet band, and you measure 150 watts reflected, what is the reflectivity of the body, and from the Table, A) what might be its composition? B) What other reflectivity measurements can you make to tell the difference between your choices?

Answer; A) The reflectivity is 100% x (150 watts/1000 watts) = 15% in the ultraviolet band. There are two candidates from the table: a surface covered with vegetation, and a surface covered with bare soil. B) By measuring the reflected power in the visible band (Vis) the difference in reflectivity is 50% for the vegetation and 30% for the bare soil, which is enough for you to be able to tell the difference.

**Problem 2** - What are the likely compositions of the areas in the map? Answer: See map below. V = Vegetation; bS = Bare Soil; C = Concrete; S = Snow and A = Aluminum Roof.

Note: The problem can be made more challenging by only giving students two out of the three band measurements for a given map 'pixel', and have the student fill -in the missing reflectivity percent and then identify the material.

**UV**

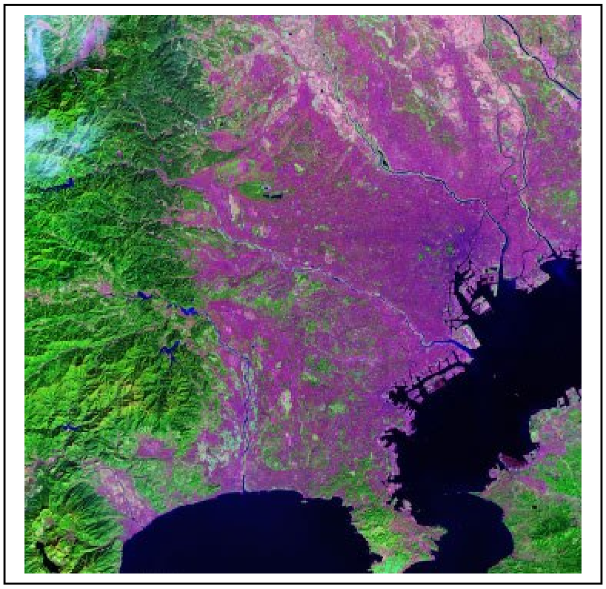
| | | |
|---|---|---|
| 15 V | 15 V | 15 bS |
| 15 V | 15 bS | 22 C |
| 22 C | 90 S | 75 A |

**VIS**

| | | |
|---|---|---|
| 50 | 50 | 30 |
| 50 | 30 | 80 |
| 80 | 80 | 74 |

**NIR**

| | | |
|---|---|---|
| 40 | 40 | 50 |
| 40 | 50 | 73 |
| 73 | 70 | 68 |



Images taken from a satellite are often used to display, both the appearance of an object and the contents of the object. For example, the Landsat image to the left shows Tokyo, Japan. The pixels that make up the image have been colorized to bring out specific details. Purple is used to represent areas that have been developed. Green is for forested areas. By obtaining images of the same scene using different filters, scientists can identify the specific 'colors' of hundreds of different surface features. Let's see how this works!

Suppose that an astronomer has obtained the first crude image of a planet orbiting another star. The satellite observatory was able to image the surface of this planet within a 8x9-pixel (rows X columns) portion of a larger image of the star and its surroundings. Images were obtained in three different color filters Red, Green and Blue, so that surface markings could be classified as water, land, snow or plants/trees. The pixel data sequences for the three images are shown below:

Red = { 0,0,0,0,5,0,0,0,0,0,0,0,5,5,5,0,0,0,0,0,0,0,0,5,0,0,0,0,0,0,5,5,5,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,5,5,5,0,0,0,0,0,0,0,5,0,0,0,0}

Blue= {0,0,0,0,5,0,0,0,0,0,0,0,5,5,5,0,0,0,0,0,5,5,5,0,5,0,0,0,5,5,0,0,0,5,5,0,
0,5,0,0,0,0,5,5,0,0,5,0,0,5,5,5,0,0,0,0,0,5,5,5,0,0,0,0,0,0,0,5,0,0,0,0 }

Green= {0,0,0,0,5,0,0,0,0,0,0,0,5,5,5,0,0,0,0,0,0,0,0,5,0,0,0,0,0,0,5,5,5,0,0,0,
0,0,5,5,5,0,0,0,0,0,0,5,5,0,0,0,0,0,0,0,0,5,5,5,0,0,0,0,0,0,0,5,0,0,0,0 }

**Problem 1** – Create an array table for each of the three images showing the pixel values in their appropriate locations assuming that the images were read-out from the top left pixel to the lower right pixel in the sequence.

**Problem 2** – By comparing the colors for each pixel, determine whether the pixel indicates dark sky S(R,B,G) = S(0,0,0 ); water W(0,5,0); ice I(5,5,5); land L(5,0,5) or plants P(0,0,5). Create a blank grid and fill in the corresponding pixels with the symbols S, W, I, L and P. If there are no matches, place a question mark in that pixel.

**Problem 3** – Using colors of your choosing, create a blank grid and color each pixel with a color suitable for the various symbols (e.g. ice = white, water = blue etc).

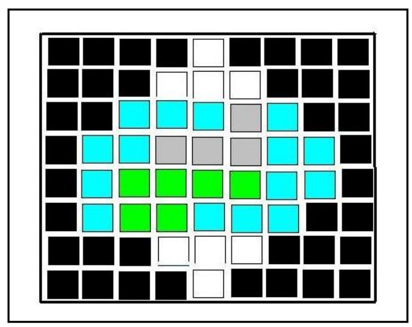
**Problem 4** – Assuming that the planet is perfectly round, draw and color an image of the planet as it might actually appear using the above surface composition information as a clue.

**Problem 1** – Create an array table for each of the three images showing the pixel values in their appropriate locations assuming that the images were read-out from the top left pixel to the lower right pixel in the sequence. Answer:

$$R = \begin{pmatrix} 0\,0\,0\,0\,5\,0\,0\,0\,0 \\ 0\,0\,0\,5\,5\,5\,0\,0\,0 \\ 0\,0\,0\,0\,0\,5\,0\,0\,0 \\ 0\,0\,0\,5\,5\,5\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,5\,5\,5\,0\,0\,0 \\ 0\,0\,0\,0\,5\,0\,0\,0\,0 \end{pmatrix} \quad B = \begin{pmatrix} 0\,0\,0\,0\,5\,0\,0\,0\,0 \\ 0\,0\,0\,5\,5\,5\,0\,0\,0 \\ 0\,0\,5\,5\,5\,0\,5\,0\,0 \\ 0\,5\,5\,0\,0\,0\,5\,5\,0 \\ 0\,5\,0\,0\,0\,0\,5\,5\,0 \\ 0\,5\,0\,0\,5\,5\,5\,0\,0 \\ 0\,0\,0\,5\,5\,5\,0\,0\,0 \\ 0\,0\,0\,0\,5\,0\,0\,0\,0 \end{pmatrix} \quad G = \begin{pmatrix} 0\,0\,0\,0\,5\,0\,0\,0\,0 \\ 0\,0\,0\,5\,5\,5\,0\,0\,0 \\ 0\,0\,0\,0\,0\,5\,0\,0\,0 \\ 0\,0\,0\,5\,5\,5\,0\,0\,0 \\ 0\,0\,5\,5\,5\,0\,0\,0\,0 \\ 0\,0\,5\,5\,0\,0\,0\,0\,0 \\ 0\,0\,0\,5\,5\,5\,0\,0\,0 \\ 0\,0\,0\,0\,5\,0\,0\,0\,0 \end{pmatrix}$$

**Problem 2** – By comparing the colors for each pixel, determine whether the pixel indicates dark sky S(R,B,G) = S(0,0,0 ); water W(0,5,0); ice I(5,5,5); land L(5,0,5) or plants P(0,0,5). Create a blank grid and fill in the corresponding pixels with the symbols S, W, I, L and P.

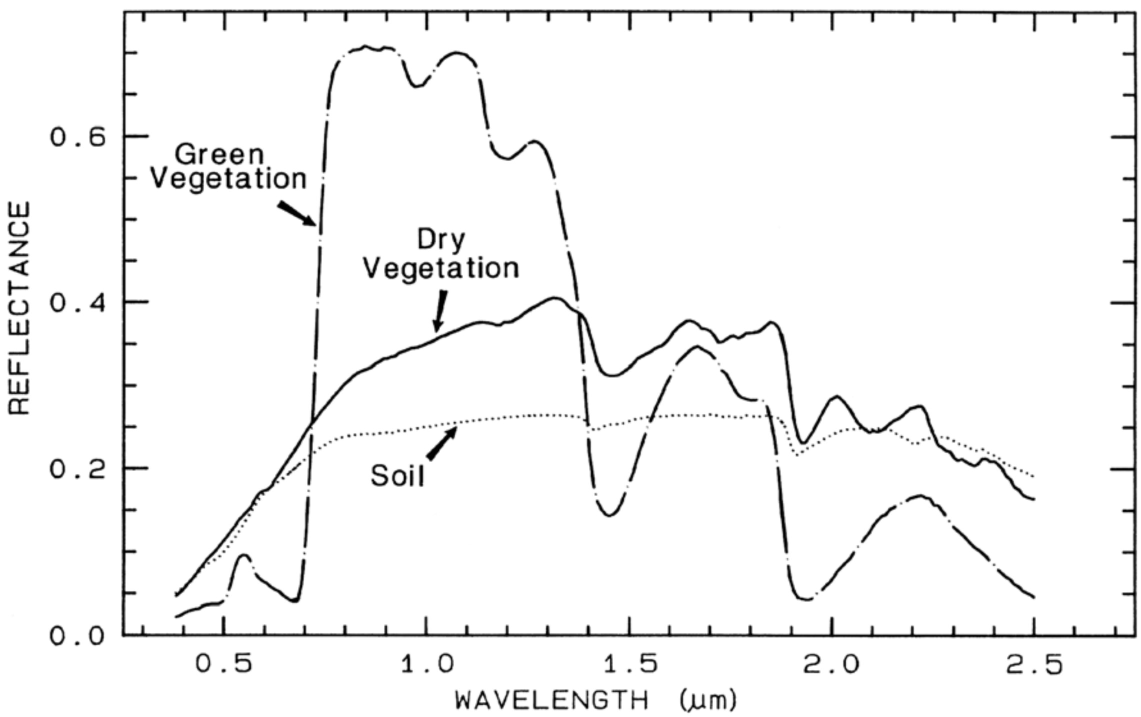
$$\begin{pmatrix} S & S & S & S & I & S & S & S & S \\ S & S & S & I & I & I & S & S & S \\ S & S & W & W & W & L & W & S & S \\ S & W & W & L & L & L & W & W & S \\ S & W & G & G & G & G & W & W & S \\ S & W & G & G & W & W & W & S & S \\ S & S & S & I & I & I & S & S & S \\ S & S & S & S & I & S & S & S & S \end{pmatrix}$$



**Problem 3** – Using colors of your choosing, create a blank grid and color each pixel with a color suitable for the various symbols (e.g. ice = white, water = blue etc). Answer: **See above**.

**Problem 4** – Assuming that the planet is perfectly round, draw and color an image of the planet as it might actually appear using the above surface composition information as a clue.
Answer: **This is a round planet with polar icecaps (white) and a landmass (grey and green) bordered by an ocean (blue).The landmass contains plant life (green) in the equatorial zone. The planet is surrounded by empty space (black)**.

# Graphical Reflectivity Measurements




Very precise measurements can be made of the reflectivity of materials that more easily reveal their subtle differences. Above is a plot of the reflectivities of green vegetation, dry vegetation and soil between wavelengths of 0.4 and 2.5 micrometers. Scientists use graphs such as these to design instruments that help them discriminate between a variety of interesting materials and mineral deposits.

**Problem 1** - An astronomer wants to map the surface of Mars with telescopes on Earth to search for plant life. What wavelength range would help her more easily discriminate between the martian soil and living vegetation?

**Problem 2** - An earth scientist measures the intensity of light between two neighboring land areas at a wavelength of 2.0 microns and 0.7 microns. Spot A appears to be 5 times brighter than Spot B in the longer wavelength band, but nearly equal in brightness in the shorter-wavelength band. What may be the difference in substances between the two spots?

**Problem 3** - The difference in the vegetation reflectivity between green vegetation and dry vegetation is that green vegetation still contains the molecule chlorophyll. What is the difference in absorption by chlorophyll molecules at a wavelength of 0.6 microns?

# Answer Key

**Problem 1** - An astronomer wants to map the surface of Mars with telescopes on Earth to search for plant life. What wavelength range would help her more easily discriminate between the martian soil and living vegetation?

Answer: The graph shows that the reflectivity of green vegetation is substantially brighter than soil between wavelengths of 0.7 to 1.4 microns. At 0.9 microns, the reflectivity of green vegetation is about 0.7 or 70%, while soil is only about 0.25 ( 25%) at the same wavelength, **so green vegetation is nearly three times as bright as soil at 0.7 microns.**

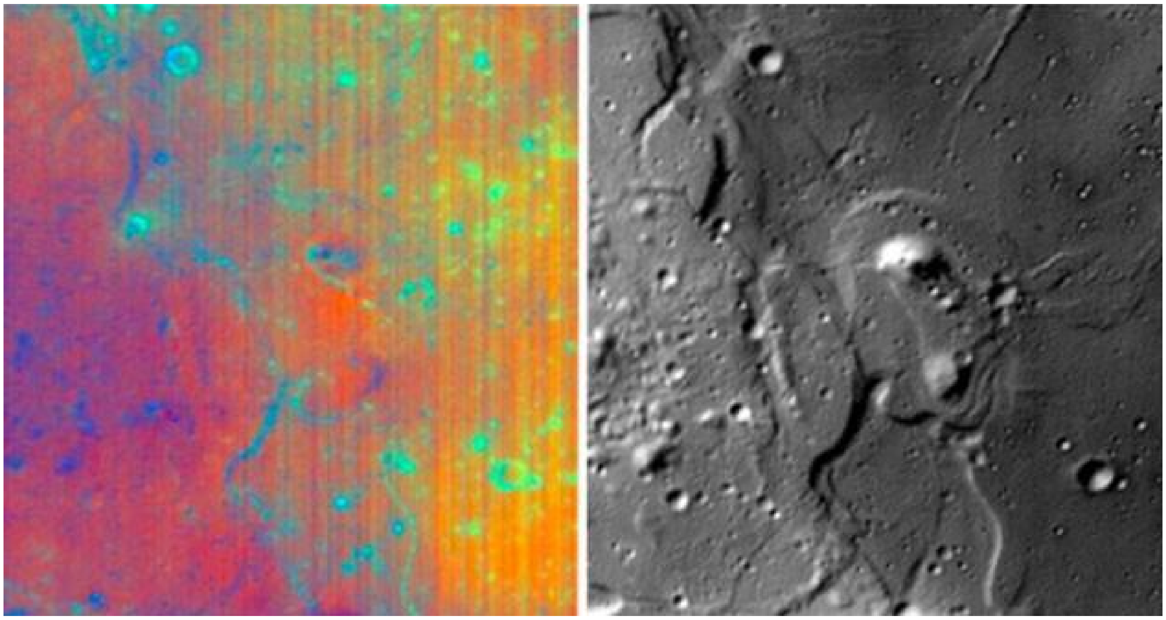
**Problem 2** - An earth scientist measures the intensity of light between two neighboring land areas at a wavelength of 2.0 microns and 0.7 microns. Spot A appears to be 5 times brighter than Spot B in the longer wavelength band, but nearly equal in brightness in the shorter-wavelength band. What may be the difference in substances between the two spots?

Answer: The graph shows that at 0.7 microns, the reflectance curves for green vegetation and dry vegetation cross, which means they are of equal reflectivity at this wavelength. At 2.0 microns, the reflectivity of dry vegetation is about 0.25 or 25%, while green vegetation is much darker and only 0.05 or 5% reflective. **Spot A appears to be 5 times brighter than Spot B so that suggests that Spot B consists of green vegetation and Spot A consists of dry vegetation.**

**Problem 3** - The difference in the vegetation reflectivity between green vegetation and dry vegetation is that green vegetation still contains the molecule chlorophyll. What is the difference in absorption by chlorophyll molecules at a wavelength of 0.6 microns?

Answer: From the graph, green vegetation has a reflectivity of about 5% while dry vegetation has a reflectivity of about 20%. Because %emission + %absorption = 100%, the absorption of green vegetation is 95% while dry vegetation absorbs only 80% of the light at this wavelength. **The difference in absorption is 15%**.

# A Mineral Study of Mare Orientale



The Moon Mineralogy Mapper on India's Chandrayaan-1 satellite measures slight reflectivity changes within 261 wavelength bands from 430 to 3000 nanometers (0.43 to 3 microns) in the light reflected from the lunar surface. The images cover a small part of the Mare Orientale region, and are each 40 km wide. The left-hand image is a false-color, coded image based on 28 separate wavelengths of light reflected from the lunar surface. Green indicates iron-bearing minerals such as pyroxene (basaltic, lava-like material) commonly found in the mare regions. Blue indicates almost pure anorthosite rock commonly found in the lunar highlands.

**Problem 1** - What is the scale of each image in meters/mm? What is the diameter of the smallest discernable crater in the right-hand image?

**Problem 2** - What type of feature is pyroxene mostly associated with?

**Problem 3** - The narrow, diagonal mountain escarpment that you see in the upper right corner of the right-hand image is not seen in the left-hand image. Why do you think this is the case?

**Problem 4** - The visible-band reflectivity of pyroxene is about 25% and anorthosite is about 63%. How much sunlight will 5 square meters of each mineral absorb on the moon's surface if the sun delivers 1300 watts per square meter of energy?

# Answer Key

**Problem 1** - What is the scale of each image in kilometers/mm? What is the diameter of the smallest discernable crater in the right-hand image? Answer; The width of the images is 76 mm which corresponds to 40 kilometers, so the scale is 40,000 m/75 mm = **530 meters/mm.** The smallest craters are about 1mm across which corresponds to **530 meters.**

**Problem 2** - What type of feature is pyroxene mostly associated with? Answer: Pyroxene shows up as green colors in the left-hand mineral map, and the green regions most commonly occur with craters in the right-hand image.

**Problem 3** - The narrow, diagonal mountain escarpment that you see in the upper right corner of the right-hand image is not seen in the left-hand image. Why do you think this is the case?

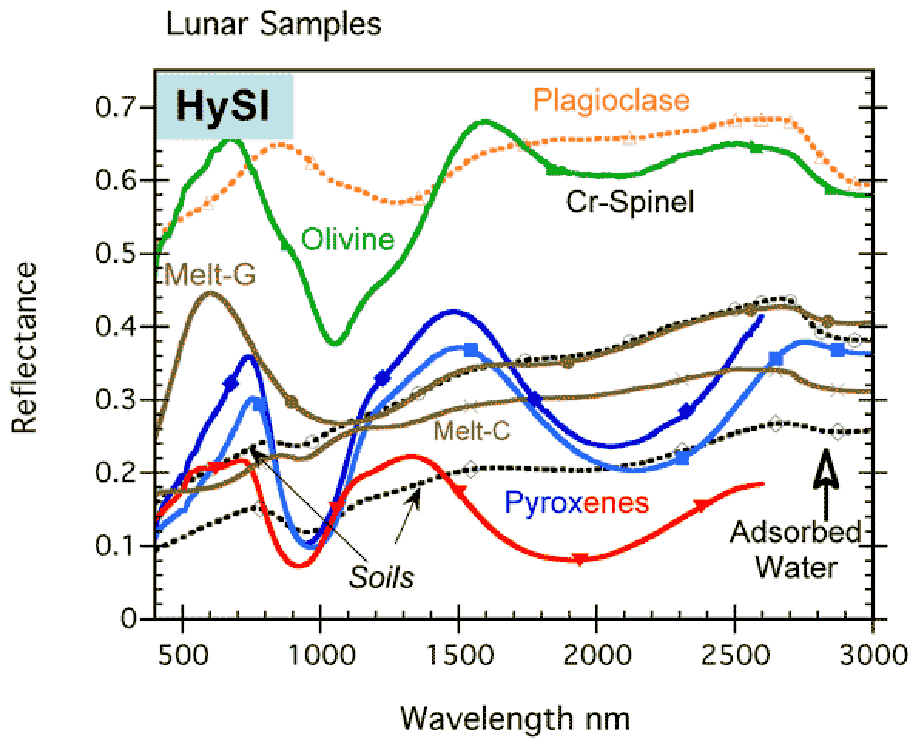
Answer: One answer might be that the material that the escarpment is made from is the same as the surrounding material, so the mineral map on the left would not pick up the difference between the escarpment and the surrounding material.

**Problem 4** - The visible-band reflectivity of pyroxene is about 25% and anorthosite is about 63%. How much sunlight will 5 square meters of each mineral absorb on the moon's surface if the sun delivers 1300 watts per square meter of energy?

Answer: The percentage of energy absorbed = 100% - the percentage reflected.

Pyroxene reflects 25% so it absorbs 75%, and so 5 square meters x 1300 watts/square meter x 0.75 = **4,900 watts.**

Anorthosite reflects 63% and absorbs 37% and so 5 square meters x 1300 watts/square meter x 0.37 = **2,400 watts.**




Lunar rock samples brought back by Apollo astronauts have been carefully examined, and represent many basic classes of minerals. The two most common are the pyroxene-A (blue line) and pyroxene-B (red line), which are found in the extensive lava fields of the lunar mare (dark areas), and plagioclase (orange line), which is found in the mountainous lunar highlands. The above graph shows the reflectivity of these common minerals.

**Problem 1** - A remote-sensing instrument, called a Multi-Spectral Reflectometer, is designed to measure the intensity of light between wavelengths of 1400 to 1500 nanometers (nm), and 2000 to 2100 nm. About what will be the reflectivity of plagioclase, Cr-spinel, Pyroxene A and B, and Melt-G in these two bands?

**Problem 2** - Which mineral will be the brightest in each band?

**Problem 3** - Which mineral has a reflectivity of 45% at 600 nm, and 26% at 1100 nm?

**Problem 4** - Two minerals have the same reflectivity at 1100 nm, and reflectivities of 28% and 40% at 2300 nm. What are the two minerals?

**Problem 5** - At 2000 nm, about 1300 watts of sunlight fall on every square meter of the lunar surface. For 3 square meters of surface area, what mineral will; A) Absorb the most solar energy in watts? B) Absorb the least solar energy in watts? and C) Which material will be the hottest on the surface?

**Problem 1** - A remote-sensing instrument, called a Multi-Spectral Reflectometer, is designed to measure the intensity of light between wavelengths of 1400 to 1500 nm, and 2000 to 2100 nm. About what will be the reflectivity of plagioclase, Cr-spinel, Pyroxene, and Melt-G in these two bands?

Answer: The values are approximate.

| Mineral | Reflectivity (1400-1500 nm) | Reflectivity (2000-2100 nm) |
|---|---|---|
| Plagioclase | **60** | **66** |
| Cr-Spinel | **60** | **60** |
| Pyroxene-A | **40** | **25** |
| Pyroxene-B | **20** | **10** |
| Melt-G | **30** | **38** |

**Problem 2** - Which mineral will be the brightest in each band?
Answer: For 1400-1500 nm, **Plagioclase or Cr-Spinel**
For 2000-2100 nm, **Plagioclase**

**Problem 3** - Which mineral has a reflectivity of 45% at 600 nm, and 26% at 1100 nm?
Answer: **Melt-G**

**Problem 4** - Two minerals have the same reflectivity at 1100 nm, and reflectivities of 28% and 40% at 2300 nm. What are the two minerals?

Answer: **Melt-G (40%) and Pyroxene-A (28%).**

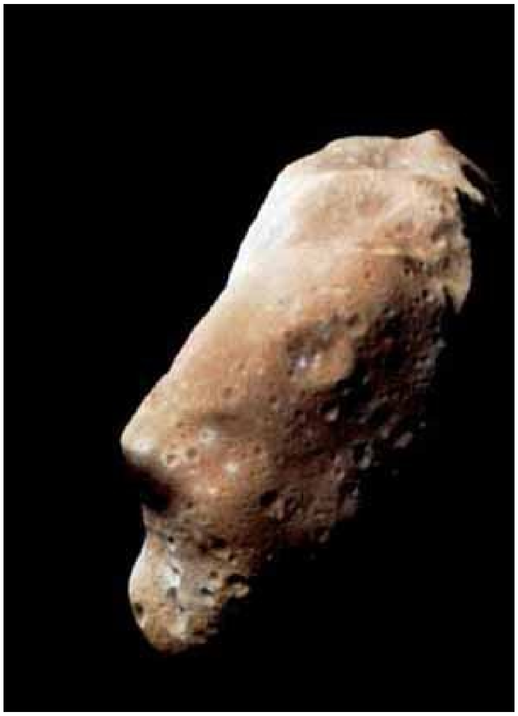
**Problem 5** - At 2000 nm, about 1300 watts of sunlight fall on every square meter of the lunar surface. For 3 square meters of surface area, what mineral will; A) absorb the most solar energy in watts? B) absorb the least solar energy in watts? and C) Which material will be the hottest on the surface?

Answer: Because Absorption + Reflectivity = 100%, the material with the lowest reflectivity will absorb the highest amount of light energy and vice versa.

A) From the possibilities in the graph, pyroxene-B has the lowest reflectivity of about 8% in this band. It therefore will absorb 92% of the light at this wavelength. Since the energy available is 1300 watts/$m^2$, the total power for this sample will be 1300 $w/m^2$ x 3 $m^2$ x 0.92 = **3,600 watts** ( rounding to two significant figures).

B) From the possibilities in the graph, plagioclase has the highest reflectivity of about 66% in this band. It therefore will absorb 34% of the light at this wavelength. Since the energy available is 1300 watts/$m^2$, the total power for this sample will be 1300 $w/m^2$ x 3 $m^2$ x 0.34 = **1,300 watts** ( rounding to two significant figures).

C) Because **plagioclase** absorbs more energy, it will be the hottest material in full sunlight.



Asteroid Gaspara

Astronomers studying the asteroid 24-Themis detected water-ice and carbon-based organic compounds on the surface of the asteroid.

NASA detects, tracks and characterizes asteroids and comets passing close to Earth using both ground and space-based telescopes.

NASA is particularly interested in asteroids with water ice because this resource could be used to create fuel for interplanetary spacecraft.

On October 7, 2009, the presence of water ice was confirmed on the surface of this asteroid using NASA's Infrared Telescope Facility. The surface of the asteroid appears completely covered in ice. As this ice layer is sublimated, it may be getting replenished by a reservoir of ice under the surface. The orbit of the asteroid varies from 2.7 AU to 3.5 AU so it is located within the asteroid belt. The asteroid is 200 km in diameter, has a mass of $1.1 \times 10^{19}$ kg, and a density of 2,800 kg/m$^3$ so it is mostly rocky material similar in density to Earth's.

By measuring the spectrum of infrared sunlight reflected by the object, the NASA researchers found the spectrum consistent with frozen water and determined that 24 Themis is coated with a thin film of ice. The asteroid is estimated to lose about 1 meter of ice each year, so there must be a sub-surface reservoir to constantly replace the evaporating ice.

**Problem 1** – Assume that the asteroid has a diameter of 200 km. How many kilograms of water ice are present in a layer 1-meter thick covering the entire surface, if the density of ice is 1,000 kg/meter$^3$? (Hint: Volume = Surface area x thickness)

**Problem 2** – Suppose that only 1% by volume of the 1-meter-thick 'dirty' surface layer is actually water-ice and that it evaporates 1 meter per year, what is the rate of water loss in kg/sec?

**Problem 1** – Assume that the asteroid has a diameter of 200km. How many kilograms of water ice are present in a layer 1-meter thick covering the entire surface, if the density of ice is 1,000 kg/meter$^3$? (Hint: Volume = Surface area x thickness)

Answer: Volume = surface area x thickness.

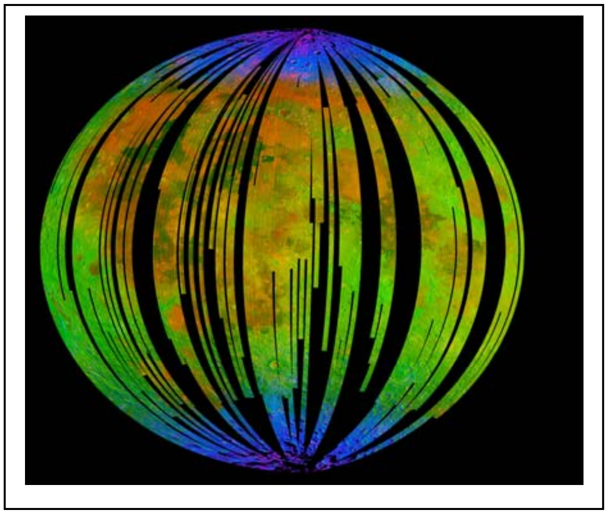
$$SA = 4 \pi r^2$$
$$= 4 (3.14) (100{,}000 \text{ meters})^2$$
$$= 1.3 \times 10^{11} \text{ meters}^2$$
$$\text{Volume} = 1.3 \times 10^{11} \text{ meters}^2 \times 1 \text{ meter}$$
$$= 1.3 \times 10^{11} \text{ meters}^3$$

$$\text{Mass of water} = \text{density} \times \text{volume}$$
$$= 1{,}000 \text{ kg/meter}^3 \times 1.3 \times 10^{11} \text{ meter}^3$$
$$= \mathbf{1.3 \times 10^{14}\ kg} \quad \text{(or 130 billion tons)}$$

**Problem 2** – Suppose that only 1% by volume of the ‘dirty’ 1-meter-thick surface layer is water-ice and that it evaporates 1 meter per year, what is the rate of water loss in kg/sec?

Answer: The mass of water in the outer 1-meter layer is 1% of $1.3 \times 10^{14}$ kg or $1.3 \times 10^{12}$ kg. Since 1 year = 365 days x 24h/day x 60m/hr x 60 sec/min = $3.1 \times 10^7$ seconds, the mass loss is just $1.3 \times 10^{12}$ kg / $3.1 \times 10^7$ sec = **42,000 kg/sec or 42 tons/sec**.

# Water on the Moon !



The debate has gone back and forth over the last 10 years as new data are found, but measurements by Deep Impact/EPOXI, Cassini and most recently the Lunar Reconnaissance Orbiter and Chandrayaan-1 are now considered conclusive. Beneath the shadows of polar craters, and clinging to the lunar regolith, billions of gallons of water are available for harvesting by future astronauts.

The image to the left created by the Moon Minerology Mapper (M3) instrument onboard Chandrayaan-1, shows deposits and sources of hydroxyl molecules. The data has been colored blue and superimposed on a lunar photo.

Complimentary data from the Deep Impact/EPOXI and Cassini missions of the rest of the lunar surface also detected hydroxyl molecules covering about 25% of the surveyed lunar surface. The hydroxyl molecule consists of one atom of oxygen and one of hydrogen, and because water is basically a hydroxyl molecule with a second hydrogen atom added, detecting hydroxyl on the moon is an indication that water molecules are also present.

How much water might be present? The M3 instrument can only detect hydroxyl molecules if they are in the top 1-millimeter of the lunar surface. The measurements also suggest that about 1 metric ton of lunar surface has to be processed to extract 1 liter (0.26 gallons) of water.

**Problem 1** – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

**Problem 2** – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter$^3$. How many metric tons of regolith are found in the surface volume calculated in Problem 1?

**Problem 3** – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the lunar surface contains water?

**Problem 4** – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters).

**Problem 1** – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Answer: The surface area of a sphere is given by $S = 4\pi r^2$ and so the volume of a layer with a thickness of L is $V = 4\pi r^2 L$ provided that L is much smaller than r.
$V = 4 \times (3.141) \times (1731000)^2 \times 0.001 =$ **$3.76 \times 10^{10}$ $m^3$**

**Problem 2** – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter$^3$. How many metric tons of regolith are found in the surface volume calculated in Problem 1? Answer: $3.76 \times 10^{10}$ $m^3 \times (3000\ kg/m^3) \times (1\ ton/1000\ kg) =$ **$1.13 \times 10^{11}$ metric tons.**

**Problem 3** – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the surface contains water? Answer: $1.13 \times 10^{11}$ tons x (1 liter water/1 ton regolith) x 1/4 = **$2.8 \times 10^{10}$ liters of water.**

**Problem 4** – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters). Answer: $2.8 \times 10^{10}$ liters x (1 gallon / 3.78 liters) = $7.5 \times 10^9$ gallons of water or about **8 billion gallons of water.**

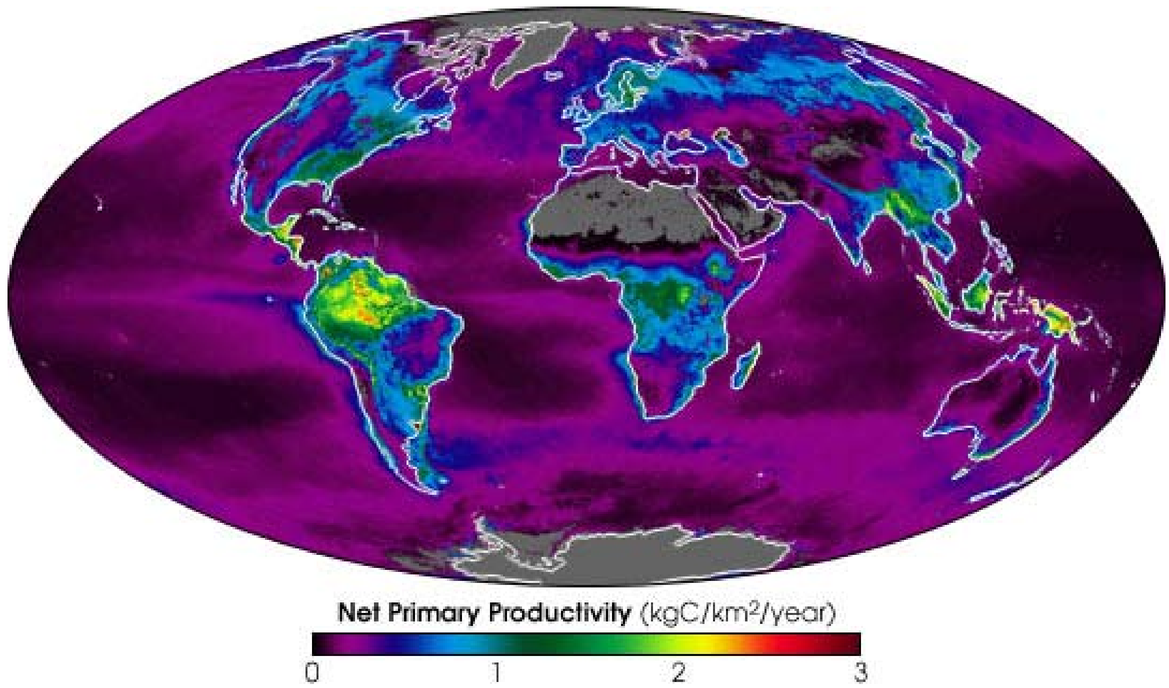
Note: This is similar to the roughly '7 billion gallon' estimate made by the M3 scientists as described in the NASA Press Release for this discovery in September 2009.

For more information, visit:

Moon Minerology Mapper News - http://moonmineralogymapper.jpl.nasa.gov/

**The front picture** of the moon is from NASA's Moon Mineralogy Mapper on the Indian Space Research Organization's Chandrayaan-1 mission. It is a three-color composite of reflected near-infrared radiation from the sun, and illustrates the extent to which different materials are mapped across the side of the moon that faces Earth. Small amounts of water and hydroxyl (blue) were detected on the surface of the moon at various locations. This image illustrates their distribution at high latitudes toward the poles. **Blue** shows the signature of water and hydroxyl molecules as seen by a highly diagnostic absorption of infrared light with a wavelength of three micrometers. **Green** shows the brightness of the surface as measured by reflected infrared radiation from the sun with a wavelength of 2.4 micrometers. **Red** shows an iron-bearing mineral called pyroxene, detected by absorption of 2.0-micrometer infrared light.

# Earth's Carbon Metabolism Revealed




NASA scientists unveiled the first consistent and continuous global measurements of Earth's "metabolism" based on data from the Terra and Aqua satellites. This new measurement is called Net Primary Production because it indicates how much carbon dioxide is taken in by vegetation during photosynthesis minus how much is given off during respiration. The false-color map shows the rate at which plants absorbed carbon out of the atmosphere during the years 2001 and 2002. The yellow and red areas show the highest rates of absorption, ranging from 2 to 3 kilograms of carbon taken out of the atmosphere per square kilometer per year. The green areas are intermediate rates, while blue and purple shades show progressively lower productivity.

**Problem 1** - According to the map, which regions are the most productive in removing carbon from the atmosphere?

**Problem 2** - Assume that the Amazon Basin has an area of 7 million square kilometers. How many metric tons of carbon does it remove from the atmosphere each year?

**Problem 3** - The oceans cover an area of 335 million square kilometers. What is the average rate of carbon removal according to the map color, and how many metric tons of carbon is removed by plant life on the oceans?

**Problem 4** - The mass of carbon dioxide is 3.7 times more than pure carbon. How many metric tons of carbon dioxide do your answers to Problem 2 and 3 represent?

**Problem 1** - According to the map, which regions are the most productive in removing carbon from the atmosphere? Answer: The most productive places on Earth show up as a yellow color on the map. These are found, in large quantities in the Amazon Basin and in Indonesia.

**Problem 2** - Assume that the Amazon Basin has an area of 7 million square kilometers. How many tons of carbon does it remove from the atmosphere each year? Answer: Its yellow color indicates a carbon removal rate of about 2 kg per square kilometer per year, so the total removal is 14 million kg per year. This equals **14,000 tons of carbon per year.**

**Problem 3** - The oceans cover an area of 335 million square kilometers. What is the average rate of carbon removal according to the map color, and how many tons of carbon is removed by plant life on the oceans? Answer: The average color is magenta, which corresponds to a rate of about 0.3 kg per square kilometer per year, so the removal rate from the world's oceans is about 0.3 x 335 million = 100 million kilograms/year or 100,000 tons/year of carbon.

**Problem 4** - The mass of carbon dioxide is 3.7 times more than pure carbon. How many tons of carbon dioxide do your answers to Problem 2 and 3 represent?

Answer: 14,000 tons of carbon/year x 3.7 = **52,000 tons/year** from the Amazon Basin, and 100,000 tons/year x 3.7 = **370,000 tons/year** of carbon dioxide from the world's oceans.

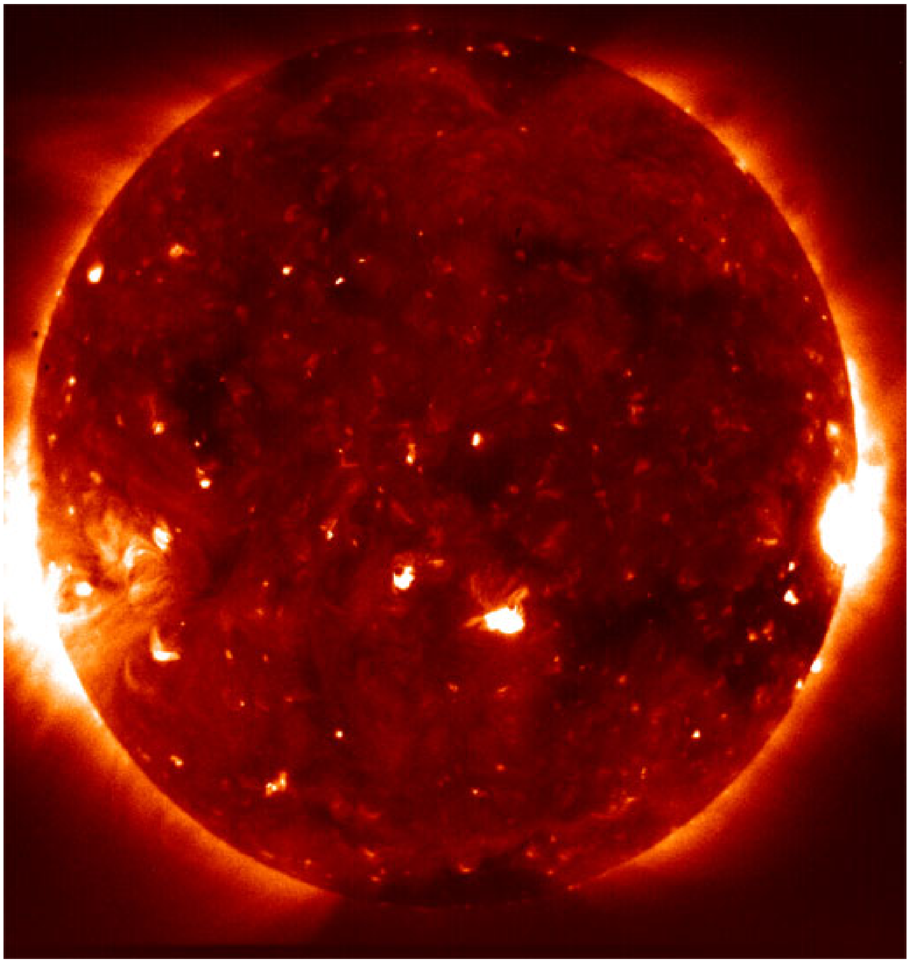


Image taken by the Hinode satellite's X-ray Telescope (XRT) using x-rays emitted by the sun at energies between 1,000 to 10,000 electron volts (1 to 10 keV).The resolution is 2 arcseconds. At these energies, only plasma heated to over 100,000 degrees K produce enough electromagnetic energy to be visible. The solar surface, called the photosphere, at a temperature of 6,000 K is too cold to produce x-ray light, and so in X-ray pictures it appears black.

The Hinode image shows for the first time that the typically dark areas of the sun can contain numerous bright 'micro-flares' that speckle the surface, releasing energy into the corona.

**Problem 1** - How big are the micro-flares compared to Earth? (Sun diameter = 1,300,000 km; Earth diameter = 12,500 km).

**Problem 2** - About how many micro-flares can you see on this hemisphere of the Sun, and how many would you estimate exist at this time for the entire solar surface?

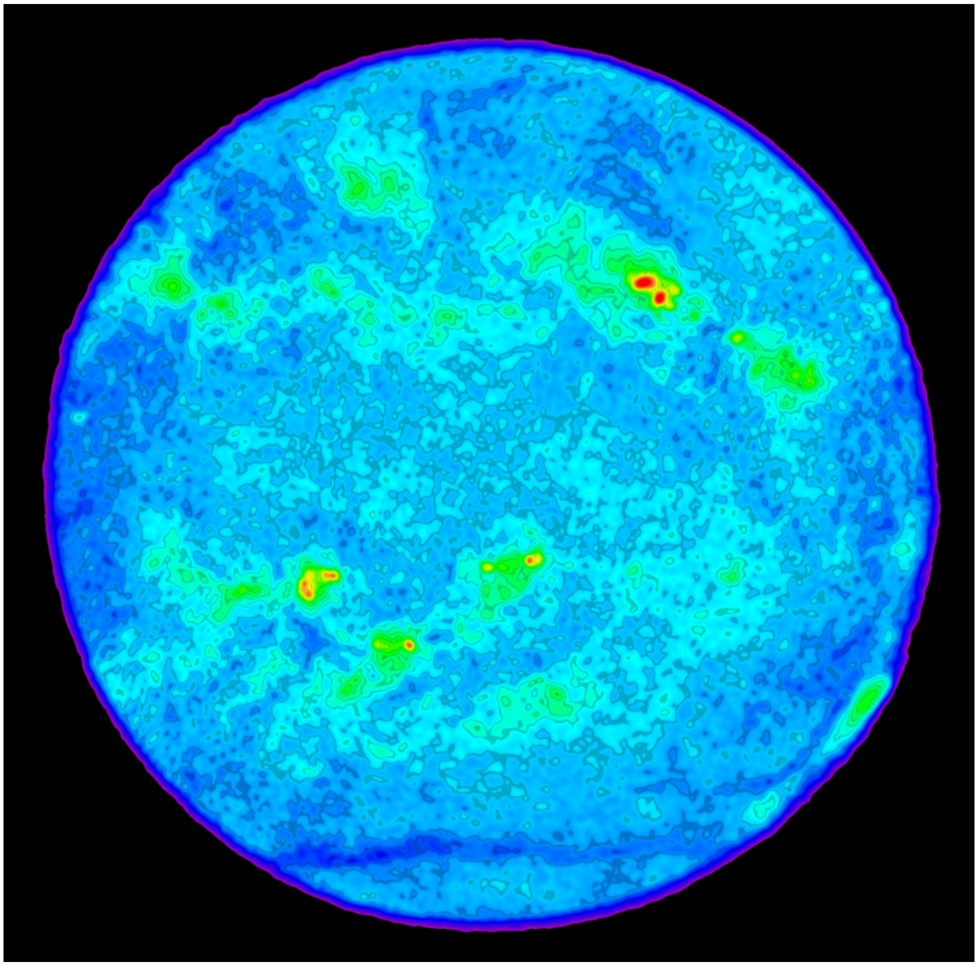
**Problem 1** - How big are the micro-flares compared to Earth? (Sun diameter = 1,300,000 km; Earth diameter = 12,500 km).

Answer: The disk of the Sun measures about 110 millimeters in diameter, so the scale is 1,300,000 km/110 mm = 12,000 km/mm. The micro-flares are just over 1 millimeter in diameter or 12,000 kilometers, which is **similar to the diameter of Earth.**

**Problem 2** - About how many micro-flares can you see on this hemisphere of the Sun, and how many would you estimate exist at this time for the entire solar surface?

Answer: A careful count should find **about 200** of these 'spots' some bright and some faint. Over the entire solar surface there would be **about 400.**

# Seeing the Sun at Radio Wavelengths



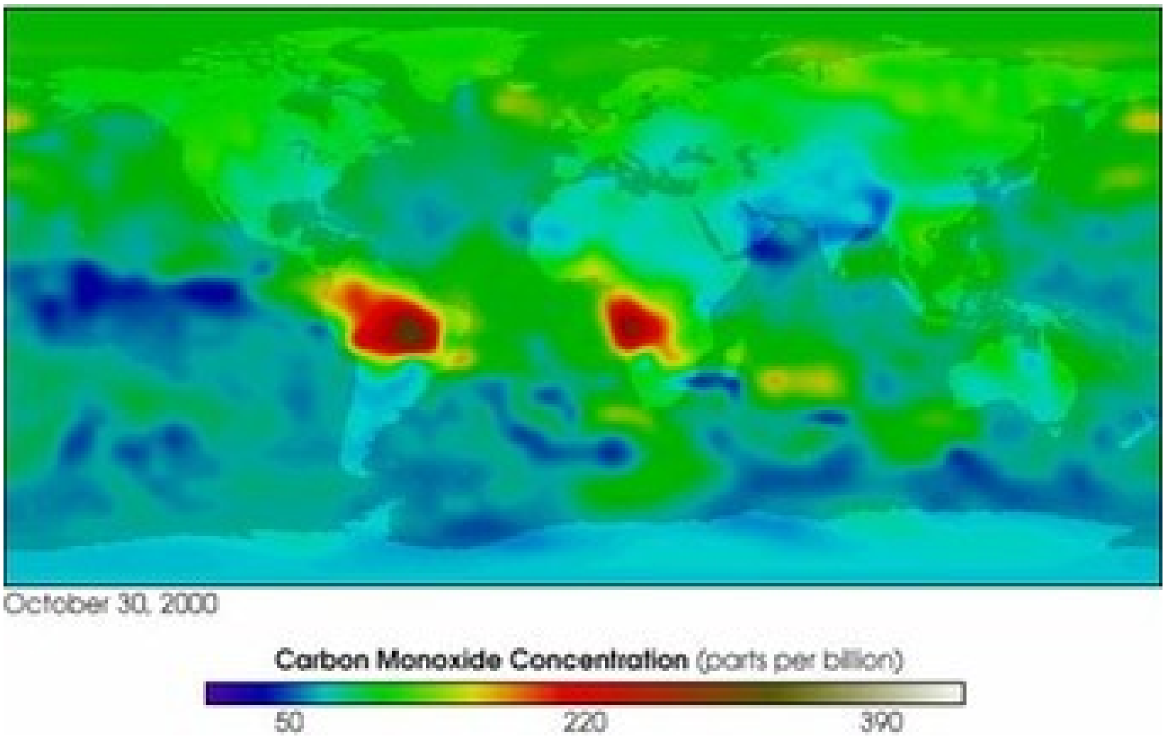
High resolution false-color image obtained at a frequency of 4.7 GHz by the Very Large Array radio telescope (VLA) of the 'quiet sun' at a resolution of 12 arcseconds, from plasma emitting at 30,000 K. The brightest features (red) in this false-color image have temperatures of about 100,000 degrees K and coincide with sunspots. The green features are cooler and show where the Sun's atmosphere is very dense. At this frequency the radio-emitting surface of the Sun has an average temperature of 30,000 K, and the dark blue features are cooler yet. (*Courtesy: Stephen White, University of Maryland, and NRAO/AUI*).

**Problem** - From the scale of this image, what is the size of the smallest feature compared to the diameter of Earth (12,500 km)? (Sun diameter = 1,300,000 km)

**Problem** - From the scale of this image, what is the size of the smallest feature compared to the diameter of Earth (12,500 km)?

Answer: The sun disk is about 115 millimeters in diameter so the scale is 1,300,000 km/115 mm = 11,300 km/mm. The smallest features are the dark blue 'freckles' which hare about 1-2 millimeters across, corresponding to a physical size of 11,000 to 23,000 kilometers. This is about **1 to 2 times the size of Earth.**

# Carbon Monoxide - A Deadly Trace Gas




The NASA Terra satellite has created this map of carbon monoxide in the lower atmosphere using the MOPITT instrument, and for the first time allows scientists to study the sources and movements of large concentrations of this harmful gas. Carbon monoxide is usually associated with forest fires in the natural setting, although fall leaf decomposition accounts for 20% of the annual natural production. The false-colors in this image show the concentration of carbon monoxide in units of parts-per-billion (ppb). They range from 390 ppb (dark brown) and 220 ppb (red) to 50 ppb (blue). One ppb means that there is one molecule of carbon monoxide (CO) for every one billion other atoms of molecules of the other atmospheric constituents (mostly nitrogen and oxygen).

**Problem 1** - Geographically, where are the largest producers of carbon monoxide on Earth, and why do you think this is the case?

**Problem 2** - The total mass of carbon monoxide in the entire atmosphere is 550 million tons (500 megatons), with an average concentration of 100 ppb as shown by the large amount of 'green' in the Terra map. The atmosphere has a surface density of 10 megatons/km$^2$. Assuming that the entire amount of CO in the anomaly was released in one day, if the total area involved in the Amazon Basin and African fires is about 20 million km$^2$, at a concentration of 230 ppb, what is the total mass of carbon monoxide released into the atmosphere by these fires in A) one day? B) one year?

**Problem 1** - Geographically, where are the largest producers of carbon monoxide on Earth, and why do you think this is the case?

Answer: **These producers are in the Amazon Basin and sub-Saharan Africa. These are known to be regions where farmers routinely burn down forests to create farmland for livestock and crops.**

**Problem 2** - The total mass of carbon monoxide in the entire atmosphere is 550 million tons (500 megatons), with an average concentration of 100 ppb as shown by the large amount of 'green' in the Terra map. The atmosphere has a surface density of 10 megatons/km$^2$. Assuming that the entire amount of CO in the anomaly was released in one day, if the total area involved in the Amazon Basin and African fires is about 20 million km$^2$, at a concentration of 230 ppb, what is the total mass of carbon monoxide released into the atmosphere by these fires in A) one day? B) one year?

Answer: The total atmospheric mass over these two regions is 10 megatons/km$^2$ x 20 million km$^2$ = 200 million million tons or 200,000 gigatons.

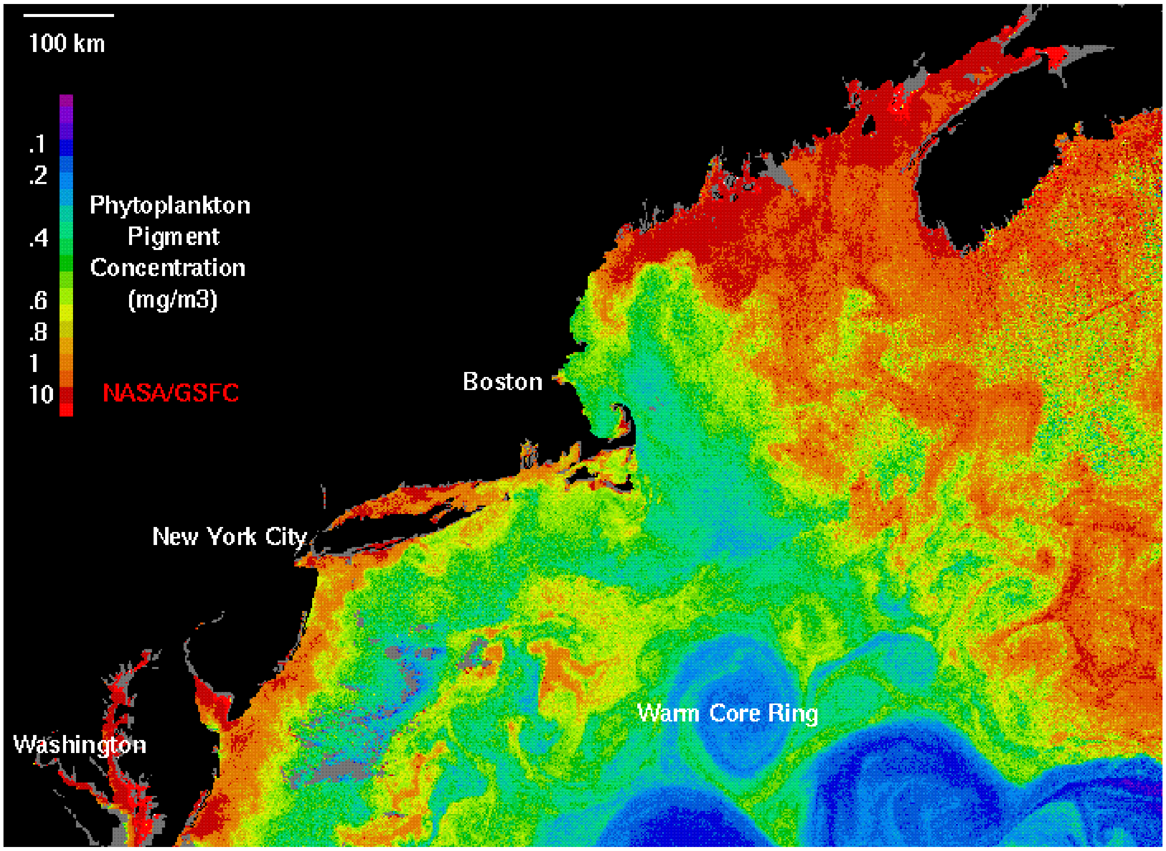
From the concentration of carbon monoxide as 230 ppb, the normal concentration is only 100 ppb, so the fires contribute 230-100 = 130 ppb additional carbon monoxide.

A) Since the total atmospheric mass over these two regions is 200,000 gigatons at an average of 100 ppb, the amount of excess carbon monoxide is

200,000 gigatons x (130 /100 ) x (1/1 billion) = **260,000 tons in one day**.

B) Multiplying the daily release in the map by 365 days/year gives 365 x 260,000 tons = **95 megatons/year**….but these fires may not burn all year long.

# Studying Ocean Plankton from Space




The purpose of the NASA Sea-viewing Wide Field-of-view Sensor (SeaWiFS) Project is to provide quantitative data on global ocean bio-optical properties to the Earth science community. Subtle changes in ocean color signify various types and quantities of marine phytoplankton (microscopic marine plants), the knowledge of which has both scientific and practical applications. Since an orbiting sensor can view every square kilometer of cloud-free ocean every 48 hours, satellite-acquired ocean color data constitute a valuable tool for determining the abundance of ocean biota on a global scale and can be used to assess the ocean's role in the global carbon cycle.

**Problem 1** - The above map gives the concentration of phytoplankton in units of milligrams per cubic meter of water. Near the coastline of the eastern United States, the concentration is about 10 milligrams per cubic meter. How much plankton, in kilograms, could you harvest by processing 1 billion gallons of seawater every day? (1 gallon equals 3.78 liters).

**Problem 2** - In which areas would you most expect to find whales and other aquatic life?

**Problem 3** - How far to the east of Cape Cod do fishing boats have to travel before they encounter areas where fishing might be economically profitable?

**Problem 1** - The above map gives the concentration of phytoplankton in units of milligrams per cubic meter of water. Near the coastline of the eastern United States, the concentration is about 10 milligrams per cubic meter. How much plankton, in metric tons, could you harvest by processing 1 billion gallons of seawater every day? (1 gallon equals 3.78 liters).

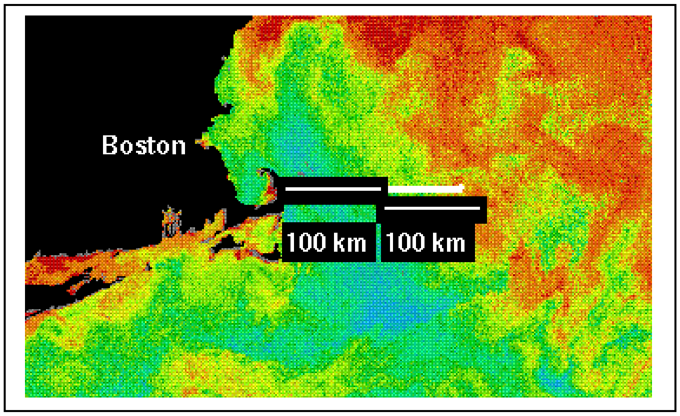
Answer: There are several approaches to this problem. First convert cubic meters to gallons, then convert the plankton concentration to grams per gallon, then multiply by the number of gallons and convert to metric tons.

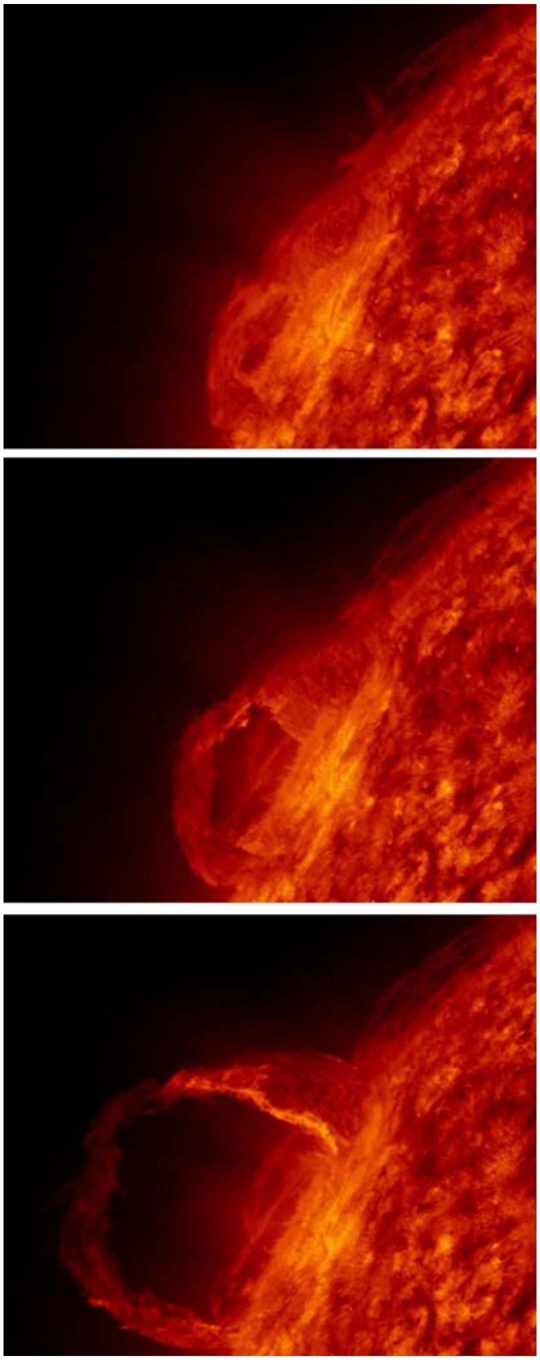
1 $m^3$ x ( 1,000,000 $cm^3/m^3$) x (1 liter/1000 $cm^3$) x (1 gallon/3.78 liters) = 265 gallons. Then 10 milligrams/$m^3$ x (1 $m^3$/265 gallons) = 0.000038 grams/gallon. Then 0.000038 grams/gallon x 1 billion gallons = 38,000 grams or **38 kilograms**.

Note: One average-sized swimming pool contains about 700,000 gallons of water!

**Problem 2** - In which areas would you most expect to find whales and other aquatic life? Answer: The areas of the highest plankton concentration are coded in red. Plankton consists of embryonic and young life forms, and whose adult forms are often co-located with the plankton. Also, plankton is on the lowest rung of the food chain and is a major food source for aquatic animals, so **we would expect that the distribution of plankton in the ocean closely matches the distribution pattern of living things that feed on the plankton**. By measuring plankton concentration from space, we can also measure, and keep track of, the location of various aquatic biomes.

**Problem 3** - How far to the east of Cape Cod do fishing boats have to travel before they encounter areas where fishing might be economically profitable? Answer: From the map below, and the scale bar that indicates '100 kilometers', we see that they have to travel about **150 to 200 kilometers** to enter the densest plankton concentrations. This region is called Georges Bank, and fishermen have to consume quite a lot of diesel fuel to get their boats out to where they can start fishing, usually under hazardous conditions.





On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the Sun. Among them was a sequence of images taken on March 30, showing an eruptive prominence ejecting millions of tons of plasma into space. The three images to the left show selected scenes from the first 'high definition' movie of this event. The top image was taken at 17:50:49, the middle image at 18:02:09 and the bottom image at 18:13:29.

**Problem 1** – The width of the image is 300,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

**Problem 2** – If the Earth has a radius of 6,378 km, how many Earths wide is this prominence?

**Problem 3** – What was the average speed of the prominence in A) kilometers/second? B) Kilometers/hour? C) Miles/hour?

For additional views of this prominence, see the NASA/SDO movies at:

**http://svs.gsfc.nasa.gov/vis/a000000/a003600/a003693/index.html**

or to read the Press Release:

**http://www.nasa.gov/mission_pages/sdo/news/first-light.html**

**Problem 1** – The width of the image is 300,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Answer: The width is 70 millimeters so the scale is 300,000 km/70 mm **= 4,300 km/mm**

**Problem 2** – If the Earth has a radius of 6,378 km, how many Earths wide is this prominence?

Answer: **The diameter of the loop is about 35 millimeters or 35mm x 4300km/mm = 150,000 km. The diameter of Earth is 13,000 km, so the loop is 12 times the diameter of Earth.**

**Problem 3** – What was the average speed of the prominence in A) kilometers/second? B) Kilometers/hour? C) Miles/hour?

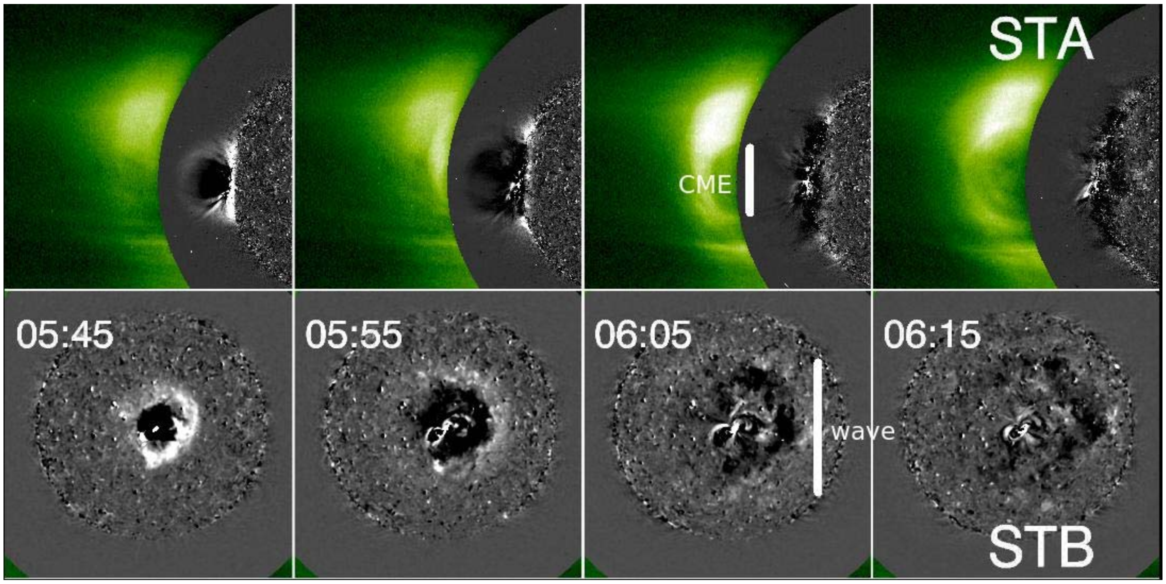
Answer: Speed = distance traveled / time elapsed.
In the bottom image, draw a straight line from the lower right corner THROUGH the peak of the coronal loop. Now draw this same line at the same angle on the other two images. With a millimeter ruler, measure the distance along the line from the lower right corner to the edge of the loop along the line. Example:
Top: 47 mm ;
Middle: 52 mm,
Bottom: 67 mm.
The loop has moved 67mm – 47mm = 20 millimeters. At the scale of the image this equals 20mm x 4,300 km/mm so D = 86,000 km.

The time between the bottom and top images is 18:13:29 – 17:50:49 or 22minutes and 40 seconds or 1360 seconds.

A) The average speed of the loop is then S = 86,000 km/1360 sec = **63 km/sec**.

B) 63 km/sec x 3600 sec/hr = **227,000 km/hour**.

C) 227,000 km/hr x 0.62 miles/km = **140,000 miles/hour**.




A solar tsunami that occurred in February 13, 2009 has recently been identified in the data from NASA's STEREO satellites. It was spotted rushing across the Sun's surface. The blast hurled a billion-ton cloud of plasma into space and sent a tsunami racing along the sun's surface. STEREO recorded the wave from two positions separated by 90 degrees, giving researchers a spectacular view of the event. Satellite A (STA) provided a side-view of the explosion, called a Coronal Mass Ejection (CME), while Satellite B (STB) viewed the explosion from directly above. The technical name is "fast-mode magnetohydrodynamic wave" – or "MHD wave" for short. The one STEREO saw raced outward at 560,000 mph (250 km/s) packing as much energy as 2,400 megatons of TNT.

**Problem 1** - In the lower strip of images, the sun's disk is defined by the mottled circular area, which has a physical radius of 696,000 kilometers. Use a millimeter ruler to determine the scale of these images in kilometers/mm.

**Problem 2** - The white circular ring defines the outer edge of the expanding MHD wave. How many kilometers did the ring expand between 05:45 and 06:15? ( Note '05:45' means 5:45 o'clock Universal Time).

**Problem 3** - From your answers to Problem 1 and 2, what was the approximate speed of this MHD wave in kilometers/sec?

**Problem 4** - Kinetic Energy is defined by the equation K.E. = 1/2 m $V^2$ where m is the mass of the object in kilograms, and V is its speed in meters/sec. Suppose the mass of the CME was about 1 billion metric tons, use your answer to Problem 3 to calculate the K.E., which will be in units of Joules.

**Problem 5** - If 1 kiloton of TNT has the explosive energy of $4.1 \times 10^{12}$ Joules, how many megatons of TNT does the kinetic energy of the tsunami represent?

**Problem 1** - In the lower strip of images, the sun's disk is defined by the mottled circular area, which has a physical radius of 696,000 kilometers. Use a millimeter ruler to determine the scale of these images in kilometers/mm.

Answer: The diameter is 31 millimeters ,which corresponds to 2 x 696,000 km or 1,392,000 km. The scale is then 1,392,000 km/31mm = **45,000 km/mm**.

**Problem 2** - The white circular ring defines the outer edge of the expanding MHD wave. How many kilometers did the ring expand between 05:45 and 06:15? ( Note '05:45' means 5:45 o'clock Universal Time).

Answer: From the scale of 45,000 km/mm, the difference in the ring radii is 12mm - 5 mm = 7mm which corresponds to 7 mm x (45,000 km/1 mm) = **315,000 kilometers**. Students answers may vary depending on where they defined the outer edge of the ring.

**Problem 3** - From your answers to Problem 1 and 2, what was the approximate speed of this MHD wave in kilometers/sec?

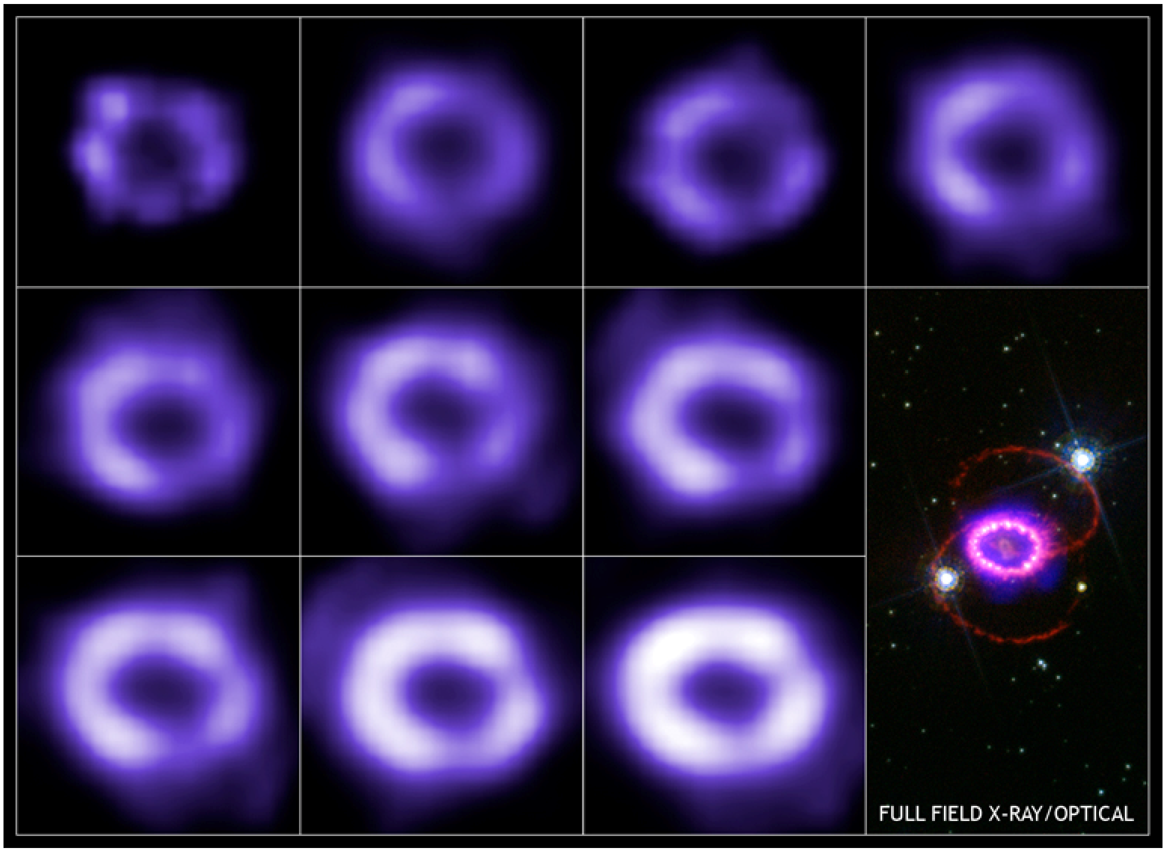
Answer: The time difference is 06:15 - 05:45 = 30 minutes. The speed was about 315,000 km / 30 minutes = 11,000 kilometers/minute, which is 11,000 km/minute x (1 minute/60 seconds) = **180 kilometers/sec**.

**Problem 4** - Kinetic Energy is defined by the equation K.E. = 1/2 m $V^2$ where m is the mass of the object in kilograms, and V is its speed in meters/sec. Suppose the mass of the CME was about 1 million metric tons, use your answer to Problem 3 to calculate the K.E., which will be in units of Joules.

Answer: The mass of the CME was 1 billion metric tons. There are 1,000 kilograms in 1 metric ton, so the mass was $1.0 \times 10^{12}$ kilograms. The speed is 180 km/sec which is 180,000 meters/sec. The kinetic energy is then about $0.5 \times 1.0 \times 10^{12} \times (180,000)^2$ = **$1.6 \times 10^{22}$ Joules.**

**Problem 5** - If 1 kiloton of TNT has the explosive energy of $4.1 \times 10^{12}$ Joules, how many megatons of TNT does the kinetic energy of the tsunami represent?

Answer: $1.6 \times 10^{22}$ Joules x (1 kiloton TNT/$4.1 \times 10^{12}$ Joules) = $3.9 \times 10^{9}$ kilotons TNT. Since 1 megaton = 1,000 kilotons, we have an explosive yield of **3,900,000 megatons TNT**. (Note; this answer differs from the STEREO estimate because the speed is approximate, and does not include the curvature of the sun).




In March, 1987 a supernova occurred in the Large Magellanic Cloud; a nearby galaxy to the Milky Way about 160,000 light years away from Earth. The site of the explosion was traced to the location of a blue supergiant star called Sanduleak -69º 202 (SK -69 for short) that had a mass estimated at approximately 20 times our own sun.The series of image above, taken by the Chandra X-ray Observatory, shows the expansion of the million-degree gas ejected by the supernova between January, 2000 (top left image) to January, 2007 (lower right image). The width of each image is 1.9 light years.

**Problem 1** - Using a millimeter ruler, what is the scale of each image in light years/millimeter?

**Problem 2** - If 1 light year = $9.5 \times 10^{12}$ kilometers, and 1 year = $3.1 \times 10^{7}$ seconds, what was the average speed of the supernova gas shell between 2000 and 2007?

**Problem 1** - Using a millimeter ruler, what is the scale of each image in light years/millimeter?

Answer: The width of each image is about 38 millimeters, so the scale is 1.9 light years / 38 mm = **0.05 light years/mm**.

**Problem 2** - If 1 light year = $9.5 \times 10^{12}$ kilometers, and 1 year = $3.1 \times 10^{7}$ seconds, what was the average speed of the supernova gas shell between 2000 and 2007?

Answer: First convert the scale to kilometers/mm to get 0.05 LY/mm x ($9.5 \times 10^{12}$ kilometers / 1 LY) = $4.8 \times 10^{11}$ kilometers/mm. Students should measure the diameter of the top left supernova ring and get about 19 millimeters. The bottom-right ring has a diameter of about 27 millimeters, so the change on radius was (27 - 19)/2 = 4 millimeters .This corresponds to a physical distance of 4 mm x ($4.8 \times 10^{11}$ kilometers/mm) = $1.9 \times 10^{12}$ kilometers. The elapsed time was (January 2007 - January 2000) = 7 years or 7 years x ($3.1 \times 10^{7}$ seconds / 1 year) = $2.2 \times 10^{8}$ seconds. The average speed is then V = $1.9 \times 10^{12}$ kilometers / $2.2 \times 10^{8}$ seconds = **8,600 kilometers/sec**.

**Note to Teacher:** Although this is the average speed, students can investigate whether the shell has been moving at a constant speed during this 7-year period, or if the gas shell has been accelerating or deceleration by measuring the speed differences between the consecutive images. The images that are shown are taken at about 9 months apart. For more information visit

**http://chandra.harvard.edu/photo/2005/sn87a/more.html** ………..for specific image dates

**http://chandra.harvard.edu/photo/2005/sn87a/index.html** ……….. supernova information

This is an image of a star field in the constellation Centaurus taken by the Hubble Space Telescope in 1994. In addition to the bright stars, the streak of a single asteroid can also be seen. The Hubble has 'accidentally' detected over 100 asteroids as its cameras have been looking at other targets. Many of the asteroids are new discoveries. The curvature of the asteroid's trail as it moved across the sky was caused by parallax changes as the telescope orbited Earth during the 40-minute exposures. The field is 2.7 arcminutes on a side, and the distance to the asteroid was estimated to be 140 million kilometers from Earth. Based on the faintness of the asteroid at this distance, it was probably only 2 kilometers across!

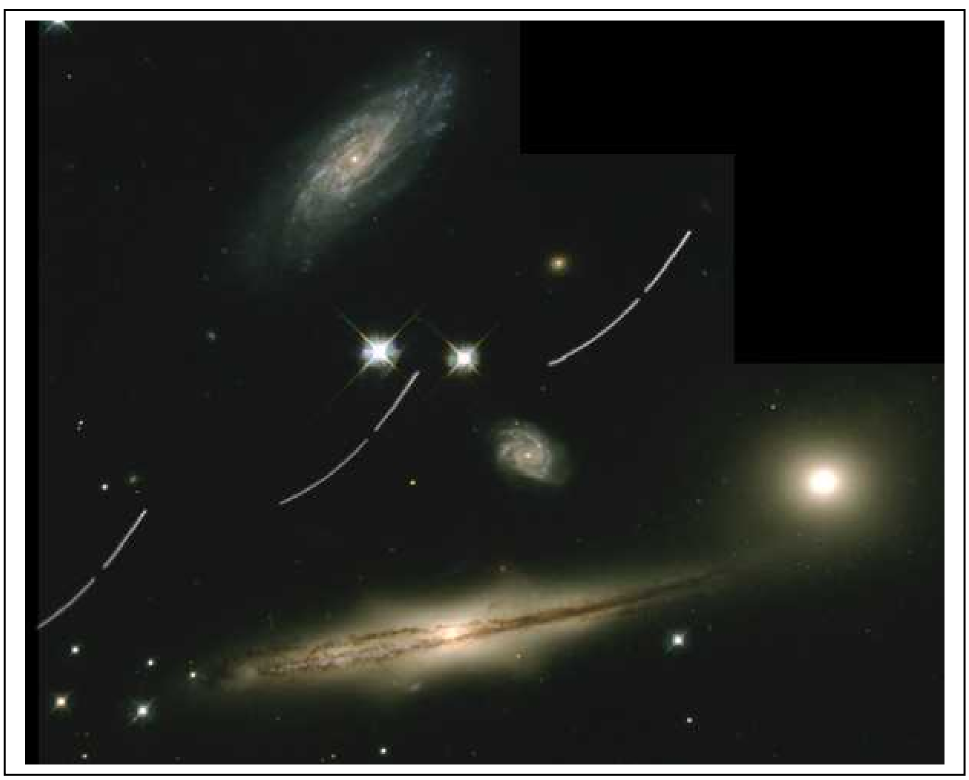
**Problem 1** - At the distance of the asteroid, this field would measure about 110,000 kilometers across. How many kilometers did the asteroid travel during the time of the exposure?

**Problem 2** - What was the approximate speed of the asteroid in kilometers/hour from the beginning to the end of the trail?

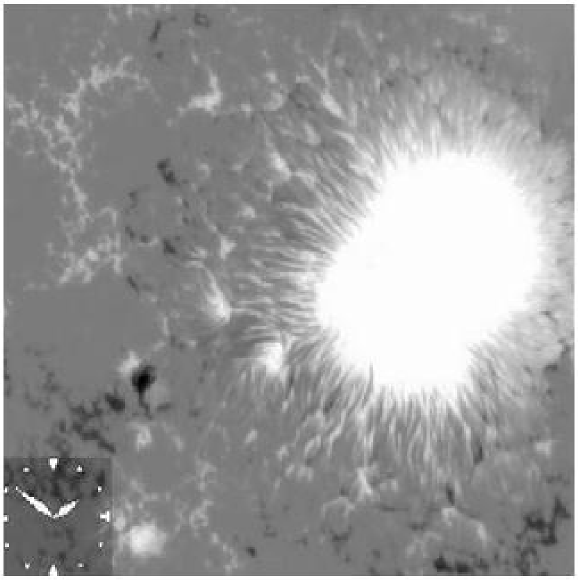
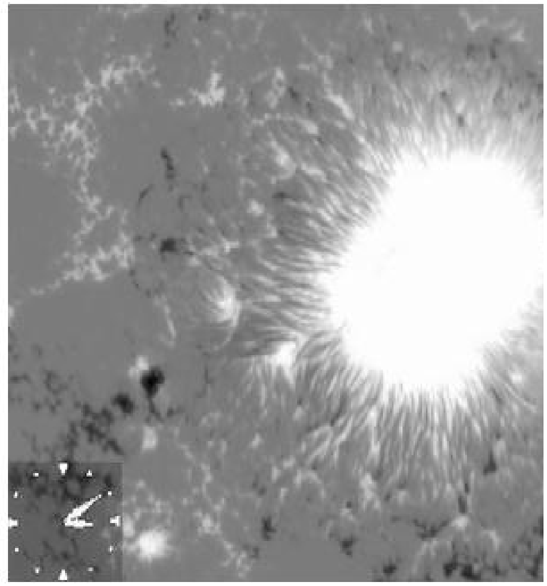
**Problem 1** - At the distance of the asteroid, this field would measure about 110,000 kilometers across. How many kilometers did the asteroid travel during the time of the exposure?
Answer: Students will have to convert the length of the streak into kilometers using the scale of the image. Use a millimeter ruler to determine the scale of the image by first measuring the width of the image to get 118 millimeters. This physical length is equal to 110,000 kilometers, so the image scale is just 110,000 km/ 118 millimeters = 932 km/mm. The length of the asteroid streak is 20 millimeters, so its length is 20 x (932 km/mm) = **18,640 kilometers**.

**Problem 2** - What was the approximate speed of the asteroid in kilometers/hour from the beginning to the end of the trail?
Answer: The paragraph says that the exposure took 40 minutes, so during that time the asteroid moved the distance indicated in Problem 1. The speed is then 18,640 Kilometers/0.66 hours = **28,200 km/hr.**

The photograph below shows another asteroid streak across a picture taken of the Hickson Galaxy Group # 87 in the constellation Capricornus. The size of the field is 132,000 km at the distance of the asteroid, and the exposure was about 6.6 hours. During this time the asteroid traveled 166,000 kilometers for a speed of about 25,000 km/hr.



# Moving Magnetic Filaments Near Sunspots

These two images were taken by the Hinode solar observatory on October 30, 2006. The size of each image is 34,300 km on a side. The clock face shows the time when each image was taken, and represents the face of an ordinary 12-hour clock.

**Problem 1** - What is the scale of each image in kilometers per millimeter?

**Problem 2** - What is the elapsed time between each image in; A) hours and minutes? B) decimal hours? C) seconds?

Carefully study each image and look for at least 5 features that have changed their location between the two images. (Hint, use the nearest edge of the image as a reference).

**Problem 3** - What direction are they moving relative to the sunspot?

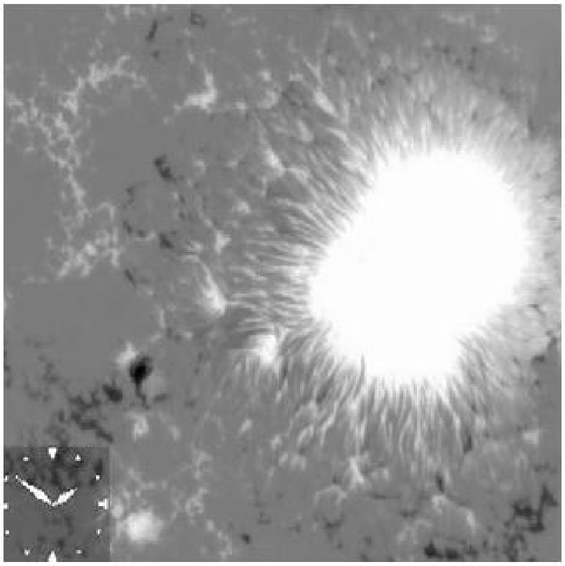
**Problem 4** - How far, in millimeters have they traveled on the image?

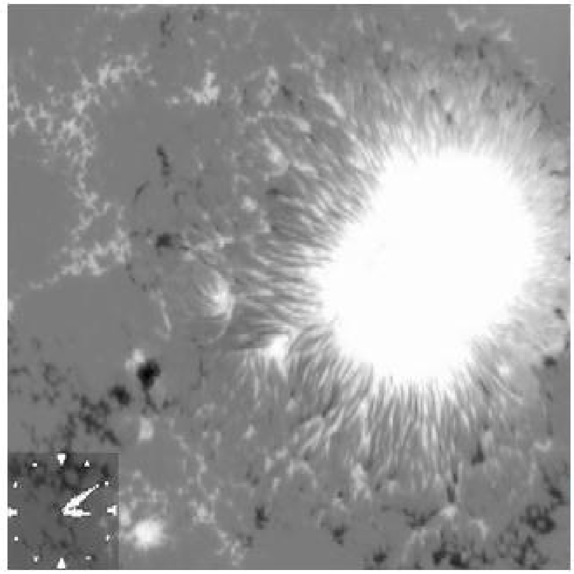
**Problem 5** - From your answers to questions 1, 2 and 4, calculate their speed in kilometers per second, and kilometers per hour.

**Problem 6** - A fast passenger jet plane travels at 600 miles per hour. The Space Shuttle travels 28,000 miles per hour. If 1.0 kilometer = 0.64 miles, how fast do these two craft travel in kilometers per second?

**Problem 7** - Can the Space Shuttle out-race any of the features you identified in the sunspot image?

## Answer Key:





**Problem 1** - What is the scale of each image in kilometers per millimeter? **Answer:** The pictures are 75 mm on a side, so the scale is 34,300 km /75mm = **457 km/mm**

**Problem 2** - What is the elapsed time between each image in;

A) hours and minutes? About 1 hour and 20 minutes.
B) decimal hours? About 1.3 hours
C) seconds? About 1.3 hours x 3600 seconds/hour = 4700 seconds

*Carefully study each image and look for at least 5 features that have changed their location between the two images. (Hint, use the nearest edge of the image as a reference). Students may also use transparent paper or film, overlay the paper on each image, and mark the locations carefully.*

**Problem 3** - What direction are they moving relative to the sunspot?
**Answer:** Most of the features seem to be moving away from the sunspot.

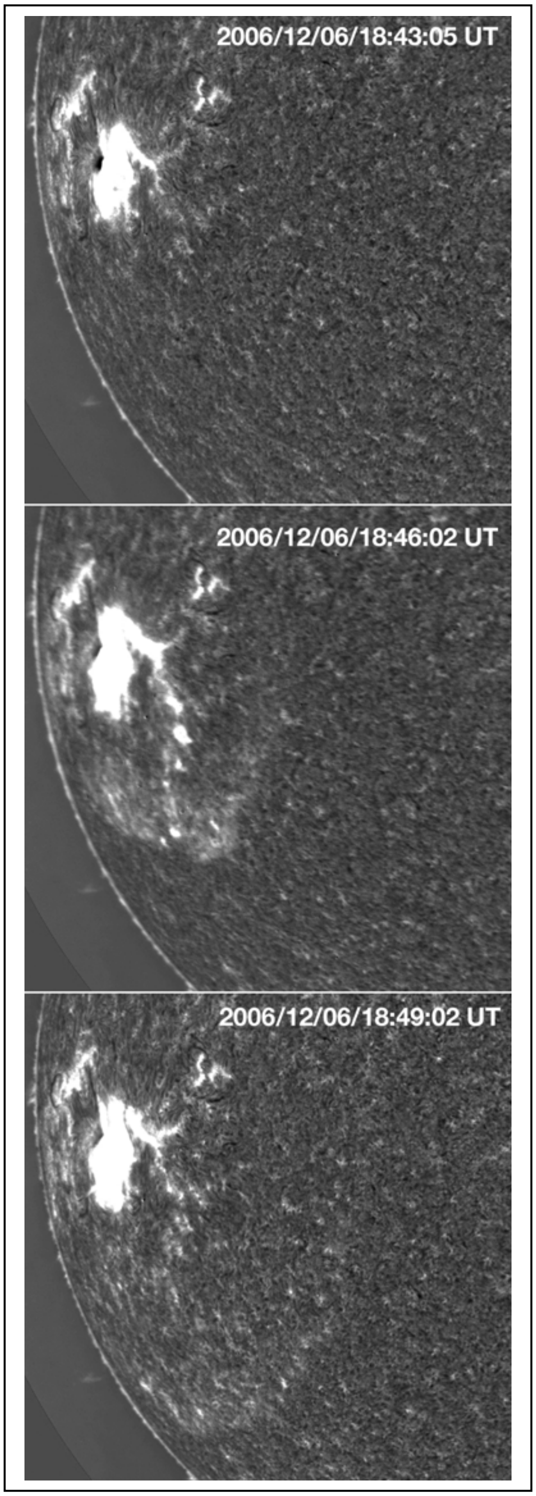
**Problem 4** - How far, in millimeters have they traveled on the image? **Answer:** The feature in the above image has moved about 2 millimeters.

**Problem 5** - From your answers to questions 1, 2 and 4, calculate their speed in kilometers per second, and kilometers per hour. **Answer:** 2 mm x 457 km/mm = 914 kilometers in 4700 seconds = 0.2 kilometers/sec or 703 kilometers/hour.

**Problem 6** - A fast passenger jet plane travels at 600 miles per hour. The Space Shuttle travels 28,000 miles per hour. If 1.0 kilometer = 0.64 miles, how fast do these two craft travel in kilometers per second? Jet speed = 600 miles/hr x ( 1 / 3600 sec/hr) x (1 km/0.64 miles) = **0.26 km/sec**. Shuttle = 28,000 x (1/3600) x (1/0.64) = **12.2 km/sec.**

**Problem 7** - Can the Space Shuttle out-race any of the features you identified in the sunspot image? **Answer:** Yes, in fact a passenger plane can probably keep up with the feature in the example above!

# Measuring the Speed of a Solar Tsunami!




Moments after a major class X-6 solar flare erupted at 18:43:59 Universal Time on December 6, 2006, the National Solar Observatory's new Optical Solar Patrol Camera captured a movie of a shock wave 'tsunami' emerging from Sunspot 930 and traveling across the solar surface. The three images to the left show the progress of this Morton Wave. The moving solar gasses can easily be seen.

Note: because the event is seen near the solar limb, there is quite a bit of fore-shortening so the motion will actually be faster than what the images suggest.

At the scale of these images, the actual disk of the sun would be a circle with a radius of 190 millimeters.

**Problem 1:** Given that the physical radius of the sun is 696,000 kilometers, what is the scale of each image in kilometers/millimeter?

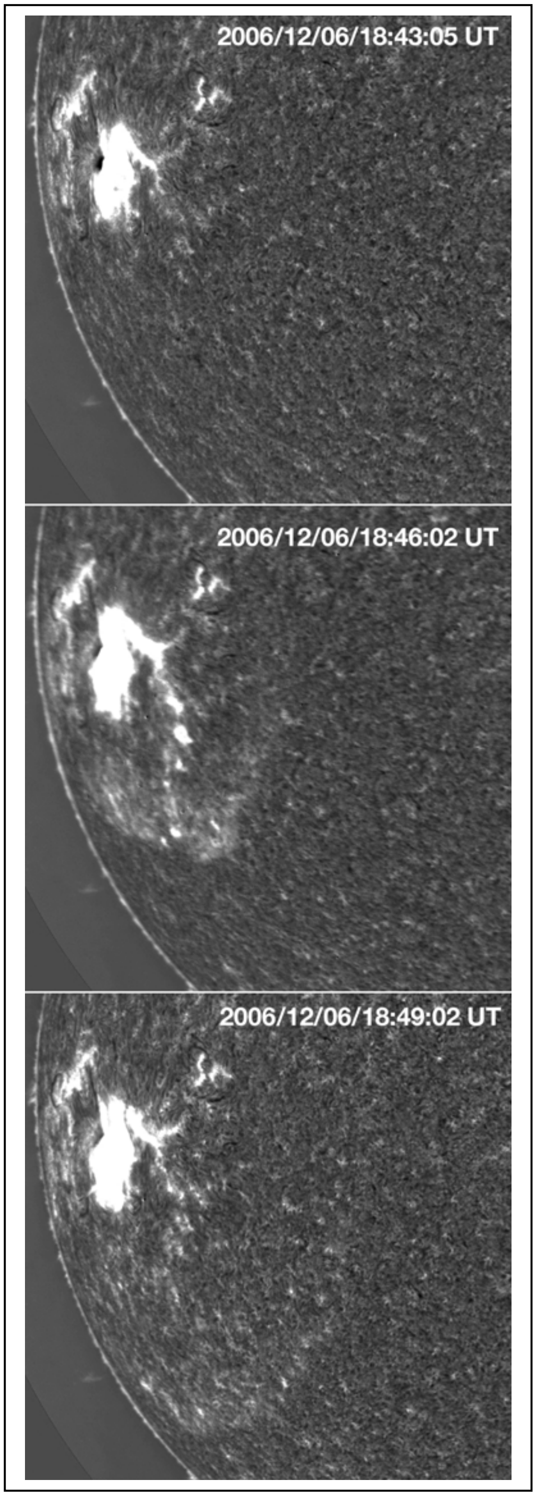
**Problem 2:** Select a spot near the center of the sunspot (large white spot in the image), and a location on the leading edge of the shock wave. What is the distance in kilometers from the center of the sunspot, to the leading edge of the shock wave in the middle and lower images?

**Problem 3:** The images were taken at 18:43:05, 18:46:02 and 18:49:02 Universal Time. How much elapsed time has occurred between these images?

**Problem 4:** From your answers to Problem 3 and 4, what was the speed of the Morton Wave in kilometers per hour between the three images? B) did the wave accelerate or decelerate as it expanded?

**Problem 5:** The speed of the Space Shuttle is 44,000 kilometers/hour. The speed of a passenger jet is 900 kilometers/hour. Would the Morton Wave have overtaken the passenger jet? The Space Shuttle?

Answer Key:




**Problem 1:** Given that the physical radius of the sun is 696,000 kilometers, what is the scale of each image in kilometers/millimeter?

Answer: 696,000/158 = **4,405 km / mm**

**Problem 2:** What is the distance in kilometers from the center of the sunspot, to the leading edge of the shock wave in each image?

Answer:

Image 2 = 27 mm = 27x4405 = **119,000 km**

Image 3 = 38 mm = **167,000 km**

**Problem 3:** The images were taken at 18:43:05, 18:46:02 and 18:49:02 Universal Time. How much elapsed time has occurred between these images?

Answer: Image 1 - Image 2 = **2 minutes 57 seconds**

Image 2 - Image 3 = **3 minutes**

**Problem 4:** From your answers to Problem 3 and 4, A) what was the speed of the Morton Wave in kilometers per hour between the three images?

Answer:

V12 = 119,000 km/2.9 min x (60 min/1 hr)

= **2.5 million km/h**

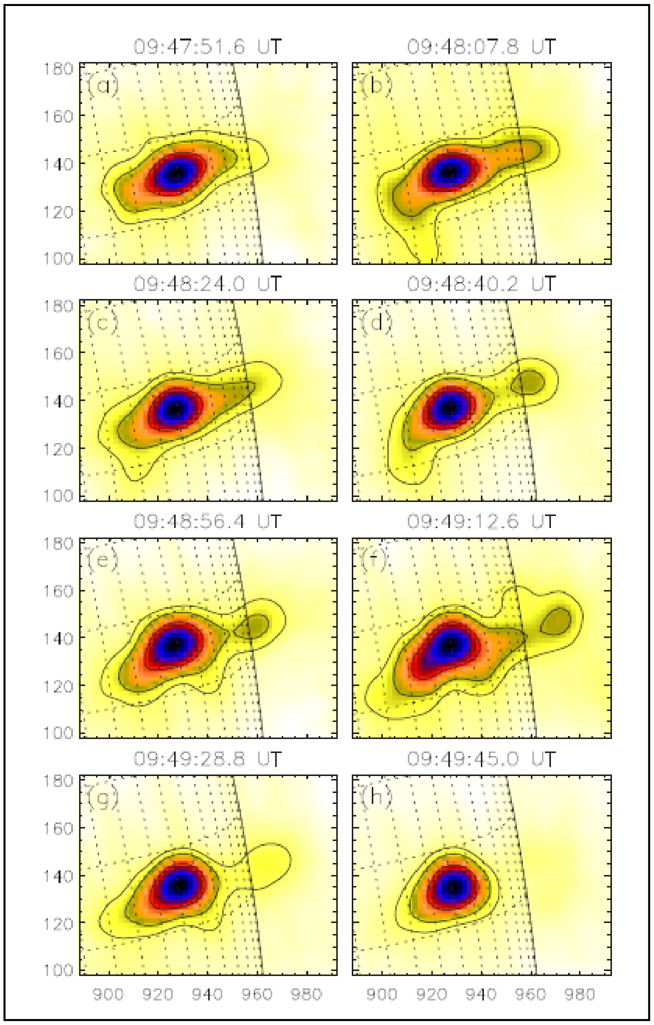
V23 = 167,000/3.0 min x (60 min/1 hr)

= **3.2 million km/h**

B) Did the speed of the wave accelerate or decelerate? Answer: **Because V23 > V12 the wave accelerated.**

Problem 5: The speed of the Space Shuttle is 44,000 kilometers/hour. The speed of a passenger jet is 900 kilometers/hour. Would the Morton Wave have overtaken the passenger jet? The Space Shuttle? Answer: It would easily have overtaken the Space Shuttle! Because of fore-shortening, the actual speed of the wave was even higher than the estimates from the images, so the speed could have been well over 4 million km/hr.

# Super-fast solar flares !




NASA's Ramaty High Energy Solar Spectroscopic Imager (RHESSI) satellite has been studying solar flares since 2002. The sequence of figures to the left shows a flaring region observed on November 3, 2003. This flare was rated as 'X3.9' making it an extremely powerful event. A detailed study of this flare by astronomer Dr. Astrid Veronig and her colleagues at the Institute of Physics of the University of Graz in Austria allowed scientists to determine the physical properties of this event.

During the 4-minute flaring event, gas temperatures of over 45 million degrees Kelvin were reached in a plasma with a density of 400 billion atoms/cc.

The figures each have a field of view of 80 second of arc x 100 seconds of arc. The diameter of the sun in these angular units is 1950 seconds of arc, and its physical diameter is 1,392,000 kilometers.

Each image shows the main flare region (blue) and Images D ,E and F show a second 'blob' being ejected by the flaring region.

*"X-ray sources and magnetic reconnection in the X3.9 flare of 2003 November 3" A. Veronig et al., Astronomy and Astrophysics, 2005 vol. 446, p.675.*

**Problem 1** - From the information in the text, what is the size of each box in kilometers?

**Problem 2** - What is the scale of each image in kilometers per millimeter?

**Problem 3** - Between Image D and Image F, how much time elapsed?

**Problem 4** - Between Image D and Image F, how far did the plasma Blob travel in kilometers?

**Problem 5** - Between Image D and Image F, what was the average speed of the Blob in kilometers per second?

**Problem 6 -** The SR-71 Blackbird holds the official Air Speed Record for a manned airbreathing jet aircraft with a speed of 3,529.56 km/h (2,188 mph). It was capable of taking off and landing unassisted on conventional runways. The record was set on July 28, 1976 by Eldon W. Joersz near Beale Air Force Base in California. Would the SR-71 have been able to out-run the plasma blob?

## Answer Key:

**Problem 1** - From the information in the text, what is the size of each box in kilometers?

Answer: (100 arc-sec/1950-arcsec) x 1,392,000 km = 71,400 km.
(80 arcsec/1950 arcsec0 x 1,392,000 km = 57,100 km.
**The boxes are 71,400 x 57,100 km in size.**

**Problem 2** - What is the scale of each image in kilometers per millimeter?
Answer: The 100-arcsec edge of a box measures 34 millimeters, so the scale is (71,400 km/34 mm) = **2,100 km/mm**

**Problem 3** - Between Image D and Image F, how much time elapsed?
Answer: 09:49:12.6 UT - 09: 48: 40.2 UT = 72.6 - 40.2 = **32.4 seconds.**

**Problem 4** - Between Image D and Image F, how far did the plasma Blob travel in kilometers?
Answer: In Image D it was 12 millimeters from the flare center. In Image F it was 15 millimeters from the flare center, for a net change of 3 millimeters or 3 mm x 2,100 km/mm = **6,300 kilometers.**

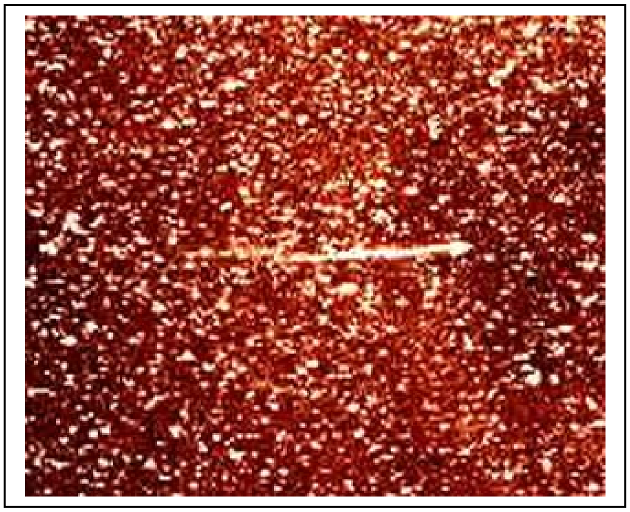
**Problem 5** - Between Image D and Image F ,what was the average speed of the Blob in kilometers per second?
Answer: The speed was 6,300 kilometers/32.4 seconds or **194 kilometers/sec**.

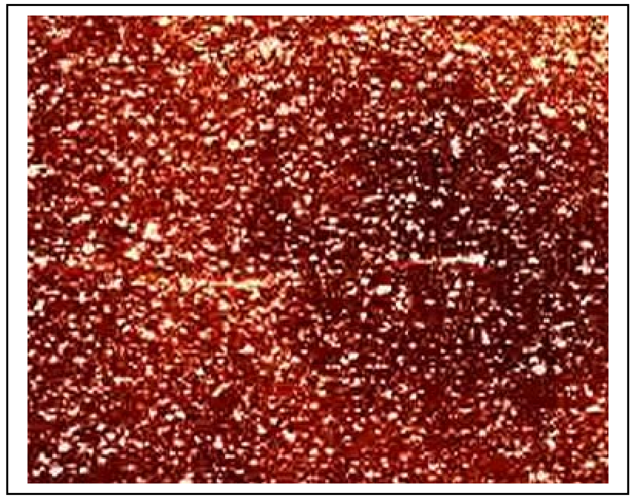
**Problem 6** - The SR-71 Blackbird holds the official Air Speed Record for a manned air-breathing jet aircraft with a speed of 3,529.56 km/h (2,188 mph). It was capable of taking off and landing unassisted on conventional runways. The record was set on July 28, 1976 by Eldon W. Joersz near Beale Air Force Base in California. Would the SR-71 have been able to out-run the plasma Blob?

Answer: The SR-71 traveled at a speed of 3,530 km/hour. There are 3,600 seconds in an hour, so the speed was 3,530 km/hr x 1 hr/3600 sec = 0.98 kilometers/sec. The solar flare blob was traveling at 194 kilometers per second or nearly 200 times faster! **The Blob Wins!!!**

# The Comet Encke Tail Disruption Event

On April 20, 2007, NASA's STEREO satellite witnessed a rare solar system event. The Comet Encke had just passed inside the orbit of Venus and was at a distance of 114 million kilometers from STEREO-A ,when a Coronal Mass Ejection occurred on the sun. The cloud of magnetized gas passed over the comet's tail at 18:50 UT, and moments later caused the tail of the comet to break into two. The two images below show two images from the tail breakup sequence. The left image was taken at 18:10 UT and the right image was taken at 20:50 UT. Each image subtends an angular size of 6.4 degrees x 5.3 degrees. For comparison, the Full Moon would correspond to a circle with a diameter of 0.5 degrees.





**Problem 1** - What is the scale of the images in arcminutes per millimeter? (1 degree=60 arcminutes)

**Problem 2** - How many seconds elapsed between the time the two images were taken by the STEREO-A satellite?

**Problem 3** - The left image shows the comet with an intact tail. The right image shows the tail separated from the head of the comet (the right-most bright feature along the comet's horizontal axis which we will call Point A), and flowing to the left. Meanwhile, you can see that the comet has already begun to reform a new tail. Carefully examine the right-hand image and identify the right-most end of the ejected tail (Call it Point B). Note that star images do not move, and are more nearly point-like than the tail gases. How far, in millimeters, is Point B from Point A?

**Problem 4** - From the image scale, convert your answer to Problem 3 into arcminutes.

**Problem 5** - The distance of the comet was 114 million kilometers, and at that distance, one arcminute of angular separation corresponds to 33,000 kilometers. How far did the tail fragment travel between the times of the two images?

**Problem 6** - What was the speed of the tail fragment?

**Problem 7** - If the comet's speed was about 40 km/sec and the CME speed was at least several hundred times faster, based on your answer to Problem 6, was the comet fragment 'left behind' or did the CME carry it off?

## Answer Key:

**Problem 1** - What is the scale of the images in arcminutes per millimeter? (1 degree=60 arcminutes)

**Answer:** horizontally, the image span 6.4 degrees x 60 minutes/degree = 384 arcminutes. The length is 77 millimeters, so the scale is 384/77 = **5.0 arcminutes/mm**

**Problem 2** - How many seconds elapsed between the time the two images were taken by the STEREO-A satellite?

**Answer:** 20:50 - 18:10 = 2 hours and 40 minutes = 160 minutes or **9600 seconds.**

**Problem 3** - The left image shows the comet with an intact tail. The right image shows the tail separated from the head of the comet (the right-most bright feature along the comets horizontal axis which we will call Point A), and flowing to the left. Meanwhile, you can see that the comet has already begun to reform a new tail. Carefully examine the right-hand image and identify the right-most end of the ejected tail (Call it Point B). Note that star images do not move, and are more nearly point-like than the tail gases. How far, in millimeters, is Point B from Point A?

**Answer:** An answer near **17 millimeters** is acceptable, but students may measure from 15 to 20 millimeters as reasonable answers.

**Problem 4** - From the image scale, convert your answer to Problem 3 into arcminutes.

**Answer:** 17 millimeters x 5 arcminutes/mm = **85 arcminutes**.

**Problem 5** - The distance of the comet was 114 million kilometers, and at that distance, one arcminute of angular separation corresponds to 33,000 kilometers. How far did the tail fragment travel between the times of the two images?

**Answer:** 85 arcminutes x 33,000 kilometers/arcminute = **2.8 million kilometers.**

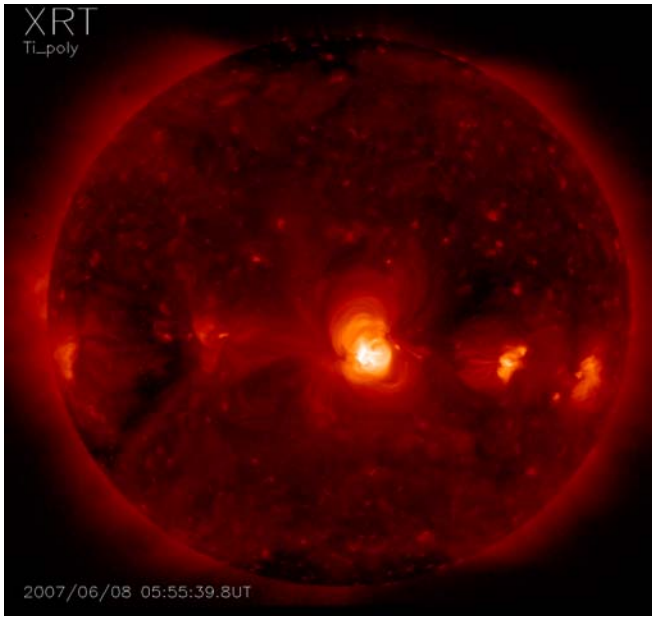
**Problem 6** - What was the speed of the tail fragment?

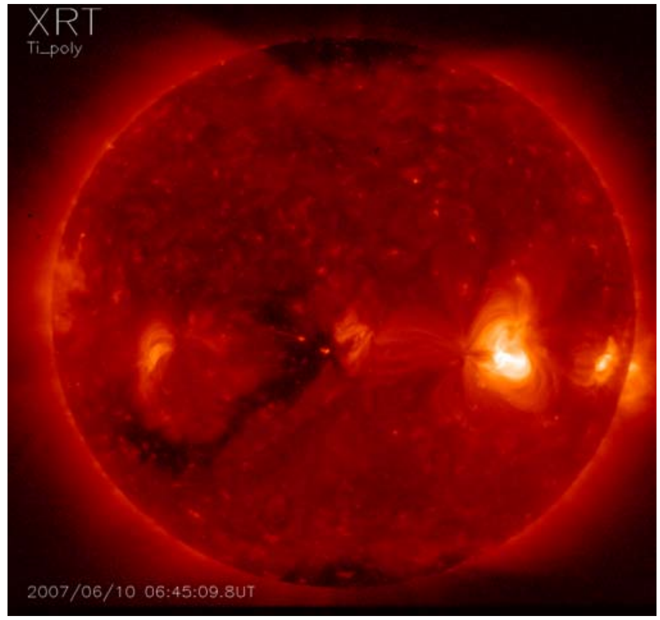
**Answer:** 2.8 million kilometers/9600 seconds = **292 kilometers/second.**

**Problem 7** - If the comet's speed was about 40 km/sec and the CME speed was at least several hundred times faster, based on your answer to Problem 6, was the comet fragment 'left behind' or did the CME carry it off?
**Answer: The speed in Problem 6 is much closer to the CME speed than the comet speed, so the fragment was carried off by the CME and not ejected by the comet.**

*This collision was studied in detail by Dr. Angelos Vourlidas and his colleagues at the Naval Research laboratory in Washington, D.C and the Rutherford Laboratory in England. They deduced from a more careful analysis that the CME speed was about 500 km/sec and the solar wind speed was about 420 km/sec. The tail fragment was carried off by the CME.* Details can be found in The Astrophysical Journal (Letters), vol. 668, pp L79-L82 which was published on October 10, 2007. A movie of the encounter may be seen at the STEREO web site ( http://stereo.gsfc.nasa.gov) in their movie gallery.

# How fast does the sun spin?




The sun, like many other celestial bodies, spins around on an axis that passes through its center. The rotation of the sun, together with the turbulent motion of the sun's outer surface, work together to create magnetic forces. These forces give rise to sunspots, prominences, solar flares and ejections of matter from the solar surface.

Astronomers can study the rotation of stars in the sky by using an instrument called a spectroscope. What they have discovered is that the speed of a star's rotation depends on its age and its mass. Young stars rotate faster than old stars, and massive stars tend to rotate faster than low-mass stars. Large stars like supergiants, rotate hardly at all because they are so enormous they reach almost to the orbit of Jupiter. On the other hand, very compact neutron stars rotate 30 times each second and are only 40 kilometers across.

The X-ray telescope on the Hinode satellite creates movies of the rotating sun, and makes it easy to see this motion. A sequence of these images is shown on the left taken on June 8, 2007 (Left); June 10 2007 (Right) at around 06:00 UT.

Although the sun is a sphere, it appears as a flat disk in these pictures when in fact the center of the sun is bulging out of the page at you! We are going to neglect this distortion and estimate how many days it takes the sun to spin once around on its axis.

The radius of the sun is 696,000 kilometers.

**Problem 1** - Using the information provided in the images, calculate the speed of the sun's rotation in kilometers/sec and in miles/hour.

**Problem 2** – About how many days does it take to rotate once at the equator?

**Inquiry Question:** What geometric factor produces the largest uncertainty in your estimate, and can you come up with a method to minimize it to get a more accurate rotation period?

# Answer Key:

**Problem 1** - Using the information provided in the images, calculate the speed of the sun's rotation in kilometers/sec and in miles/hour.

First, from the diameter of the sun's disk, calculate the image scale of each picture in kilometers per millimeter.

Diameter= 76 mm. so radius = 38 mm. Scale = (696,000 km)/38 mm = 18,400 km/mm

Then, find the center of the sun disk, and using this as a reference, place the millimeter ruler parallel to the sun's equator, measure the distance to the very bright 'active region' to the right of the center in each picture. Convert the millimeter measure into kilometers using the image scale.

Picture 1: June 8 distance = 4 mm d = 4 mm (18,400 km/mm) = 74,000 km
Picture 2; June 10 distance = 22 mm d = 22 mm (18,400 km/mm) = 404,000 km

Calculate the average distance traveled between June 8 and June 10.

Distance = (404,000 - 74,000) = 330,000 km

Divide this distance by the number of elapsed days (2 days)…………….. 165,000 km/day

Convert this to kilometers per hour……………………………………….. 6,875 km/hour

Convert this to kilometers per second……………………………………….. 1.9 km/sec

Convert this to miles per hour ……………………………………………… **4,400 miles/hour**

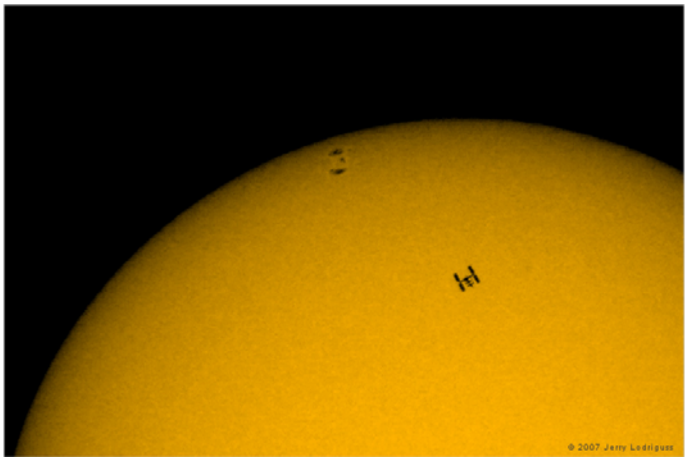
**Problem 2** – About how many days does it take to rotate once at the equator?

The circumference of the sun is 2 $\pi$ (696,000 km) = 4,400,000 kilometers.

The equatorial speed is 66,000 km/day so the number of days equals
4,400,000/165,000 = **26.6 days.**

**Inquiry Question:**
Because the sun is a sphere, measuring the distance of the spot from the center of the sun on June 10 gives a distorted linear measure due to foreshortening.

The sun has rotated about 20 degrees during the 2 days, so that means a full rotation would take about (365/20) x 2 days = 36.5 days which is closer to the equatorial speed of the sun of 35 days.




(Photo courtesy Jerry Lodriguss (Copyright 2007, http://www.astropix.com/HTML/SHOW_DIG/055.HTM )

The relationship between the distance to an object, **R**, the objects size, **L**, and the angle that it subtends at that distance, $\theta$, is given by:

$$\theta = 57.29 \frac{L}{R} \quad \text{degrees}$$

$$\theta = 3{,}438 \frac{L}{R} \quad \text{arcminutes}$$

$$\theta = 206{,}265 \frac{L}{R} \quad \text{arcseconds}$$

To use these formulae, the units for length, L, and distance, R, must be identical.

**Problem 1** - You spot your friend (L = 2 meters) at a distance of 100 meters. What is her angular size in arcminutes?

**Problem 2** - The Sun is located 150 million kilometers from Earth and has a radius of 696.000 kilometers, what is its angular diameter in arcminutes?

**Problem 3** - How far away, in meters, would a dime (1 centimeter) have to be so that its angular size is exactly one arcsecond?

**Problem 4** - The spectacular photo above shows the International Space Station streaking across the disk of the Sun. If the ISS was located 379 kilometers from the camera, and the ISS measured 73 meters across, what was its angular size in arcseconds?

**Problem 5** - The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) what was the angle, in arcminutes, that it moved through in one second as seen from the location of the camera? B) What was its angular speed in arcminutes/second?

**Problem 6** - Given the diameter of the Sun in arcminutes (Problem 2), and the ISS angular speed (Problem 5) how long, in seconds, did it take the ISS to travel across the face of the sun?

**Problem 1** - Answer: Angle = 3,438 x (2 meters/100 meters) = **69 arcminutes**.

**Problem 2** - Answer: 3,438 x (696,000/150 million) = 15.9 arcminutes in radius, so the diameter is 2 x 15.9 = **32 arcminutes**.

**Problem 3** - Answer: From the second formula R = 3438 * L/A = 3438 * 1 cm/1 arcsecond so R = 3,438 centimeters or a distance of **34.4 meters**.

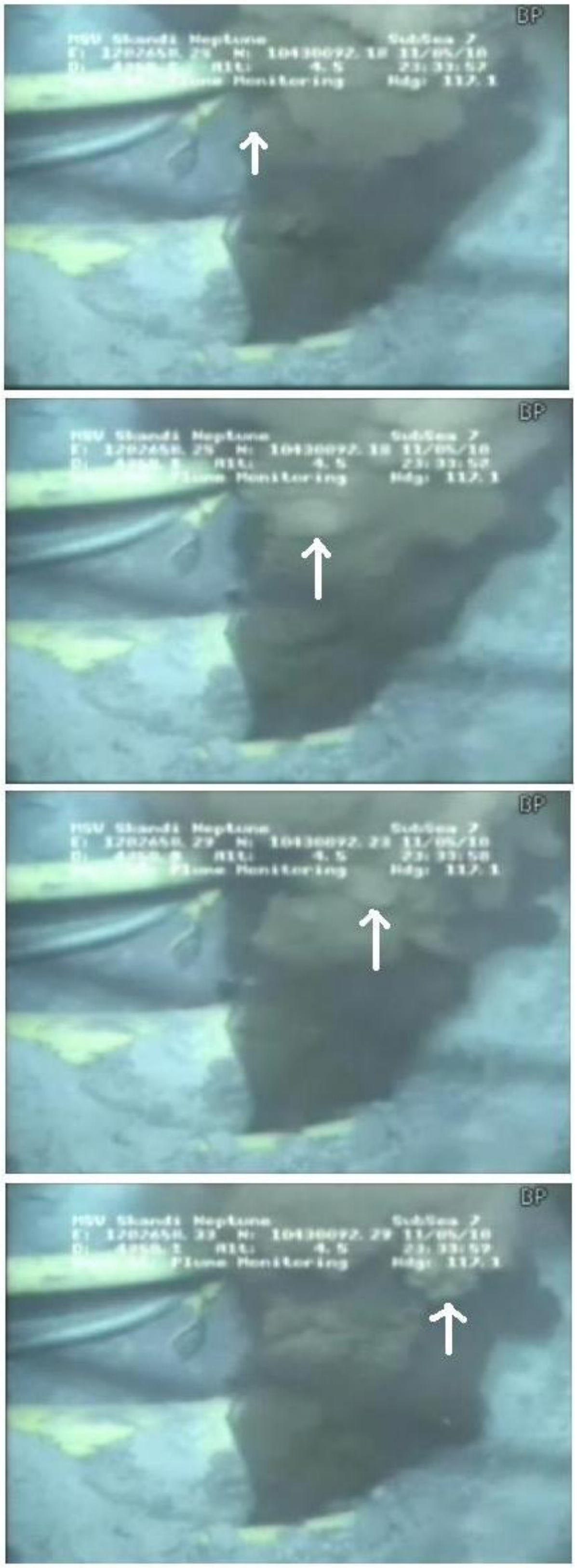
**Problem 4** - Answer: From the third formula, Angle = 206,265 * (73 meters/379,000 meters) = **40 arcseconds**.

**Problem 5** - Answer: The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) The ISS traveled L = 7.4 kilometers so from the second formula Angle = 3,438 * (7.4 km/379 km) = **67 arcminutes**. B) The angular speed is just **67 arcminutes per second**.

**Problem 6** - Answer: The time required is T = 31.8 arcminutes/ (67 arcminutes/sec) = **0.47 seconds.**

The spectacular photo by Jerry Lodriguss had to be taken with careful planning beforehand. He had to know, to the second, when the Sun and ISS would be in the right configuration in the sky as viewed from his exact geographic location. Here's an example of the photography considerations in his own words:

*" I considered trying to monitor the transit visually with a remote release in my hand and just firing (the camera) when I saw the ISS in my guidescope. Then I worked out the numbers. I know that my reaction time is 0.19 seconds. This is actually quite good, but I make my living shooting sports where this is critical, so I better be good at it. I also know that the Canon 1D Mark IIn has a shutter lag of 55 milliseconds. Adding these together, plus a little bit of a fudge factor, the best I could hope for was about 1/4 of a second from when I saw it to when the shutter opened. Since the entire duration of the transit was only 1/2 of a second, in theory, I could capture the ISS at about the center of the disk if I fired as soon as I saw it start to cross. This was not much of a margin for error. I could easily blink and miss the whole thing... Out of 49 frames that the Mark IIn recorded, the ISS is visible in exactly one frame."*



The April 14, 2010 BP Gulf Oil Leak has been in the news for nearly one month, and experts predict that it may rank as one of the most environmentally costly accidents in recent history. Considerable debate continues as to the actual rate at which the leaky British Petrolium (BP) well is leaking oil. Initial estimates from the observed surface oil slick suggested 210,000 gal/day. Following the release of actual videos of the leak, experts now estimate from 3 to 4 million gallons/day.

The images to the left were extracted from the May 12, 2010 video between 23:33:57 and 23:33:58. The arrow shows how far a portion of the billowing oil moved during this time.

The diameter of the pipe fragment shown in the image is 21 inches.

**Problem 1** - From the scale of the images, how many inches did the oil spot move in the time between the first and last images?

**Problem 2** - What is the area of the open circular pipe in square-feet?

**Problem 3** - If the oil is emerging at the same speed as you derived in Problem 1, how many cubic-feet of oil is leaving the pipe each second?

**Problem 4** - If 1 cubic foot equals 7.5 gallons, what do you estimate as the rate in gallons/day at which oil is leaving the pipe if A) 100% of the dark material is oil? B) 50% is oil and 50% is gas?

**Problem 1** - From the scale of the images, how many inches did the oil spot move in the time between the first and last images?

Answer: Using a metric ruler, the diameter of the 21-inch pipe is 18 millimeters, so the scale of the image is 1.2 inches/mm. The distance between the arrow in the top image and the bottom image is about 25 millimeters or 25 x 1.2 = **30 inches**.

**Problem 2** - What is the area of the open circular pipe in square-feet?
Answer: Assuming a circular aperture, and a diameter of 21/12 = 1.8 feet,
$A = \pi (1.8/2)^2$ = **2.5 feet$^2$**.

**Problem 3** - If the oil is emerging at the same speed as you derived in Problem 1, how many cubic-feet of oil is leaving the pipe each second?

Answer: The speed of the flow is 30 inches/1 second or 30 inches/sec. This can be converted to feet/sec to get S = 2.5 feet/sec. Flow = Area x Speed so Flow = 2.5 feet$^2$ x 2.5 feet/sec so **Flow = 6.3 feet$^3$/sec**.

**Problem 4** - If 1 cubic foot equals 7.5 gallons, what do you estimate as the rate in gallons/day at which oil is leaving the pipe if A) 100% of the dark material is oil? B) 50% is oil and 50% is gas?

Answer: A) 100% x 6.3 feet$^3$/sec x (3600 sec/hour) x (24 hour/day) x (7.5 gallons / 1 feet$^3$) so Rate = **4 million gallons/day.**
B) At a 50% mixture, by volume, of oil and gas, Rate = **2 million gallons/day**.

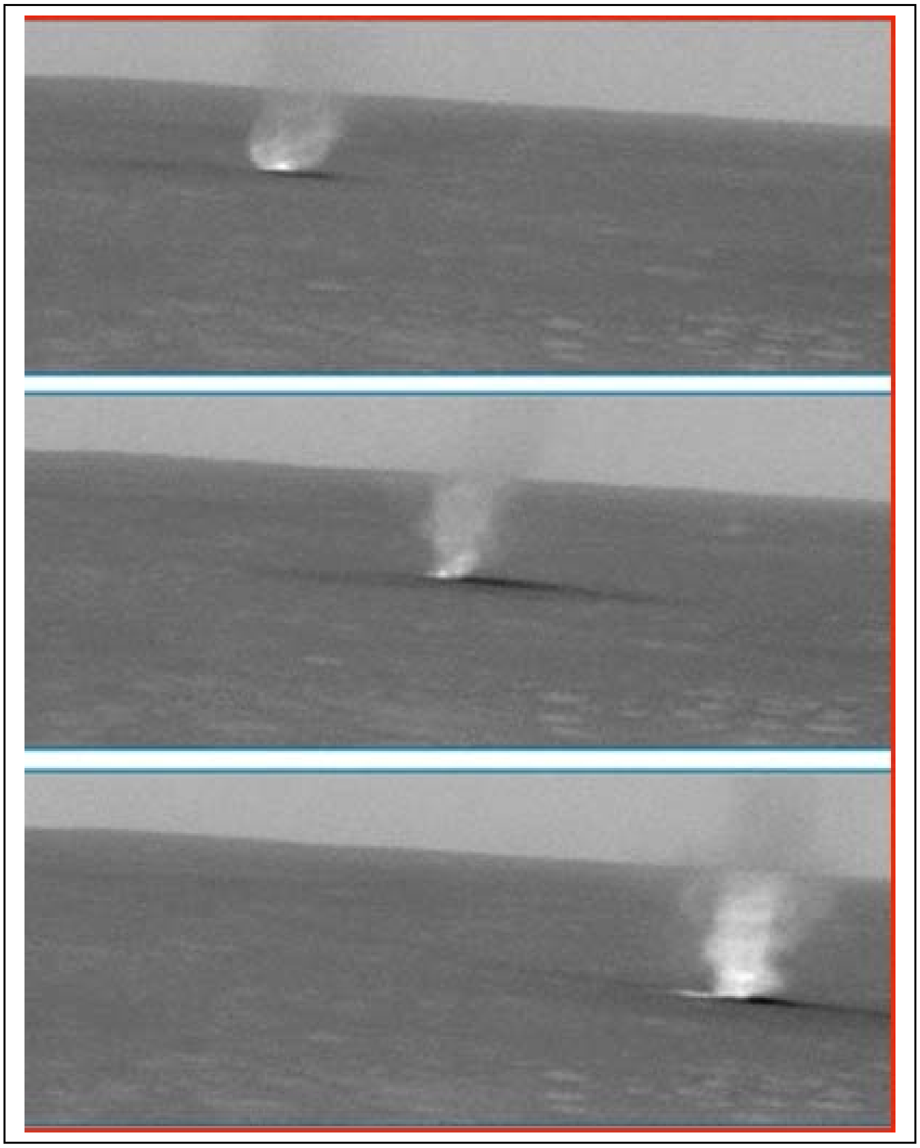
**Note to Teacher:** Students may view the actual video at:

http://www.necn.com/05/13/10/Video-Gulf-oil-leak-at-the-source/landing_scitech.html?blockID=234064&feedID=4213

There are many other websites that archive the BP oil leak video. Students may examine other portions of the videos to obtain additional estimates .They may also discuss the issue of the actual concentration of the oil in the outflowing material seen in the videos, and also how to obtain better speed estimates by following 'blobs' in the video. What are some of the problems with using this video? Are there any geometric effects that have to be taken into account because the camera/pipe/cloud are tilted relative to the image?

Comparing this daily flow rate with the rate estimated from the size of the surface oil seen by the Terra satellite, why do you think that scientists believe that there is a significant amount of oil below the surface of the ocean that has not been accounted for yet?

# The Martian Dust Devils



A dust devil spins across the surface of Gusev Crater just before noon on Mars.

NASA's Spirit rover took the series of images with its navigation camera on the rover's martian day, or sol, 486 (March 15, 2005).

The images were taken at:

11:48:00 (T=top)
11:49:00 (M=middle)
11:49:40 (B=bottom)

based upon local Mars time.

The dust devil was about 1.0 kilometer from the rover at the start of the sequence of images on the slopes of the "Columbia Hills."

**Problem 1** - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top (T) and bottom (B) frames?

**Problem 2** - What was the time difference, in seconds, between the images T-M, M-B and T-B?

**Problem 3** - What was the distance, in meters, traveled between the images T-M and M-B?

**Problem 4** - What was the average speed, in meters/sec, of the dust devil between T-B?

**Problem 5** - What were the speeds during the interval from T-M, and the interval M-B?

**Problem 6** - Was the dust devil accelerating or decelerating between the times represented by T-B?

# Answer Key

**Problem 1** - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top and bottom frames? Answer: The location of the dust devil in frame B when placed in image T is a shift of about 65 millimeters, which at a scale of 7.4 meters/mm equals **about 480 meters**.

**Problem 2** - What was the time difference between the images T-M, M-B and T-B? Answer: T-M = 11:49:00 - 11:48:00 - 1 minute or 60 seconds. For M-B the time interval is 40 seconds. For T-B the time interval is **100 seconds**.

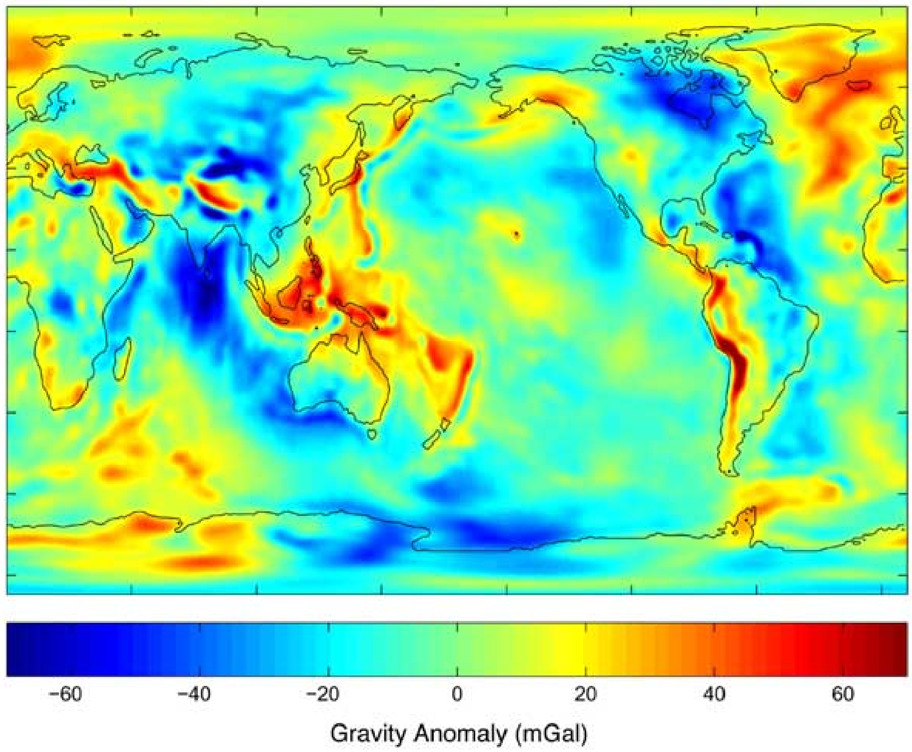
**Problem 3** - What was the distance traveled between the images T-M and M-B? Answer: T-M = about 30 mm or 222 meters; M-B = about 35 mm or **259 meters**.

**Problem 4** - What was the average speed, in meters/sec, of the dust devil between T-B? Answer: Speed = distance/time so 480 meters/100 seconds = **4.8 meters/sec**.

**Problem 5** - What were the speeds during the interval from T-M, and the interval M-B? Answer: Speed(T-M) = 222 meters/60 seconds = 3.7 meters/sec. Speed(M-B) = 259 meters/40 seconds = **6.5 meters/sec**.

**Problem 6** - Was the dust devil accelerating or decelerating between the times represented by T-B? Answer: because the speed increased from 4.8 meters/sec to 6.5 meters/sec, the dust devil was **accelerating** during this time interval from 11:48:00 to 11:49:40.

# GRACE: Exploring Earth's Gravity Field




The joint NASA-German Aerospace Center Gravity Recovery and Climate Experiment (GRACE) mission has created the most accurate map yet of Earth's gravity field. The map shows how the acceleration of gravity at Earth's surface varies from the standard g=9.8067 meters/sec$^2$ (32 feet/sec$^2$) in units of milliGals. One thousand milliGals equals 9.8067 meters/sec$^2$, so 1 milliGal = 0.0098067 meters/sec$^2$. In the map, for example, an orange color means the acceleration of gravity, g, is +40 mGals larger than 9.8067 or g= (9.8067 + 20(0.0098)) = +10.0027 meters/sec$^2$. Regions where the crust is dense, or rich in iron deposits, will tend to have higher than average strengths.

**Problem 1** - About what is the average acceleration of gravity across the continental United States?

**Problem 2** - About where is the red 'gravity anomaly' located in the continental United States?

**Problem 3** - The period of a 1-meter pendulum in seconds, T, is given by the formula $T^2 = 4\pi^2 L / g$ where g is the acceleration of gravity in meters/sec$^2$. From the map value for g and L = 1.0, what is T for a pendulum in: A) California? B) Hawaii? C) The middle of the Indian Ocean?

# Answer Key

**Problem 1** - About what is the average acceleration of gravity across the continental United States?

Answer: The average color is light-blue which corresponds to about -20 milliGals or -0.020 g which equals a difference of -0.020 x 9.8067 m/sec$^2$ or -0.196 m/sec$^2$. The total acceleration is then 9.8067 - 0.196 = **9.611 m/sec$^2$**.

**Problem 2** - About where is the red 'gravity anomaly' located in the continental United States?

Answer: **The gravity anomaly is in Utah**. Students may attempt a more exact answer from scaled measurements and 'triangulation' with other geographic features in the GRACE map.

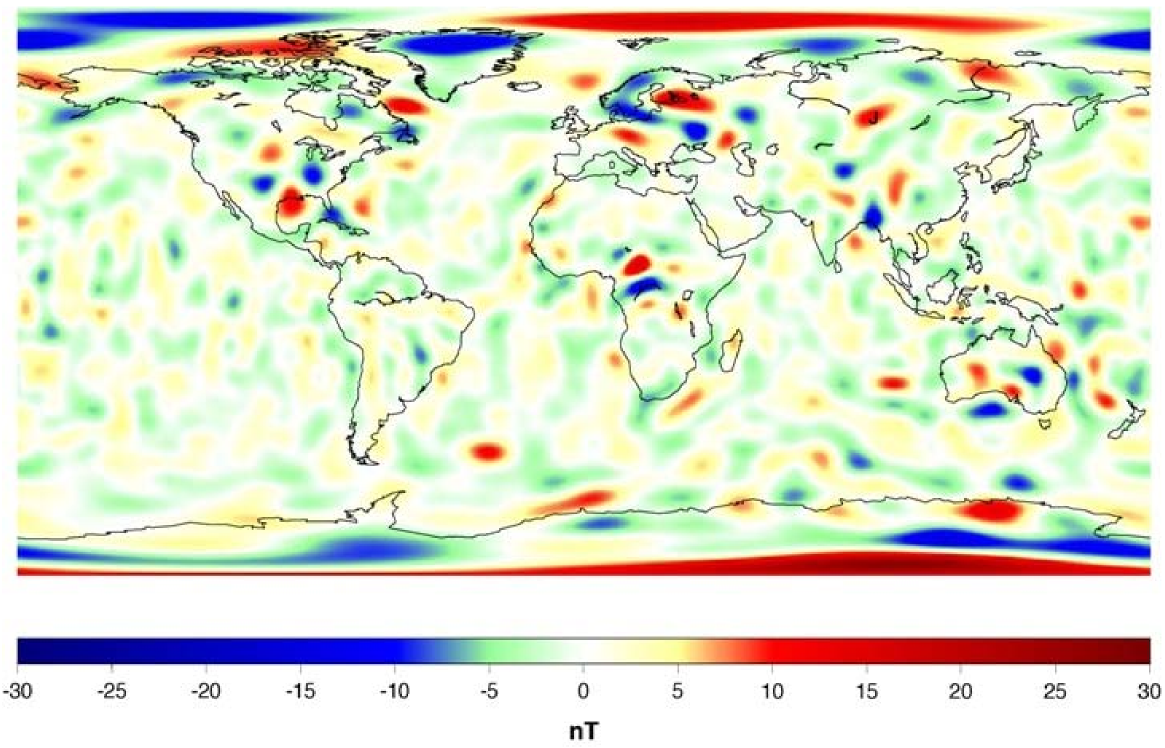
**Problem 3** - The period of a 1-meter pendulum in seconds, T, is given by the formula $T^2 = 4\pi^2 L / g$ where g is the acceleration of gravity in meters/sec$^2$. From the map value for g and L = 1.0, what is T for a pendulum in: A) California? B) Hawaii? C) The middle of the Indian Ocean?

Answer: A) In California on the map, the average color is light-blue which corresponds to about -20 milliGals or -0.020 g which equals a difference of -0.020 x 9.8067 m/sec$^2$ or -0.196 m/sec$^2$. The total acceleration is then 9.8067 - 0.196 = 9.611 m/sec$^2$. From the formula $T^2$ = 39.463 / g we get $T^2$ = 39.463/9.611 = 4.106 and so taking the square-root we get **T = 2.026 seconds.**

B) Hawaii is the small red dot on the map in the middle of the Pacific Ocean. The average color is red which corresponds to about +60 milliGals or +0.060 g which equals a difference of +0.060 x 9.8067 m/sec$^2$ or +0.588 m/sec$^2$. The total acceleration is then 9.8067 + 0.588 = 10.395 m/sec$^2$. From the formula $T^2$ = 39.463 / g so $T^2$ = 39.463/10.395 = 3.796 and so taking the square-root we get **T = 1.948 seconds**.

C) The average color is dark-blue which corresponds to about -60 milliGals or -0.060 g which equals a difference of -0.060 x 9.8067 m/sec$^2$ or -0.588 m/sec$^2$. The total acceleration is then 9.8067 - 0.588 = 9.219 m/sec$^2$. From the formula $T^2$ = 39.463 / g so $T^2$ = 39.463/9.219 = 4.281 and so taking the square-root we get **T = 2.069 seconds**.

# CHAMP: Magnetic Earth and the Lithosphere




The map was constructed using data collected from a variety of different spacecraft orbiting about 400 km above the Earth, including NASA's Magsat mission and Polar Orbiting Geophysical Observatory, the German CHAMP satellite, and the Danish Oersted satellite. The average magnetic field of Earth's surface has a strength of 70,000 nanoTeslas and is shown as a white color in the map scaling. The map shows variations in Earth's surface magnetism so that a variation of +30 nanoTeslas (dark red) means an actual surface strength of 70,000 + 30 = 70,030 nanoTeslas. The variations are related to deposits of iron-rich ores in the lithosphere.

For the following problems, use a Mercator map of the Earth to determine latitude, longitude coordinates and distances.

**Problem 1** - About how large, in kilometers, are the magnetic anomalies that can be detected in this map?

**Problem 2** - What European country has the largest magnetic anomaly compared to the area of the country?

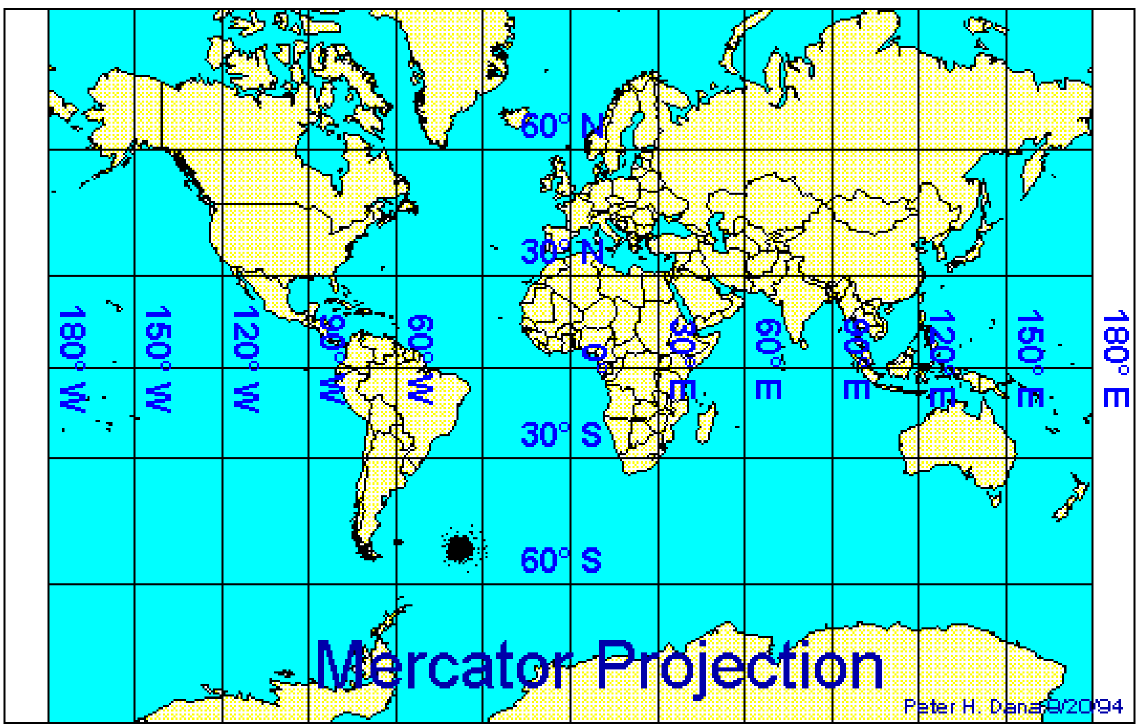
**Problem 3** - At about what latitude and longitude is the South Atlantic magnetic anomaly located?

# Answer Key

**Problem 1** - About how large, in kilometers, are the magnetic anomalies that can be detected in this map? Answer: The red and blue spots are about **300 to 500 kilometers across**

**Problem 2** - What European country has the largest magnetic anomaly compared to the area of the country? Answer: **Norway** is nearly completely covered by a magnetic anomaly with a strength of -30 nanoTeslas. This mountainous country is known for its iron mining.

**Problem 3** - At about what latitude and longitude is the South Atlantic magnetic anomaly located? Answer: Students may estimate the latitude and longitude by comparing the anomaly map with a conventional map that includes a coordinate grid as shown below. The approximate location of the magnetic anomaly is indicated by the dark spot. The coordinates are approximately **Latitude = -50 degrees and Longitude = 40 degrees West. This is a relatively barren part of the Atlantic Ocean.**



# How many stars are there?



On a clear night in the city you might be able to see a few hundred stars. In the country, far away from city lights, perhaps 5000 can be seen. Telescopes can see literally millions of stars. But how do we accurately count them? This exercise will show you the basic method!

This image was taken by the 2MASS sky survey. It is a field that measures 9.0 arcminutes on a side.

**Problem 1** – By using a millimeter ruler, divide this star field into an equally-spaced grid that is 3 x 3 cells.

**Problem 2** – Select 3 of these cells and count the number of star images you can see in each cell. Calculate the average number of stars in a cell.

**Problem 3** – A square degree measures 60 arcminutes x 60 arcminutes in area. The full sky has an area of 41,253 square degrees. What are the total number of stars in A) one square degree of the sky; B) the number of stars in the entire sky.

**Problem 4** – Why do you think we needed to average the numbers in Problem 2?

# Answer Key

**Problem 1** – By using a millimeter ruler, divide this star field into an equally-spaced grid that is 3 x 3 cells. Answer: **An example is shown below**.

**Problem 2** – Select 3 of these cells and count the number of star images you can see in each cell. Calculate the average number of stars in a cell. Answer: Using the cells in the top row you may get: 159, 154 and 168. The average is 481/3 = **160 stars**.

**Problem 3** – Use the information in the text to convert your answer into the total number of stars in one square degree of the sky.

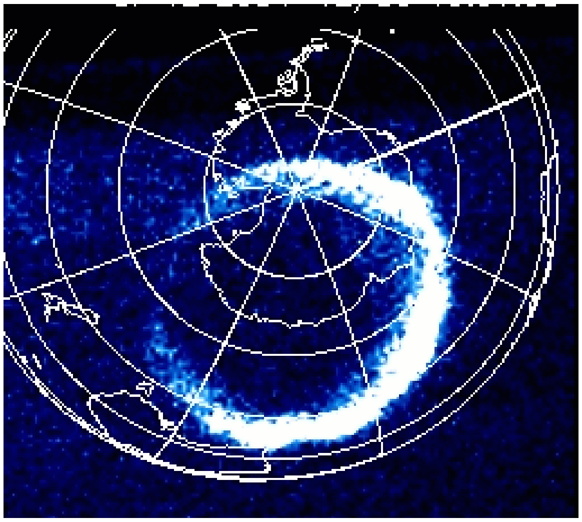
**Answer**: A) The answer from 3 is the number of stars in one cell. The area of that cell is 3 x 3 = 9 square arcminutes. One degree contains 60 arcminutes, so a square degree contains 60 x 60 = 3600 square arcminutes. Your estimated number of stars in one square degree is then 3600/9 = 400 times the number of stars you counted in one average one cell. For the answer to Problem 2, the number in a square degree would be 160 X 400 = **64,000 stars**.

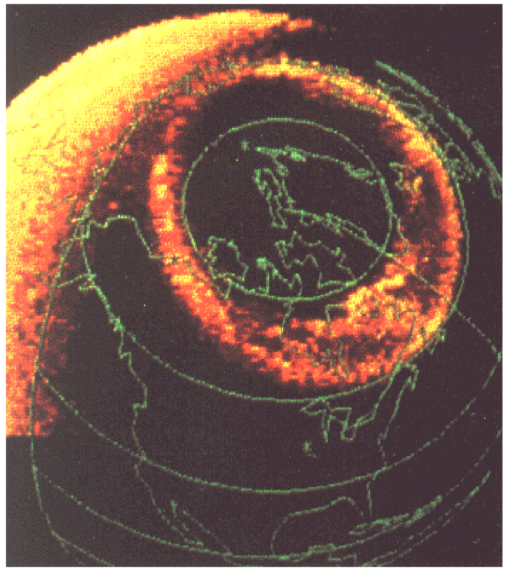
B) The text says that there are 41,253 square degrees in the full sky, so from your answer to Problem 3, you can convert this into the total number of stars in the sky by multiplying the answer by 41,253 to get 64,000 x 41,253 = **2,640,000,000 stars**!

**Problem 4** – Why do you think we needed to average the numbers in Problem 2? Answer: Because stars are not evenly spread across the sky, so you need to figure out the average number of stars.

The aurora form a glowing halo of light above Earth's North and South Polar Regions. Because aurora are caused by charged particles that are affected by Earth's magnetic field, the Auroral Ovals are centered in Earth's magnetic poles, not its geographic poles about which the planet rotates.

The photos below, were taken of the two polar aurora by the IMAGE FUV (Left) and the Polar (right) instruments. The data has been colorized to bring out details of interest to scientists.





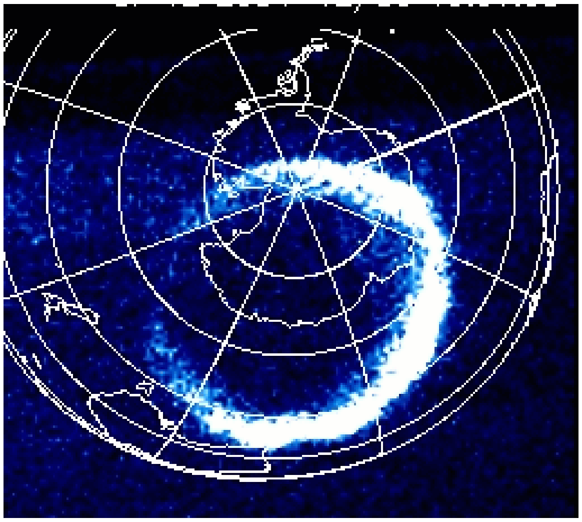
**Problem 1** - The South Magnetic Pole is located in the Northern Hemisphere. From the appropriate image above, locate this magnetic pole on a map.

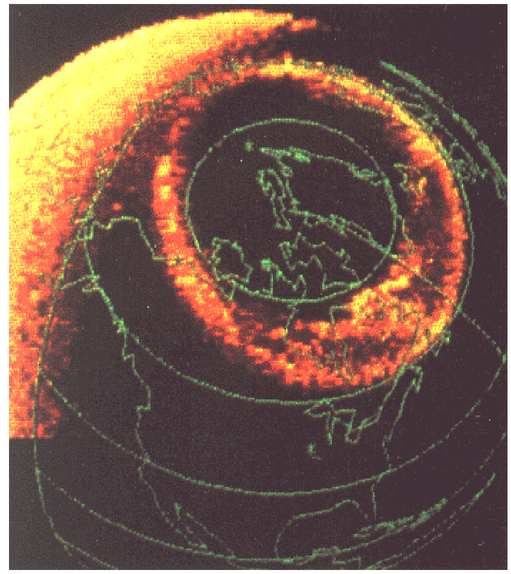
**Problem 2** -- The North Magnetic Pole is located in the Southern Hemisphere. From the appropriate image above, locate this magnetic pole on a map.

**Problem 3:** -- From the geographic clues in the map, estimate the diameter of both auroral ovals in kilometers. (Hint: The radius of Earth is 6,378 kilometers)

**Problem 4:** – What interesting geographic features would you find if you traveled to each of the magnetic poles? If you were going to undertake an expedition to each pole, describe your journey starting from your city or town and mention any special or unusual gear you would bring.

**Problem 5:** Using a compass, and the idea that likes repel and opposites attract, why don't the names of the magnetic poles match the hemispheres they are in?





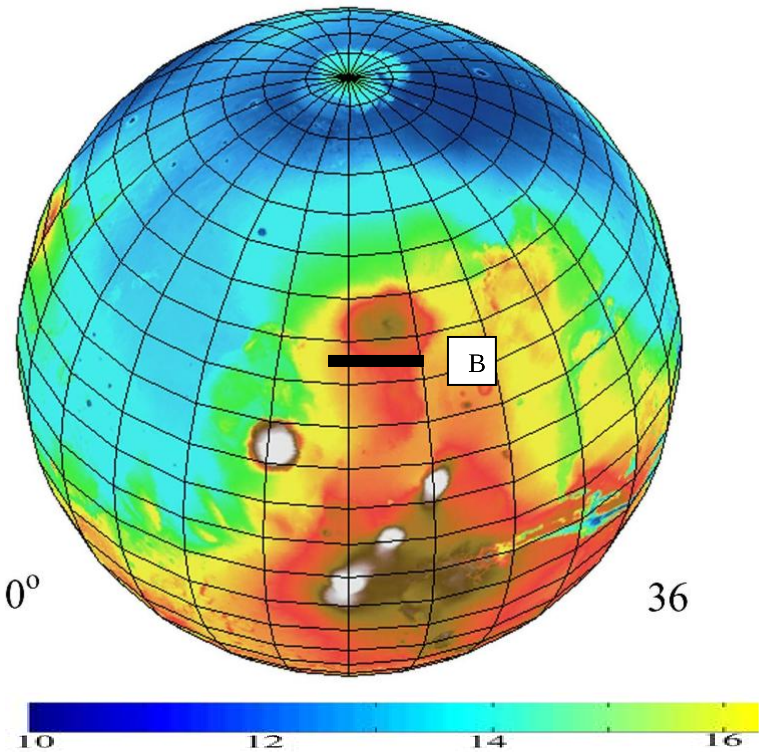
**Problem 1** - The right-hand image from the Polar satellite shows the Arctic Region and the contours of Greenland and North America/Canada. From a world map, students can estimate that the center of the auroral oval is **near longitude 105 West and latitude 83 North)**

**Problem 2** -- The left-hand image from the IMAGE satellite shows Antarctica. The center of the auroral oval is **near longitude 110 West and latitude 72 South**.

**Problem 3:** The diameter of each coordinate grid covers Earth, so the diameter of the grid is the diameter of Earth. Calculate the scale of each image (kilometers per millimeter) and multiply by the diameter of each auroral oval in millimeters. For the north polar aurora, its diameter is about **6400 kilometers**. For the south polar aurora, the diameter is about **6,000 kilometers**.

**Problem 4: Accept any reasonable answer** related to icy climates (Antarctica – permanent ice) or the north polar sea (ice or water location depending on season)

**Problem 5:** In the Northern Hemisphere, the 'N' on the compass is a north-type magnet by the way we defined the naming convention for magnets, so it will be attracted to a south-type pole, which is therefore the polarity of the pole in the Northern Hemisphere.




Mars has virtually no atmosphere, and this means that, unlike Earth, its surface is not protected from solar and cosmic radiation. On Earth, the annual dosage on the ground is about 0.35 Rem/year, but can vary from 0.10 to 0.80 Rem/year depending on your geographic location, altitude, and lifestyle.

This figure, created with the NASA, MARIE instrument on the Odyssey spacecraft orbiting Mars, shows the unshielded surface radiation dosages, ranging from a maximum of 20 Rem/year (brown) to a minimum of 10 Rem/year (deep blue).

Astronauts landing on Mars will want to minimize their total radiation exposure during the 540 days they will stay on the surface. The Apollo astronauts used spacesuits that provided 0.15 gm/cm$^2$ of shielding. The Lunar Excursion Module provided 0.2 gm/cm$^2$ of shielding, and the orbiting Command Module provided 2.4 gm/cm$^2$. The reduction in radiation exposure for each of these was about 1/4, 1/10 and 1/50 respectively. Assume that the Mars astronauts used improved spacesuit technology providing a reduction of 1/8, and that the Mars Excursion Vehicle provided a 1/20 radiation reduction.

The line segment on the Mars radiation map represent an imaginary 1,000 km exploration track that ambitious astronauts might attempt with fast-moving rovers, and not a lot of food! Imagine a schedule where they would travel 100 kilometers each day. Suppose they spent 20 hours a day within a shielded rover, and they studied their surroundings in spacesuits for 4 hours each day.

**Problem 1** - Convert 10 Rem/year into milliRem/day.

**Problem 2** - What is the astronauts radiation dosage per day if they stayed in a region (brown) where the ambient background produces 20 Rem/year?

**Problem 3** - What is the approximate total dosage to an astronaut in milliRems (mRems), given the exposure times and shielding information provided above?

# Having a Hot Time on Mars!

**Problem 1** - Convert 10 Rem/year into milliRem/hour.

**Answer: (10 Rem/yr) x (1 year/ 365 days) x (1 day/24 hr) = 1.1 milliRem/hour**

**Problem 2** - What is the astronauts radiation dosage per day if they stayed in a region (brown) where the ambient background produces 20 Rem/year?

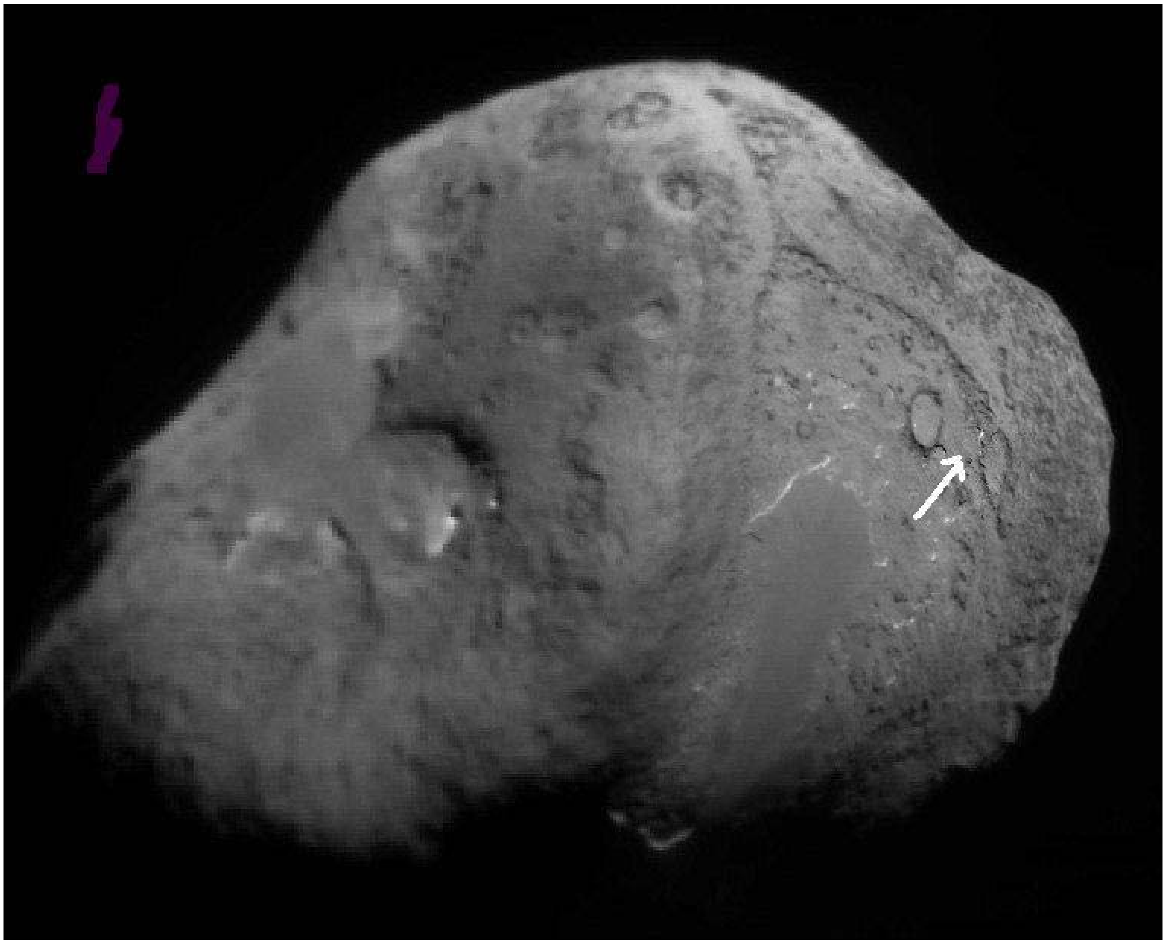
**Answer: From Problem 1, 20 Rem/year = 2.2 milliRem/hour. Then Rover shielding is 1/20 and the spacesuit outside is 1/8, so**
**Dosage = Inside + Outside**
**= 20 hours x (1/20) x 1.1 milliRem/hr + 4 hours x (1/8) x 1.1 milliRem/hr**
**= 1.1 + 0.55**
**= 1.65 milliRem/day**

**Problem 3** - What is the approximate total dosage to an astronaut in milliRems (mRems), given the exposure times and shielding information provided above?

Answer:

Dosage = **2x (16 Rems/yr x 3.3days + 18 Rems/yr x 3.3days + 20 Rems/yr x 3.3days)(1.65/20)**
**= 2x(14.8)**
**= 29.6 mRem**

# Tempel 1 - Close-up of a Comet!



On July 4, 2005, the Deep Impact spacecraft flew within 500 km of comet Tempel 1. This composite image of the surface was put together from images taken by the Impactor probe as it plummeted towards the comet before finally hitting it and excavating a crater. The width of this picture is 8.0 kilometers.

**Problem 1** - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus by the arrow? D) What is the size of some of the smallest details you can see in the picture?

**Problem 2** - The white streak near the center of the picture is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

**Problem 3** - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of the arrow. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

**Problem 1** - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus? D) What is the size of some of the smallest details you can see in the picture?

Answer: By using a millimeter ruler, what is the scale of this image in meters per millimeter? Answer: A) Width = 153 millimeters, so the scale is 8000 meters/153 mm = **52 meters/mm**. B) Width = 147 mm x 110 mm or **7.6 km x 5.7 km**. C) Although the craters are foreshortened, the maximum size gives a better indication of their 'round' diameters of about 7mm or **360 meters**. D) **Students may find features about 1 millimeter across or 50 meters.**

**Note to Teacher**: Depending on the quality of your printer, the linear scale of the image in millimeters may differ slightly from the 153 mm stated in the answer to Problem 1. Students may use their measured value as a replacement for the '153 mm' stated in the problem.

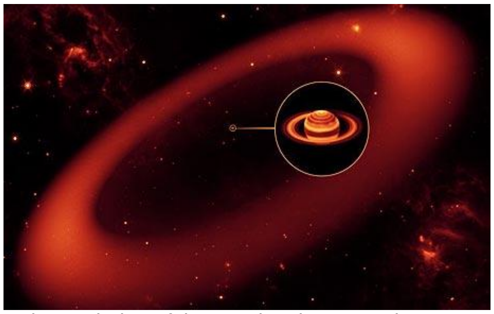
**Problem 2** - The white streak near the center of the picture is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

Answer: The width of the irregular white feature is about 0.5 millimeters or 26 meters. The length is about 15 millimeters or 15x52 = **780 meters**.

**Problem 3** - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of the arrow. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

Answer: The picture shows that the shortest distance to the edge of the nucleus is about 20 millimeters to the right, so this is a distance of about 20 x 52 = **1 kilometer**!

**Note to Teacher**:
Since the distance to the Earth was about 100 million kilometers, the spacecraft orbit had to be calculated to better than 1 part in 100 million over this distance in order for the probe to hit Tempel-1 as planned.



Artist rendering of the new ice ring around Saturn detected by the Spitzer Space Telescope.

"This is one supersized ring," said one of the authors, Professor Anne Verbiscer, an astronomer at the University of Virginia in Charlottesville. Saturn's moon Phoebe orbits within the ring and is believed to be the source of the material.

The thin array of ice and dust particles lies at the far reaches of the Saturnian system. The ring was very diffuse and did not reflect much visible light but the infrared Spitzer telescope was able to detect it. Although the ring dust is very cold -316F it shines with thermal 'heat' radiation. No one had looked at its location with an infrared instrument until now.

"The bulk of the ring material starts about 6.0 million km from the planet, extends outward about another 12 million km, and is 2.6 million km thick. The newly found ring is so huge it would take 1 billion Earths to fill it." (CNN News, October 7, 2009)

Many news reports noted that the ring volume was equal to 1 billion Earths. Is that estimate correct? Let's assume that the ring can be approximated by a washer with an inner radius of r, an outer radius of R and a thickness of h.

**Problem 1** - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

**Problem 2** - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

**Problem 3** - If $r = 6 \times 10^6$ kilometers, $R = 1.2 \times 10^7$ kilometers and $h = 2.4 \times 10^6$ kilometers, what is the volume of the new ring in cubic kilometers?

**Problem 4** - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

**Problem 5** - About how many Earths can be fit within the volume of Saturn's new ice ring?

**Problem 6** - How does your answer compare to the Press Release information? Why are they different?

**Problem 1** - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

Answer: The area of the large circle is given by $\pi R^2$ minus area of small circle $\pi r^2$ equals **$A = \pi (R^2 - r^2)$**

**Problem 2** - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Answer: Volume = Area x height so **$V = \pi (R^2 - r^2) h$**

**Problem 3** - If $r = 6 \times 10^6$ kilometers, $R = 1.2 \times 10^7$ kilometers and $h = 2.4 \times 10^6$ kilometers, what is the volume of the new ring in cubic kilometers?

Answer: $V = \pi (R^2 - r^2) h$

$= (3.141) [ (1.2 \times 10^7)^2 - (6.0 \times 10^6)^2] 2.4 \times 10^6$

` $=$ **$8.1 \times 10^{20}$ km$^3$**

Note that the smallest number of significant figures in the numbers involved is 2, so the answer will be reported to two significant figures.

**Problem 4** - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Answer: Volume of a sphere $V = 4/3 \pi R^3$ so for Earth,

$V = 1.33 \times (3.14) \times (6.378 \times 10^3)^3$

$=$ **$1.06 \times 10^{12}$ km$^3$**

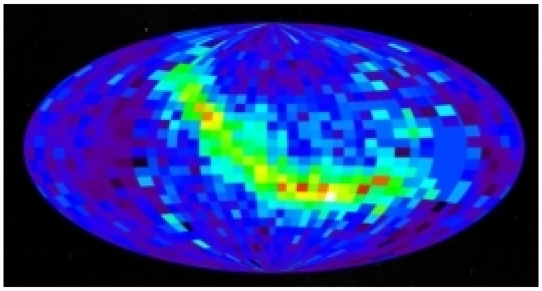
Note that the smallest number of significant figures in the numbers involved is 3, so the answer will be reported to three significant figures.

**Problem 5** - About how many Earths can be fit within the volume of Saturn's new ice ring?

Answer: Divide the answer for Problem 3 by Problem 4 to get

$8.1 \times 10^{20}$ km$^3$ / $(1.06 \times 10^{12}$ km$^3)$ = **$7.6 \times 10^8$ times**

**Problem 6** - How does your answer compare to the Press release information? Why are they different? Answer: **The Press Releases say 'about 1 billion times' because it is easier for a non-scientist to appreciate this approximate number. If we rounded up $7.6 \times 10^8$ times to one significant figure accuracy, we would also get an answer of '1 billion times'.**



NASA's IBEX satellite recently made headlines by creating a picture of the entire sky, not using light but by using cosmic particles called ENAs (Energetic Neutral Atoms). These fast-moving atoms flow through the solar system. Some of them reach Earth, where they can be captured by the IBEX satellite. By counting how many of these ENAs the satellite sees in different directions in the sky, IBEX can create a unique 'picture' of where ENAs are coming from in space.

The big surprise was that they were not coming from all over the sky as expected. They were also coming from a specific band of directions as we see in the image to the left. This image has the same kind of geometry as the map of the Earth below it! It is called a Mollweide Projection, except that instead of graphing geographic points on Earth, the IBEX image shows points in outer space!

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

Data String:
A5, E2, B2, D4, C3, A1, E4, C3, D4, B2, D4, B3, C4, E5, D5, D4, C2, D3, B1, E5, A2, C3, D5, C5, D4, E4, D3, C4, B4, D2, E3, C1, B5, A3, E1, A4, D1, B3, C2, E3

The IBEX satellite detected a series of particles entering its ENA instrument, and was able to determine the direction that each particle came from in the sky. The grid above shows a portion of the sky as a 5x5 grid with columns labeled by their letter and rows by their number. The data string to the right shows the detections of individual ENA particles with their direction indicated by their cell. 'A5, E2, B2…' means that the first ENA particle came from the direction of cell 'A5', the second from cell 'E2' and the third from cell 'B2' and so on. In some ways this process is like the 'call out' during a Bingo game, except that you keep track of the particle 'tokens' in each square to build a picture! Let's look at an example of constructing an ENA image.

**Problem 1** - From the hypothetical data string, tally the number of particles detected in each sky cell in the grid. Select colors to represent the number of ENAs to create an 'image' of the sky in ENAs! How many particles were reported by this data string?

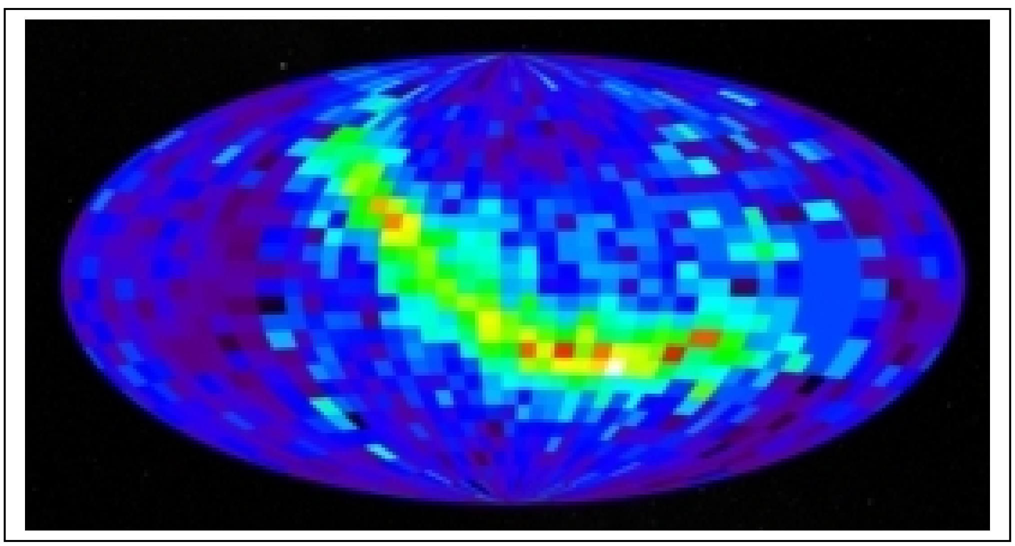
**Problem 2** - Suppose that cell B2 is in the direction of the constellation Auriga, cell C3 is towards Taurus and cell D4 is towards Orion, from which constellation in the sky were most of the ENAs detected?

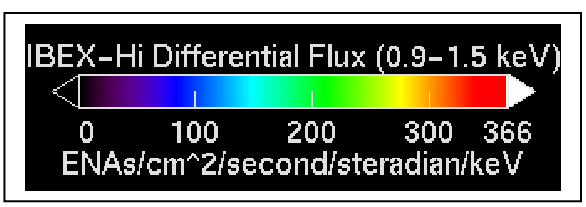
| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 1 | 1 |
| 3 | 1 | 2 | 3 | 2 | 2 |
| 4 | 1 | 1 | 2 | 5 | 2 |
| 5 | 1 | 1 | 1 | 2 | 2 |

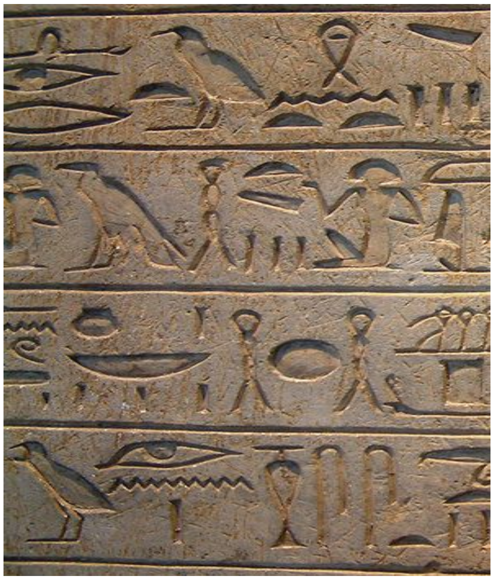
**Problem 1** - Answer above. Students can select yellow = 5, orange=3, red=2 and blue=1 as an example and colorize the table. This is an example of using 'false color' to highlight data in order to reveal patterns. This technique is used by scientists, but should never be confused with the 'actual' color of an object. If you add up the number of ENAS in the data stream you get 40 ENAs being reported.

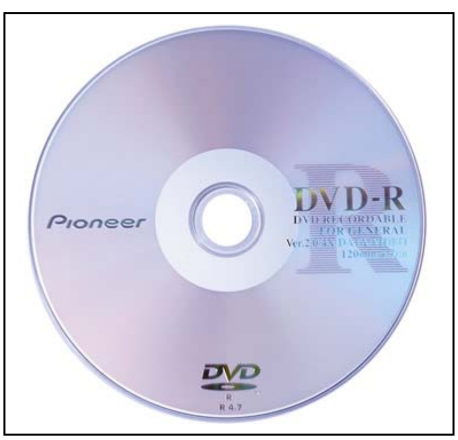
**Problem 2** - The most ENAs recorded in any of the cells is '5' from Cell D4, which is in the direction of the constellation Orion.

The image below shows a close-up of actual IBEX data revealing the individual sky cells that make up the image. The color bar below shows the relationship between the color used, and the number of ENAs detected in each sky cell (called the particle flux).










How do we record measurements and observations today, and preserve them for future research in 10, 50 or 100 years from now? This is one of the biggest challenges facing scientists in the 21$^{st}$ Century.

The images to the left show three styles of recording information for posterity. In each case, we can identify: 1) a recording medium, 2) the density of information on the surface, 3) the technology used to write the information, 4) the technology used to read the information, 5) and the technology used to interpret the information.

Each of these five items determines how long the data will remain useful.

**Problem 1** – In the example of Egyptian hieroglyphics (top) ca 2000 BC, assume the complete inscription is 500 characters long, and that each character can be stored in a 1-byte word. If the inscription covered an area of 0.5 square meters, what is the density of the information in bytes/meter$^2$?

**Problem 2** – The Gutenberg Bible (middle) was printed in ca 1550 AD at about 2500 characters per page. If 1 byte codes one character, and each page measures about 300x450mm, what was the information density in bytes/meter$^2$?

**Problem 3** – For each of the three examples of information, can you identify what properties and items go along with the five items above?

**Problem 1** – In the example of Egyptian hieroglyphics, assume the complete inscription is 500 characters long, and that each character can be stored in a 1-byte word. If the inscription covered an area of 0.5 square meters, what is the density of the information in bytes/meter$^2$? Answer: 500 characters x 1 byte/character) = 500 bytes. Information density = 500 bytes/(0.5 meters$^2$) = **1000 bytes/meter$^2$**.

**Problem 2** – The Gutenberg Bible was printed at about 2500 characters per page. If 1 byte codes one character, and each page measures about 300x450 mm, what was the information density in bytes/meter$^2$? Answer: The area of the page is 0.3 meters x 0.45 meters = 0.135 meters$^2$, then the information density is 2500 bytes/0.135 meters$^2$ = **19,000 bytes/meter$^2$**.

**Problem 3** – For each of the three examples of information, can you identify what properties and items go along with the five items above? Answer: Here is an example.

| Hieroglyphics | Gutenberg Bible | DVD |
|---|---|---|
| Stone | Paper/Vellum | Plastic |
| 1000 | 19,000 | 1 trillion |
| Chisel | Printing press | Laser DVD writer |
| Human eye | Human eye | Laser DVD reader |
| Brain | Brain | Computer software |

As new technologies for storing and retrieving information become common through commercial expansion, data stored in older technologies must be transferred to the newer technologies or risk being lost.

This process is called data migration, and is an essential operation in any archive where scientific data is being preserved for the next generation of scientists.

**Problem 1** – NASA is replacing 10,000 of its older 9-track tape archives by data storage on DVD disks. The tapes store at an information density of 800 bytes/inch and are 2,500 feet long. The DVD disks store 5 gigabytes of information. How many DVD disks will be required to store the tape archive?

**Problem 2** – Apollo-11 photographs taken on 35-millimeter film are to be digitized and temporarily stored on 1 terabyte hard drives. About 1,000 photos were taken. Each photo is digitally scanned and converted into 25 megabyte files. How many terabyte hard drives will be required to store the images?

**Problem 3** – The Solar Dynamics Observatory generates 1.5 terabytes of data each day, and will continue operation for 5 years. At the present time, the SDO data is being stored on 1-terabyte hard drives for immediate use. Each unit costs $50.00. DVD disks cost $0.25 for each 4 gigabytes of data. A) What will data storage cost on the terabyte hard drives? B) How much will the entire SDO archive cost to store on DVD disks (4.7 gigabytes/disk)?

**Problem 1** – NASA is replacing 10,000 of its older 9-track tape archives by data storage on DVD disks. The tapes store at an information density of 800 bytes/inch and are 2,500 feet long. The DVD disks store 5 gigabytes of information. How many DVD disks will be required to store the tape archive?

Answer: 10000 tapes x (2500 feet/tape) x (12 inches/foot) x (800 bytes/inch) x(1 DVD/5 billion bytes) **= 48 DVD disks.**

**Problem 2** – Apollo-11 photographs taken on 35-millimeter film are to be digitized and temporarily stored on 1 terabyte hard drives. About 1,000 photos were taken. Each photo is digitally scanned and converted into 25 megabyte files. How many terabyte hard drives will be required to store the images?

Answer: 1000 photos x ( 25 megabytes/photo) x ( 1 hard drive/1 terabytes) = 0.025 hard drives…or **1 hard drive**.

**Problem 3** – The Solar Dynamics Observatory generates 1.5 terabytes of data each day, and will continue operation for 5 years. At the present time, the SDO data is being stored on 1-terabyte hard drives for immediate use. Each unit costs $50.00. DVD disks cost $0.25 for each 4 gigabytes of data. A) What will data storage cost on the terabyte hard drives? B) How much will the entire SDO archive cost to store on DVD disks (4.7 gigabytes/disk)?

**Answer**: 5 years x (365 days/1 year)x (1.5 terabytes/1 day) = 2,738 terabytes.

A) 2,738 terabytes x $50.00/terabyte = **$136,900**

B) A DVD disk has a capacity of 4.7 gigabytes, so you will need
2,738 terabytes x (1,000 gigabytes/1 terabyte) x ( 1 DVD/4.7 gigabytes)
582,554 DVD disks x $0.25/disk = **$145,639**

Note: The hard drive data can be immediately accessed, while the DVD data has to be robotically mounted on a DVD drive after the proper disk is selected from the 'jukebox' archive.


There are many ways in which recorded data can be physically damaged. A simple scratch can render some modern media unreadable.

This is a major problem for archiving because modern storage media have a far higher information density than earlier storage media.

This means that defects or scratches can make data migration a risky business with the potential loss of considerable information encoded at the new data recording densities.

An Egyptian hieroglyphic inscription has been enscribed on a stone at a density of 1000 bytes/meter$^2$. Each symbol can be coded as a 1-byte data word. An archeologist wants to migrate this data to a modern storage medium in which data is stored on a DVD disk at a density of 1 trillion bytes/meter$^2$.

**Problem 1** – How large, in square centimeters, is a single byte of data recorded on the stone and on the DVD?

**Problem 2** – A scratch appears on the two storage media that is 2 cm long and 0.1 cm wide. How many bytes of information are lost A) on the stone inscription? B) on the DVD?

**Problem 1** – How large, in square centimeters, is a single byte of data recorded on the stone and on the DVD?

Answer: Stone: $1/1000 \times 1 \text{ meter}^2 \times (10000 \text{ cm}^2/1 \text{ meter}^2) =$ **$10 \text{ cm}^2$**.

DVD: $1/1\text{trillion} \times 1 \text{ meter}^2 \times (10000 \text{ cm}^2/1 \text{ meter}^2) =$ **$10^{-8} \text{ cm}^2$**

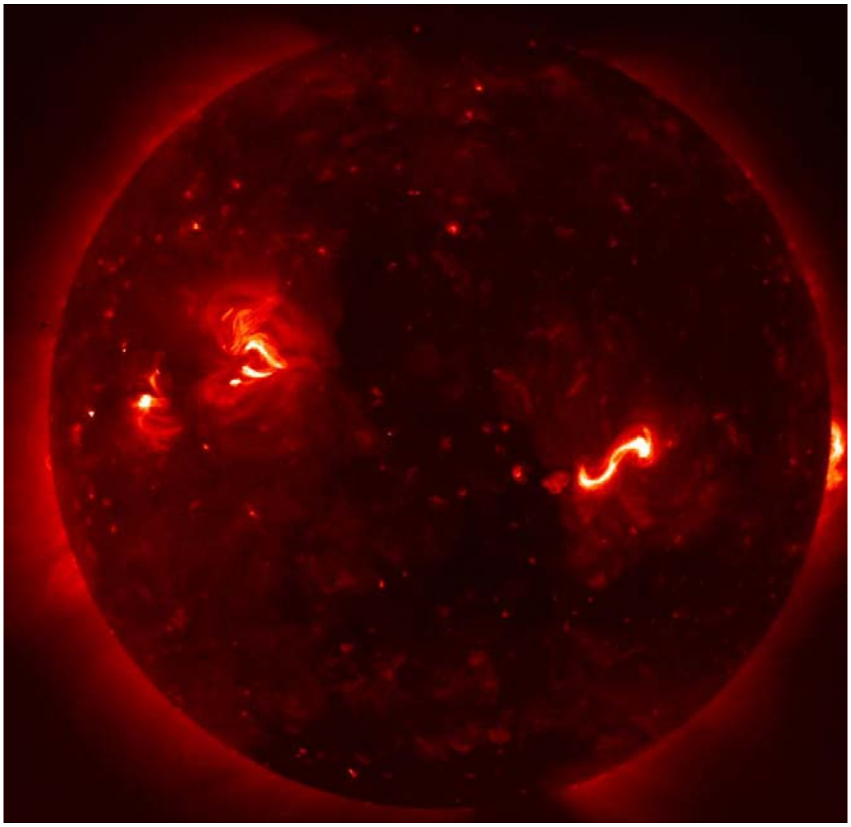
**Problem 2** – A scratch appears on the two storage media that is 2 cm long and 0.1 cm wide. How many bytes of information are lost A) on the stone inscription? B) On the DVD?

Answer: Area of scratch $A = 2 \text{ cm} \times 0.1 \text{ cm} = 0.2 \text{ cm}^2$.

A) Stone: $1 \text{ byte} /10 \text{ cm}^2 \times 0.2 \text{ cm}^2/1 \text{ scratch} =$ **0.02 bytes per scratch**. Note: Since 1 hieroglyphic symbol covers 2 $\text{cm}^2$, the scratch did not damage a single symbol so no data was lost.

B) DVD: $1 \text{ byte} /10^{-8} \text{ cm}^2 \times 0.2 \text{ cm}^2/1 \text{ scratch} =$ **$2\times10^7$ bytes per scratch**

Note: The ancient stone inscriptions are not affected by scratches and hardly any information is lost over their 4000+ year lifetimes. For DVDs, it is conceivable that a single scratch could have destroyed all of the combined hieroglyphic inscriptions from all Egyptian sources had they been digitized and the original stone inscriptions lost.



This image of the sun was obtained on February 12, 2007 by the Hinode Soft X-ray Telescope (XRT), which photographs the sun using the x-ray light that its hot gases (called plasmas) produce.

The image shows three large 'active regions' that are related to sunspot groups with complex magnetic fields. The larger fields can be seen as individual filaments

(Courtesy JAXA/CFA/NASA)

The first thing to do is to understand the scale of an image so that, through image analysis (once called photogrammetry) one can quantitatively determine the sizes of the various features of interest.

**Problem 1** – Using a millimeter ruler, and the fact that the diameter of the sun is 1,380,000 kilometers, what is the scale of this X-ray image of the sun in kilometers/mm?

**Problem 2** – The diameter of Earth is about 13,000 km. At the scale of this X-ray image, how many millimeters in diameter would be a properly-scaled circle representing Earth?

**Problem 3** – The small bright spots in the image are called microflares, and at any given moment, thousands of them cover the entire surface of the sun. What is A) the size of a microflare in kilometers? B) The diameter of a microflare region compared to Earth?

**Problem 4** – How long is the S-shaped filament in A) kilometers? B) Earth diameters?

**Problem 1** – Using a millimeter ruler, and the fact that the diameter of the sun is 1,380,000 kilometers, what is the scale of this X-ray image of the sun in kilometers/mm?

Answer: The diameter of the disk is about 100 millimeters, so the scale is just 1,380,000 km/100mm = **14,000 km/mm**.

**Problem 2** – The diameter of Earth is about 13,000 km. At the scale of this X-ray image, how many millimeters in diameter would be a properly-scaled circle representing Earth?

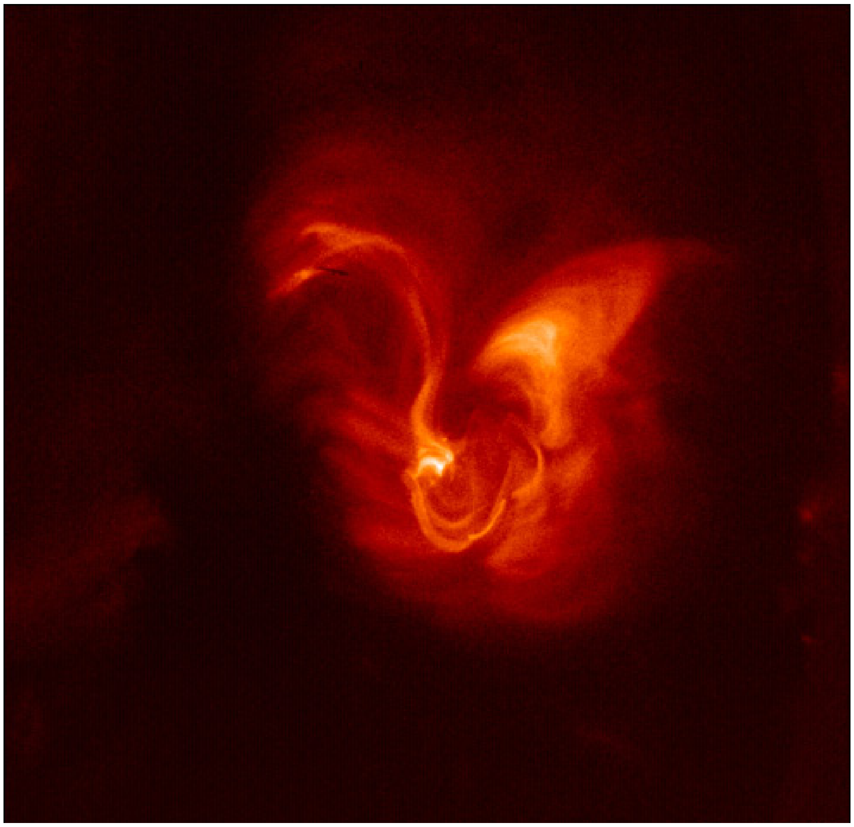
Answer: **About 1 millimeter**!

**Problem 3** – The small bright spots in the image are called microflares, and at any given moment, thousands of them cover the entire surface of the sun. What is A) the size of a microflare in kilometers? B) The diameter of a microflare region compared to Earth?

Answer: A) Depending on which microflare is measured, diameters between 1 to 2 mm are appropriate and correspond to physical sizes from **14,000 to 28,000 km**. B) Ranges from **one to two Earth-diameters** are acceptable.

**Problem 4** – How long is the S-shaped filament in A) kilometers? B)Earth diameters?

Answer: A) The filament spans a distance of about 10 millimeters or equivalently **140,000 kilometers**. B) About **ten times Earth's diameter**.



This image of a solar flare on the sun was obtained on February 5, 2007 by the Hinode Soft X-ray Telescope (XRT), which images the sun using the x-ray light that its hot gases (called plasmas) produce.

This image shows the magnetic structure of active region AR-10940. Numerous magnetic filaments are easily seen at this resolution.
(Credit: JAXA/NASA/SAO)

Solar flares are created in the vicinity of sunspots and other 'active regions' on the solar surface as magnetic fields release their stored energy. Individual parts of a sunspot group are mobile, and carried around by the convecting solar surface. Eventually, magnetic fields become so tangled up by this motion that they reform into simpler shapes: a process called magnetic reconnection. Because of the speed of the motions involved, most of the magnetic rearrangement occurs at scales of 10,000 kilometers or less. High-resolution images of these regions, using the high-energy X-rays that they emit, allow scientists to see the flare initiation process up close for the first time.

**Problem 1** – The width of this image is 512 pixels, with each pixel subtending an angle of 1 arcsecond. At the distance of the sun from the satellite, 1 arcsecond = 780 kilometers. Using a millimeter ruler, what is the scale of this image in kilometers/mm?

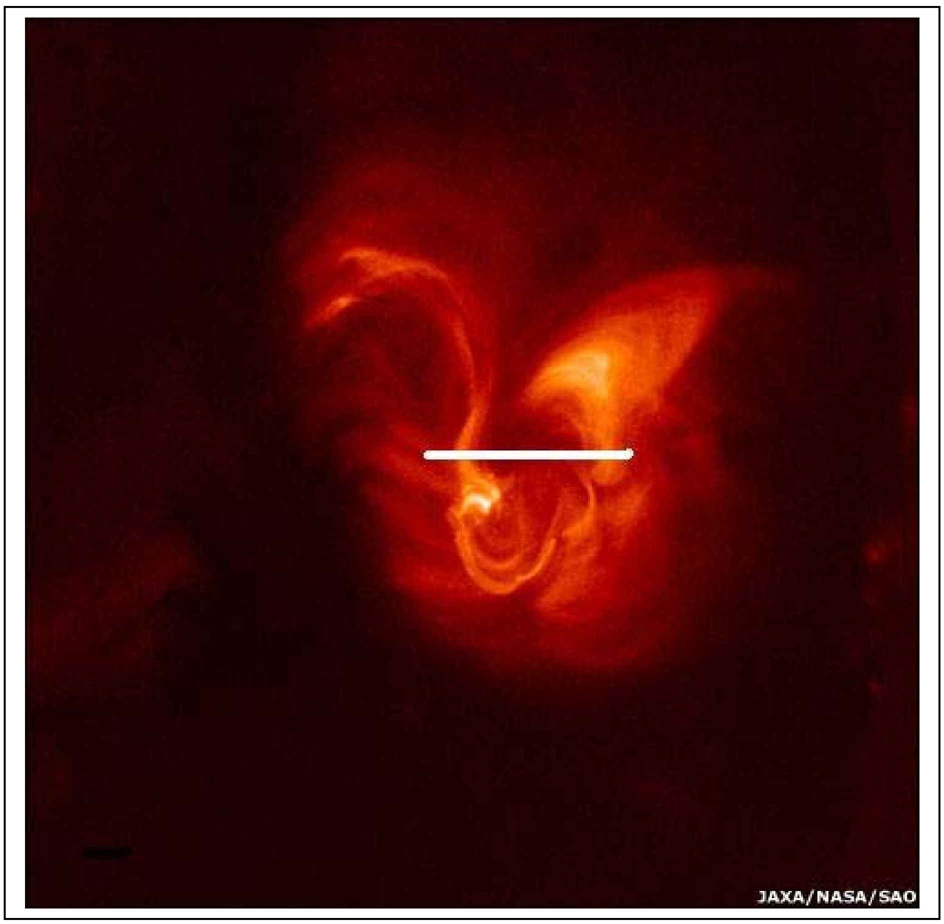
**Problem 2** – The plasma ejected from this active region travels at 80 km/sec. How far will it travel across this active region plasma system in the typical time of a solar flare taking 20 minutes?

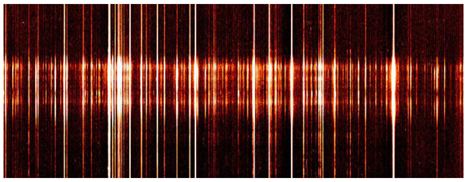
**Problem 1** – The width of this image is 512 pixels, with each pixel subtending an angle of 1 arcsecond. At the distance of the sun from the satellite, 1 arcsecond = 780 kilometers. Using a millimeter ruler, what is the scale of this image in kilometers/mm?

Answer: The physical span of this image is 512 pixels x (780 kilometers/1 pixel) = 400,000 kilometers. The millimeter ruler indicates about 112 millimeters, so the scale is about 400,000 km/112 mm = **3,600 kilometers/mm**

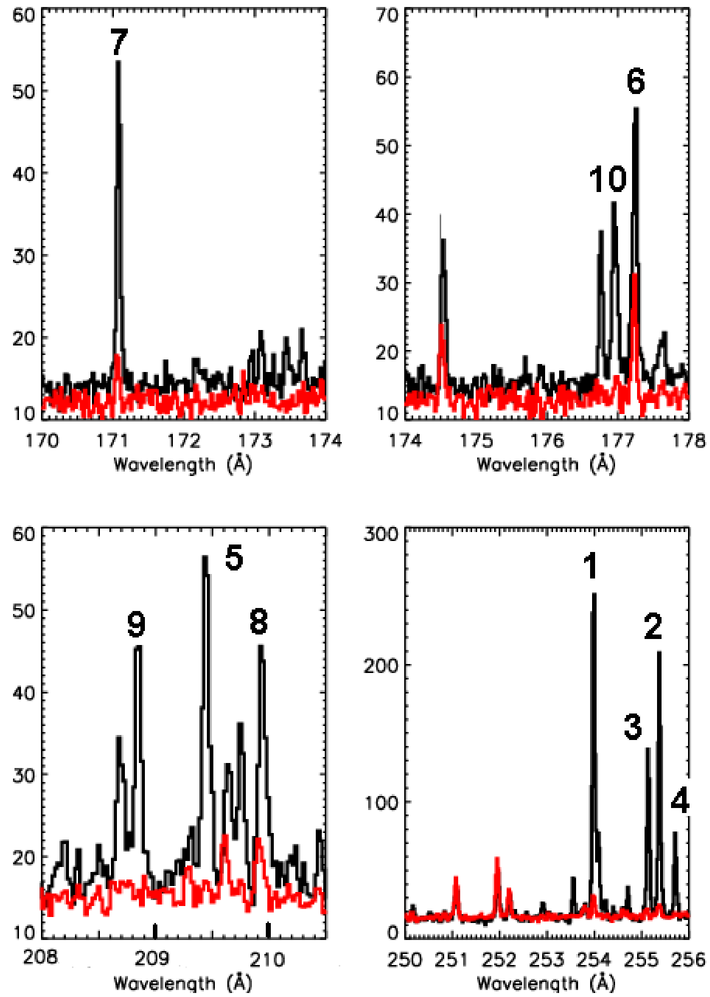
**Problem 2** – The plasma ejected from this active region travels at about 80 km/sec. How far will it travel across this active region plasma system in the typical time of a solar flare taking 20 minutes?

Answer: The distance traveled is 80 km/sec x (20 minutes) x (60 sec/1 minute) = 96,000 kilometers in 20 minutes. At the image scale this is 96,000 km x ( 1 mm/3600 km) = **27 millimeters**. The illustration below gives a sense of how the flaring time relates to the size of the region involved.





Above is a spectrum taken by the Hinode satellite of a specific 'pixel' location on the solar surface. To the right we see a graph of the intensities of the various 'lines' in a portion of this 'bright line' spectrum.




**Problem 1** – For the four graphs combined, determine the peak intensity of each of the 10 strongest spectral lines.

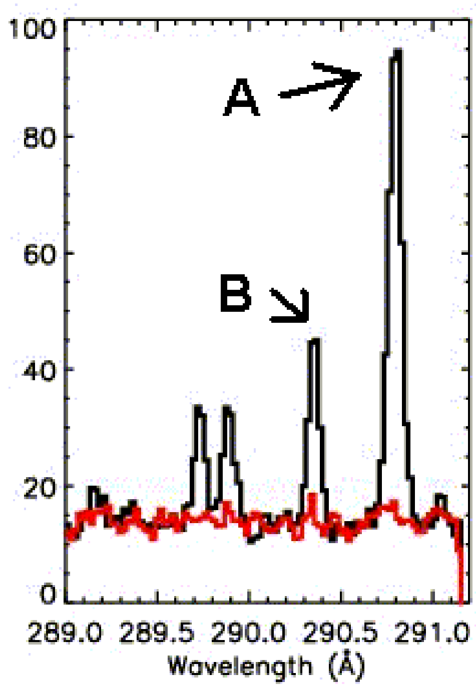
**Problem 2** – Complete the table below listing the wavelength, in Angstroms, and the intensity of each of the 10 spectral lines seen in the collection of four spectra.

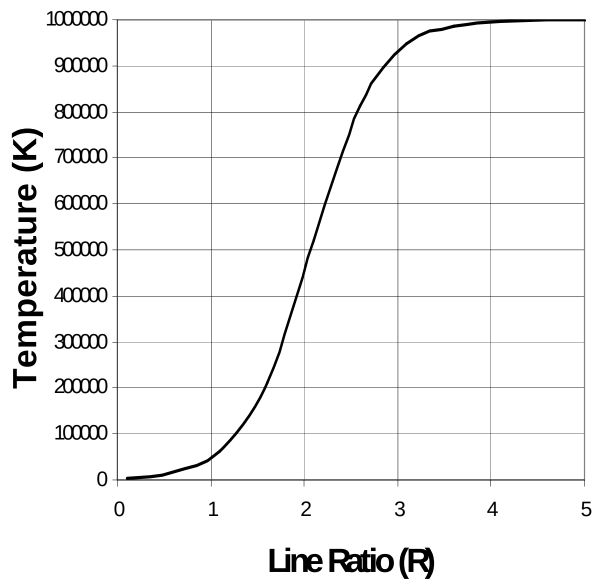
| Line Number | Wavelength (A) | Intensity |
|---|---|---|
| 1 | | |
| 2 | 255.4 | 210 |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |

**Problem 1** – For the four graphs combined, determine the peak intensity of each of the 10 strongest spectral lines. Answer: **See table below.**

**Problem 2** – Complete the table below listing the wavelength, in Angstroms, and the intensity of each of the 10 spectral lines seen in the collection of four spectra. Answer: **See table below**

| Line Number | Wavelength (A) | Intensity |
|---|---|---|
| 1 | 254.0 | 250 |
| 2 | 255.4 | 210 |
| 3 | 255.2 | 140 |
| 4 | 255.7 | 80 |
| 5 | 209.4 | 57 |
| 6 | 177.2 | 56 |
| 7 | 171.1 | 54 |
| 8 | 209.9 | 46 |
| 9 | 208.8 | 45 |
| 10 | 176.9 | 42 |




The light produced by heated gas is emitted in the specific spectral lines that the particular atoms and molecules are permitted to emit according to the rules of quantum mechanics. This also means that some specific lines can be used as a thermometer to determine the temperature of the gas they are a part of. Scientists can do this by selecting the specific atomic lines that work well as 'thermometers' and perform a simple calculation using the ratio of the line intensities.

The spectrum plot to the top-left was obtained by the Hinode satellite Extreme-Ultraviolet Imaging Spectrometer (EIS) of a specific 'pixel' location on the solar surface. To the top-right is a graph that relates the ratio of the brightness of the two spectral lines, R=A/B, to the temperature of the gas producing them.

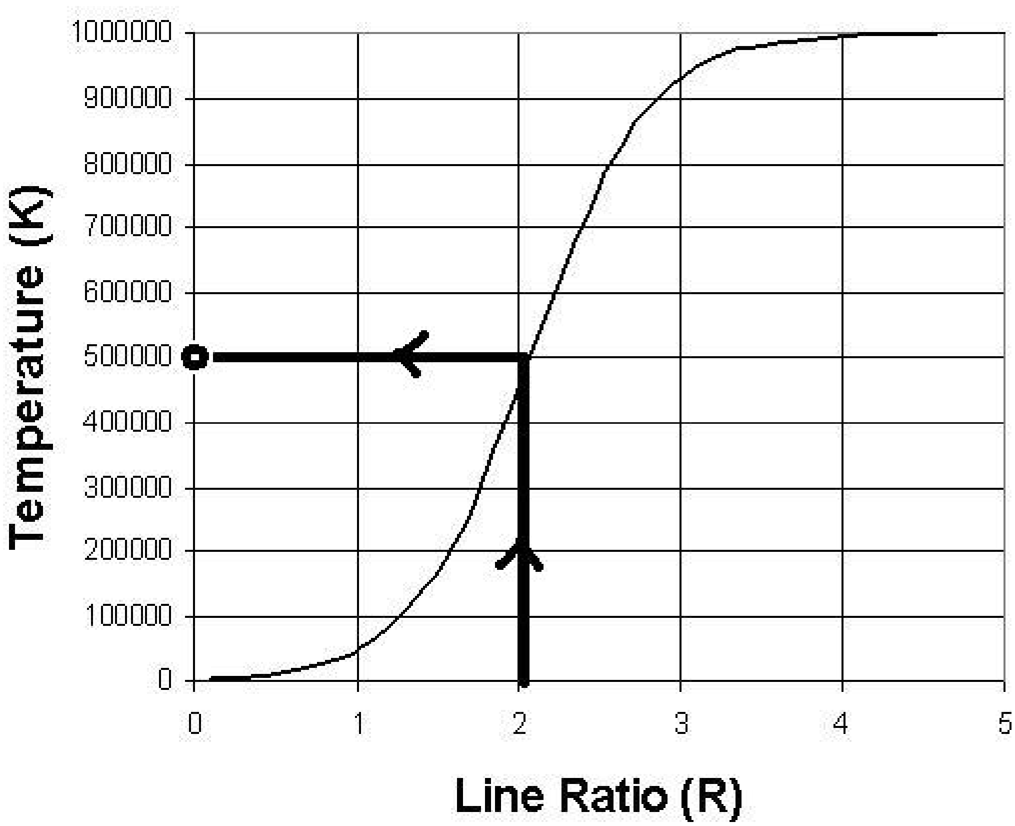
**Problem 1** – Measure the intensities of the two lines, A and B in the spectrum. What is the ratio of the line intensities defined as R = A/B ?

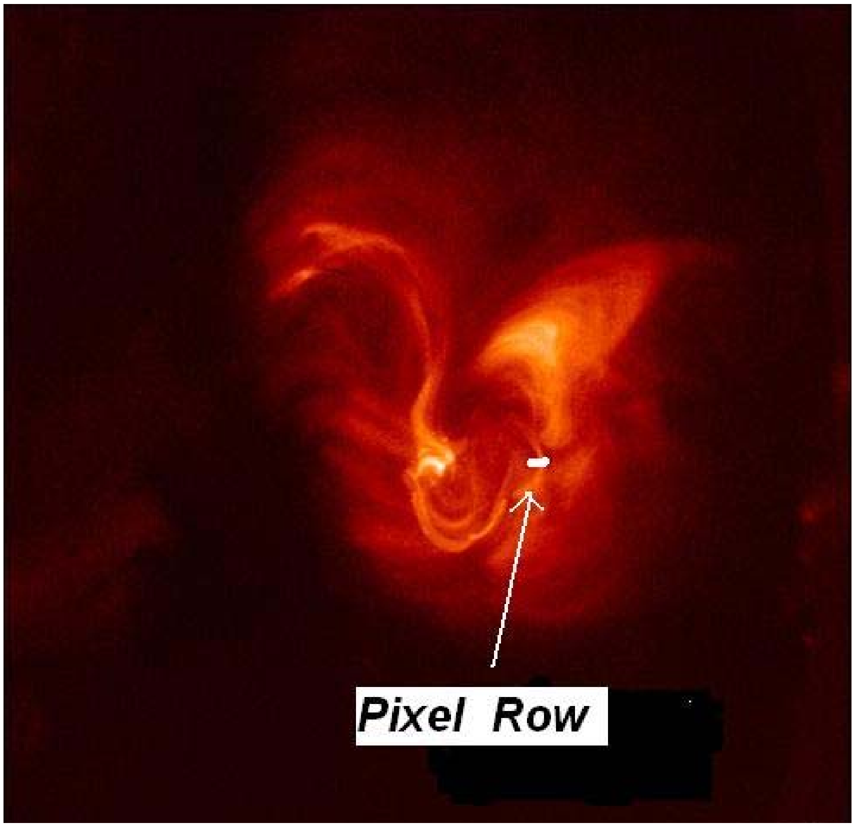
**Problem 2** – From the graph for T(R), what is the estimated temperature of the plasma producing these spectral lines?

**Problem 1** – Measure the intensities of the two lines, A and B in the spectrum. What is the ratio of the line intensities defined as R = A/B ?

Answer: A = 95 and B = 45 so R = 95/45 **R= 2.1**

**Problem 2** – From the graph for T(R), what is the estimated temperature of the plasma producing these spectral lines? Answer: **500,000 K. See graph below**






This image of Active Region AR10940 was obtained on February 5, 2007 by the Hinode Soft X-ray Telescope (XRT), which photographs the sun using the x-ray light that its hot gases (called plasmas) produce.

The image shows hot plasma trapped temporarily in loops of magnetic field associated with a sunspot group.

**(Credit: JAXA/NASA/SAO)**

The white line identifies a string of 10 pixels in the image for which x-ray intensity is measured in the table below. The first row is a measurement of the x-ray light emitted by iron atoms ionized 15 times (FeXVI) and the second row is the light emitted by iron atoms ionized 16 times (FeXVII).

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| FeXVI | 10 | 15 | 15 | 15 | 40 | 30 | 15 | 15 | 5 | 5 |
| FeXVII | 40 | 60 | 45 | 45 | 80 | 60 | 45 | 45 | 10 | 10 |
| Ratio | 4 | | | | | | | | | |

**Problem 1** – The width of the image is 512 pixels and corresponds to a physical distance of 400,000 kilometers. What is the scale of the image in km/pixel?

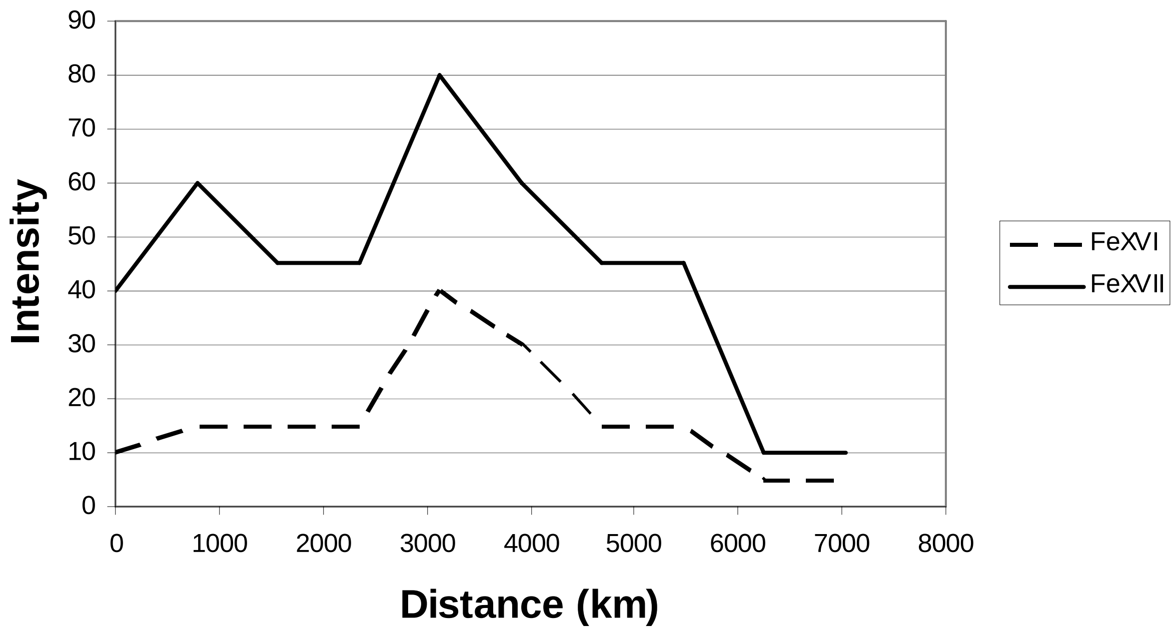
**Problem 2** – Starting from the 'origin' at Pixel 1, create a graph whose horizontal axis is the distance in kilometers from Pixel 1, and whose vertical axis spans the intensity range from 0 to 100. On the same graph, plot the intensities of each of the two iron lines as two separate curves.

**Problem 3** - Complete the table above by calculating the ratio of the intensity of the Fe XVII line to the Fe XVI line to 2 significant figures. (See example in above table)

**Problem 1** – The width of the image is 512 pixels and corresponds to a physical distance of 400,000 kilometers. What is the scale of the image in km/pixel?

Answer: 400,000/512 = **781 km/pixel**

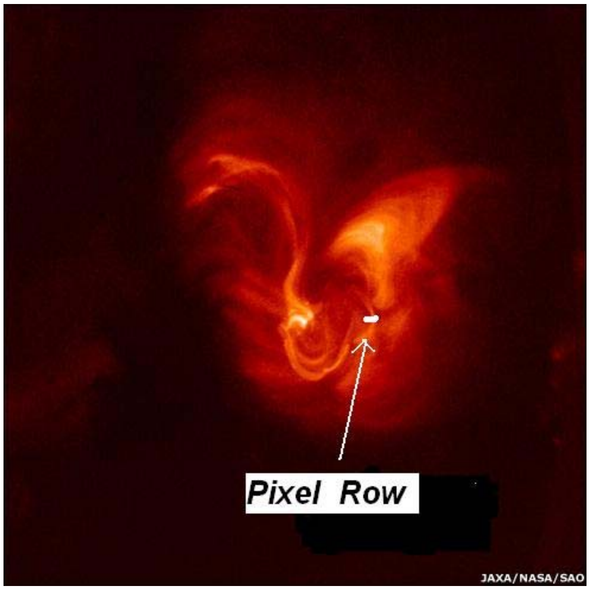
**Problem 2** – Starting from the 'origin' at Pixel 1, create a graph whose horizontal axis is the distance in kilometers from Pixel 1, and whose vertical axis spans the intensity range from 0 to 100. On the same graph, plot the intensities of each of the two iron lines as two separate curves.

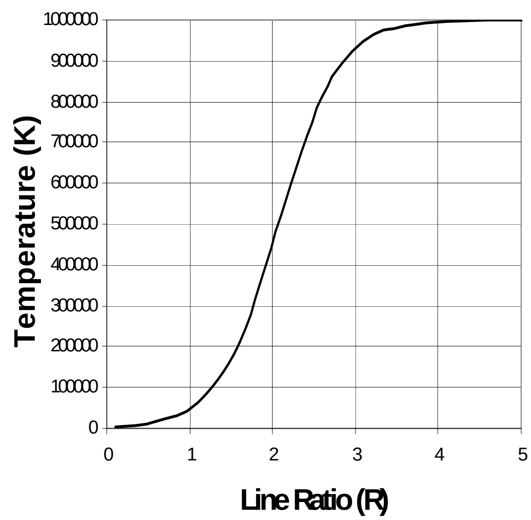



**Problem 3** - Complete the table above by calculating the ratio of the intensity of the Fe XII line to the Fe XVI line to 2 significant figures.

Answer:

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| FeXVI | 10 | 15 | 15 | 15 | 40 | 30 | 15 | 15 | 5 | 5 |
| FeXVII | 40 | 60 | 45 | 45 | 80 | 60 | 45 | 45 | 10 | 10 |
| Distance | **0** | **781** | **1,562** | **2,343** | **3,124** | **3,905** | **4,686** | **5,467** | **6,248** | **7,029** |
| Ratio | **4** | **4** | **3** | **3** | **2** | **2** | **3** | **3** | **2** | **2** |




This image of Active Region AR10940 was obtained on February 5, 2007 by the Hinode Soft X-ray Telescope (XRT), which photographs the sun using the x-ray light that its hot gases (called plasmas) produce. The image shows hot plasma trapped temporarily in loops of magnetic field associated with a sunspot group. **(Credit: JAXA/NASA/SAO)**

The white line identifies a string of 10 pixels in the image for which x-ray intensity is measured in the table below. The first row is a measurement of the x-ray light emitted by iron atoms ionized 15 times (FeXVI) and the second row is the light emitted by iron atoms ionized 16 times (FeXVII).

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| FeXVI | 10 | 15 | 15 | 15 | 40 | 30 | 15 | 15 | 5 | 5 |
| FeXVII | 40 | 60 | 45 | 45 | 80 | 60 | 45 | 45 | 10 | 10 |
| Distance | **0** | **781** | **1,562** | **2,343** | **3,124** | **3,905** | **4,686** | **5,467** | **6,248** | **7,029** |
| Ratio | | | | | | | | | | |
| Temp. | | | | | | | | | | |

**Problem 1** – From the line intensities in each pixel, calculate the line ratio R = FeXVI/FeXVII.

**Problem 2** – From the plotted curve and the line ratio R, determine the temperature of each pixel, T, in units of 100,000 K, and to one significant figure.

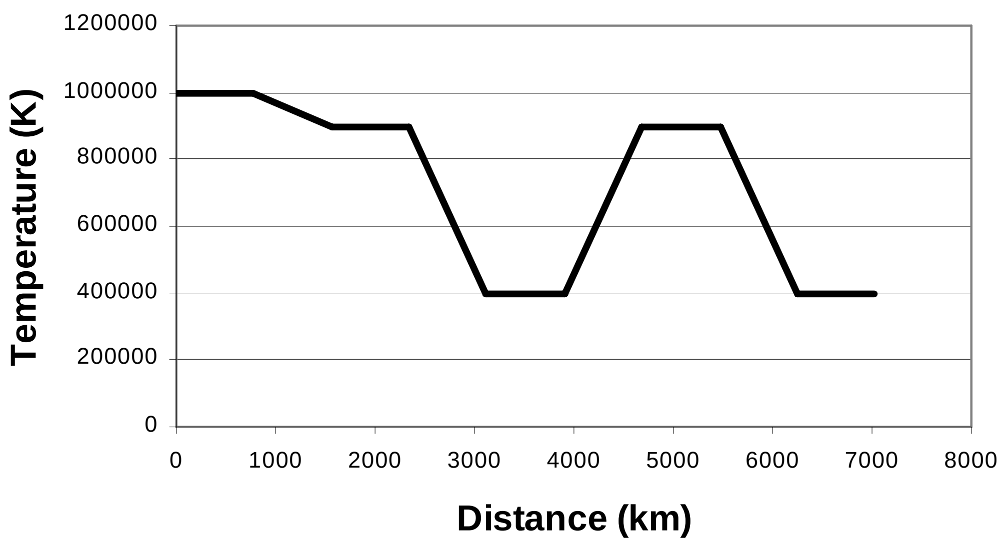
**Problem 3** – Graph the temperature of the pixel in degrees K, as a function of its distance along the active region image.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| FeXVI | 10 | 15 | 15 | 15 | 40 | 30 | 15 | 15 | 5 | 5 |
| FeXVII | 40 | 60 | 45 | 45 | 80 | 60 | 45 | 45 | 10 | 10 |
| Distance | **0** | **781** | **1,562** | **2,343** | **3,124** | **3,905** | **4,686** | **5,467** | **6,248** | **7,029** |
| Ratio | **4** | **4** | **3** | **3** | **2** | **2** | **3** | **3** | **2** | **2** |
| Temp. | **10** | **10** | **9** | **9** | **4** | **4** | **9** | **9** | **4** | **4** |

**Problem 1** – From the line intensities in each pixel, calculate the line ratio R = FeXVI/FeXVII. Answer: **See table above**

**Problem 2** – From the plotted curve and the line ratio R, determine the temperature of each pixel, T, in units of 100,000 K, and to one significant figure. Answer: **See table above**

**Problem 3** – Graph the temperature of the pixel in degrees K, as a function of its distance along the active region image. Answer: **See below**



| Band | Center Wavelength (nm) | Bandwidth (nm) |
|---|---|---|
| 1 | 482 | 450-515 |
| 2 | 565 | 525-605 |
| 3 | 660 | 630-690 |
| 4 | 825 | 750-900 |
| 5 | 1,650 | 1550-1750 |
| 6 | 11,450 | 10400-12500 |
| 7 | 2,220 | 2090-2350 |
| Pan | 710 | 520-900 |

Band 6 resolution = 60 meters
Panchromatic Band = 15 meters.

The Landsat satellite is in an orbit that parallels lines of longitude so that its imaging system scans Earth's surface at a resolution of 30-meters per pixel. Each spot on the ground is also imaged in seven different filters or 'spectral bands' shown in the table to the left. This way, when the images are created from the data, the seven separate images can be combined to thematically classify every pixel in terms of its spectral composition: rock, soil, water, forest, grass, etc. Each of these substances has a distinct spectral fingerprint determined by the amount of light that it reflects in each of the seven bands.

**Creating a small atlas of standard substance spectra:**

**Problem 1** – A pixel covers a spot in the middle of San Francisco Bay and an on-the-spot study confirms that it coincided with pure ocean water. The pixel intensities in Bands 1-7 are given by the ordered set (86,62,47,15, 14,113,15). On a linear scale, graph the spectrum of this 'water' calibration over the band domain and data range X:[1,7] on the X-axis and Y:[0,255] on the Y-axis.

**Problem 2** – A pixel covers a spot in the middle of downtown Oakland and an on-the-spot study confirms that it coincided with modern office buildings. The pixel intensities in Bands 1-7 are given by the ordered set (155,140,150,74,123,140,115). On a linear scale, graph the spectrum of this 'building' calibration over the band domain and data range X:[1,7] and Y:[0,255].

**Problem 3** – A pixel covers a spot in the middle of Redwood Forest and an on-the-spot study confirms that it coincided with conifer trees. The pixel intensities in Bands 1-7 are given by the ordered set (75,63,45,100,83,130,40 ). On a linear scale, graph the spectrum of this 'tree' calibration over the band domain and data range X:[1,7] and Y:[0,255].

**Problem 4** – A pixel covers a spot on Alcatraz Island and an on-the-spot study confirms that it coincided with pure rock. The pixel intensities in Bands 1-7 are given by the ordered set (175,150,155,110,140,120,120). On a linear scale, graph the spectrum of this 'rock' calibration over the band domain and data range X:[1,7] and Y:[0,255].

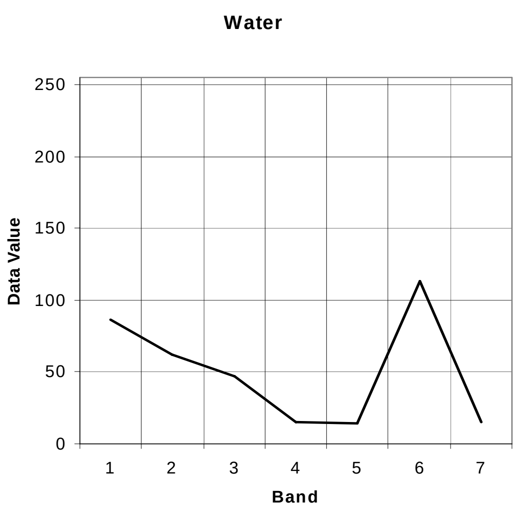
**Problem 1** – water: (86,62,47,15, 14,113,15).

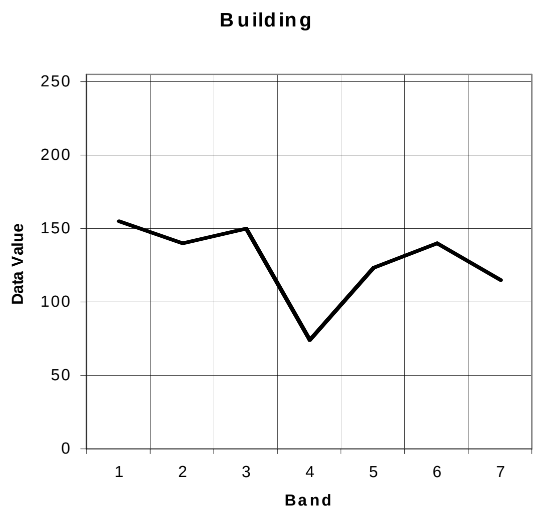
**Problem 2** – buildings: (155,140,150,74,123,140,115).

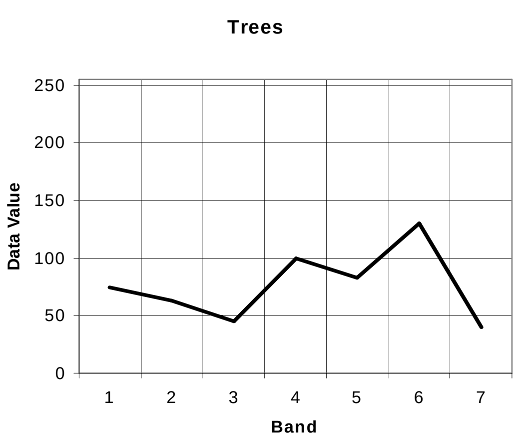
**Problem 3** – trees: (75,63,45,100,83,130,40 ).

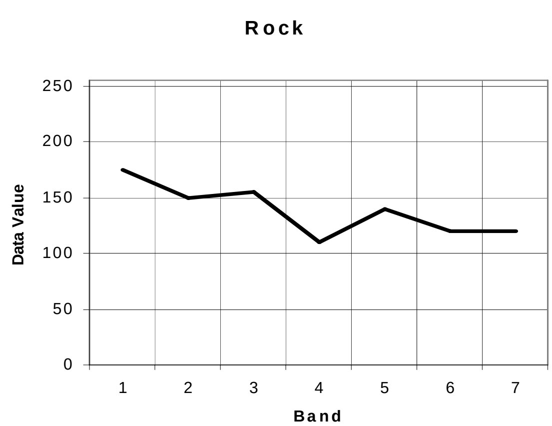
**Problem 4** – rock: (175,150,155,110,140,120,120).

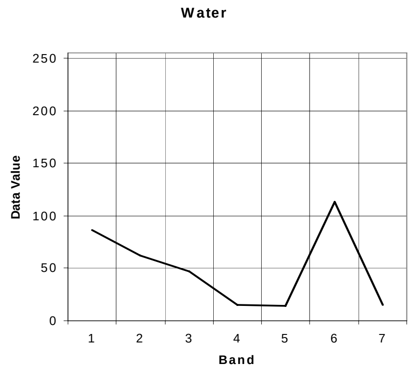
Graphs:

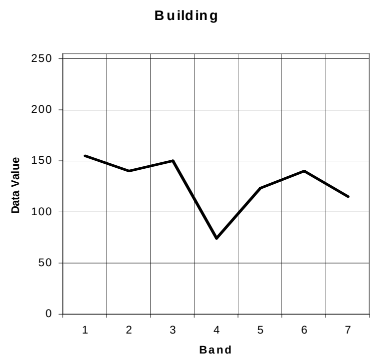


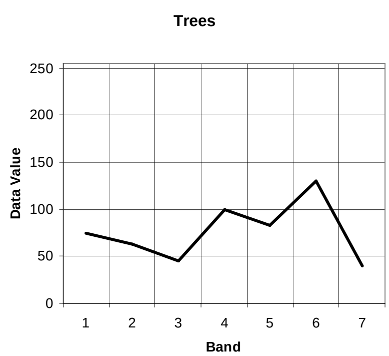



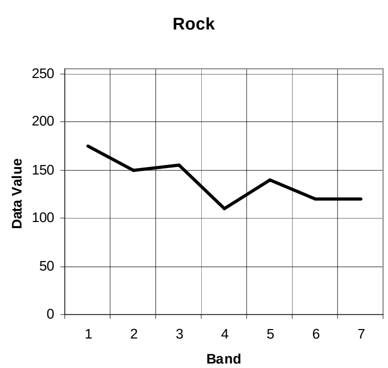






Landsat's high resolution, multiwavelength imaging system has mapped nearly all of Earth's surface, returning huge amounts of data on regions of the globe that are remote and inaccessible for ground study. By using calibrated thematic spectra of known substances, the composition of inaccessible regions can be classified.

The five basic spectra to the left represent a small number of the hundreds of common surface materials that have been 'fingerprinted'. In the problems below, graph the pixel spectra in standard form over the band domain [1,7] and data range [0,255]. Use the five basic spectra and surface types to identify the composition of the 30mx30m area covered by each image pixel.

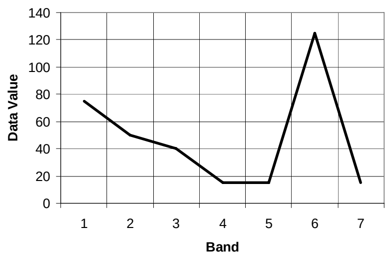
**Problem 1** – This pixel is in the middle of downtown Oakland. What surface in your catalog is most similar to it?
(75,50,40,15,15,125,15)

**Problem 2** – This pixel is located in San Francisco Bay. What surface in your catalog is most similar to it?
(175,150,155,110,140,120,120)

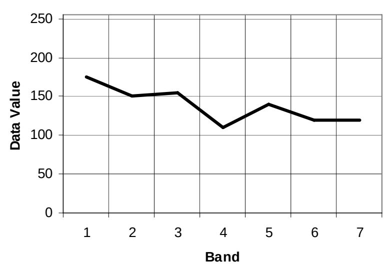
**Problem 3** – This pixel is in the middle of the city of San Francisco. What surface in your catalog is most similar to it?
(70,60,50,80,80,140,40)

**Problem 4** – This pixel is in the middle of Walnut Creek. What surface in your catalog is most similar to it?
(120,100,120,50,100,140,70)

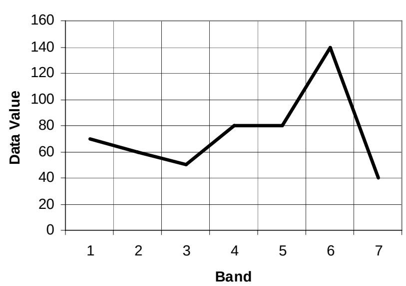
**Problem 1** – This pixel is in the middle of downtown Oakland. What is it?
(75,50,40,15,15,125,15) **This spectrum resembles the 'Water' calibration**



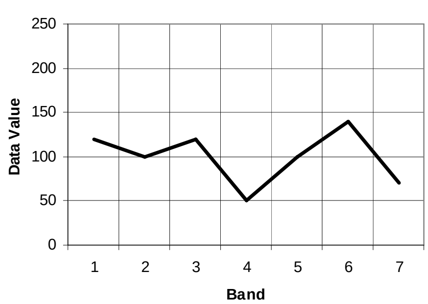

**Problem 2** – This pixel is located in San Francisco Bay. What is it?
(175,150,155,110,140,120,120) **This spectrum resembles the 'Rock' calibration**

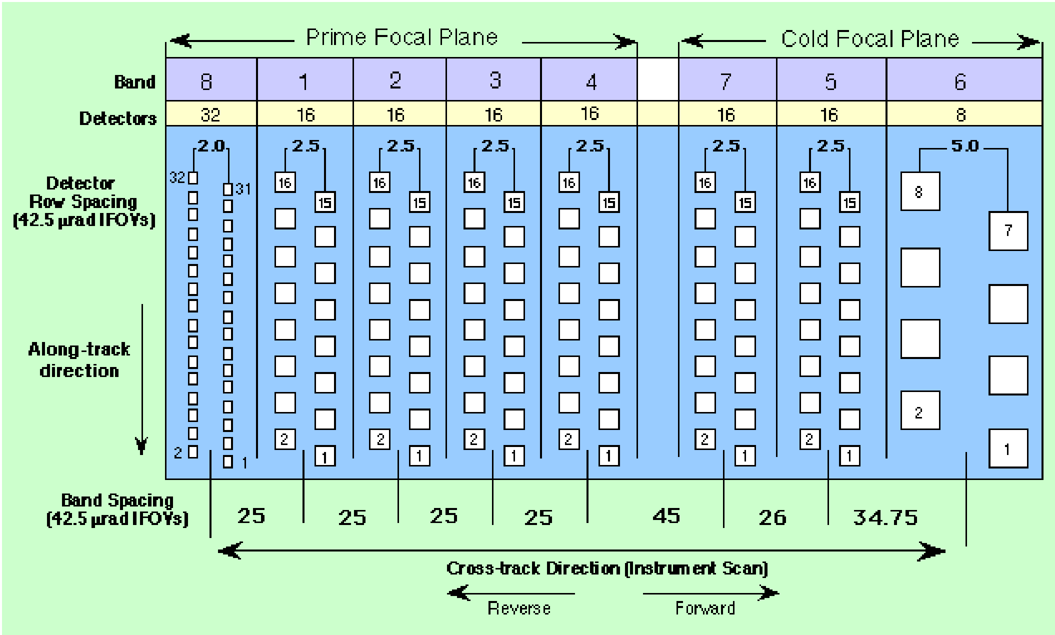



**Problem 3** – This pixel is in the middle of the city of San Francisco. What is it?
(70,60,50,80,80,140,40) **This spectrum resembles the 'Trees' calibration**




**Problem 4** – This pixel is in the middle of Walnut Creek. What is it?
(120,100,120,50,100,140,70) **This spectrum resembles the 'Building' calibration**






Landsat's imaging system is unlike the CCD array in a common digital camera. Instead of the light sensors (pixels) being directly next to each other, there are far fewer of them, and they are located in staggered lies. This set-up is required because the satellite cannot 'target' a specific spot on Earth, but is constantly sweeping its field-of-view across the surface.

As the satellite orbits Earth, the center of its field-of-view, called the focal plane, sweeps across Earth's surface at 7500 meters/sec. The diagram above shows a scaled representation of the pixels in each of the 8 bands. The angular units are given in micro-radians (µrad) so that 1.0 µrad = 0.2 arcseconds. At the altitude of Landsat (705 km), 1.0 arcsecond corresponds to 3.4 meters.

**Problem 1** – If each square sensor in Bands 1 sees an instantaneous area 30 meters on a side, in the above diagram for the sensor geometry, what is the total vertical (along-track) and horizontal (cross-track) length of the Band-1 array?

**Problem 2** – Suppose that the cross-track surface speed of the array from left to right is 7500 meters/sec. How long does it take for the same pixel on the ground to be scanned by Band 1 and Band 2?

**Problem 3** – The sensors measure ground brightness continuously in a data stream. If Pixel 16 is measured in Band 1 at a time of 12:34:56.001 when will the measurement of this pixel in the data stream occur in Band 2?

**Problem 1** – If each square sensor in Bands 1 sees an instantaneous area 30 meters on a side, in the above diagram for the sensor geometry, what is the total vertical (along-track) and horizontal (cross-track) length of the Band-1 array?

Answer: The 16 square detectors are staggered into two columns horizontally, along the vertical direction they completely fill the vertical space with no gaps in between them. So the vertical distance is 30 meters x 9 sensors = **270 meters long**. In the horizontal direction, the scanning 'swath' only provided continuous coverage for a one-pixel-wide vertical strip ,so the width is **30 meters** cross-track.

**Problem 2** – Suppose that the cross-track surface speed of the array from left to right is 7500 meters/sec. How long does it take for the same pixel on the ground to be scanned by Band 1 and Band 2 ?

Answer: The band separation is 25 microradians .This corresponds to 25 x 0.2 arcseconds x 3.4 meters/arcsecond = 17 meters separation on the ground. At 7500 meters/sec, the time interval is T = 17/7500 = **0.002 seconds**.

**Problem 3** – The sensors measure ground brightness continuously in a data stream. If Pixel 16 is measured in Band 1 at a time of 12:34:56.001 when will the measurement of this pixel in the data stream occur in Band 2? Answer: The time interval between repeat measurements is 0.002 seconds, so the time would be **12:34:56.003**

# The Landsat 2-d Images

Band 1

| | | |
|---|---|---|
| 75 | 75 | 75 |
| 175 | 175 | 175 |
| 86 | 86 | 86 |

Band 2

| | | |
|---|---|---|
| 63 | 63 | 63 |
| 150 | 150 | 150 |
| 62 | 62 | 62 |

Band 3

| | | |
|---|---|---|
| 45 | 45 | 45 |
| 155 | 155 | 155 |
| 47 | 47 | 47 |

Band 4

| | | |
|---|---|---|
| 100 | 100 | 100 |
| 110 | 110 | 110 |
| 15 | 15 | 15 |

Band 5

| | | |
|---|---|---|
| 83 | 83 | 83 |
| 140 | 140 | 140 |
| 14 | 14 | 14 |

Band 6

| | | |
|---|---|---|
| 130 | 130 | 130 |
| 120 | 120 | 120 |
| 113 | 113 | 113 |

Band 7

| | | |
|---|---|---|
| 40 | 40 | 40 |
| 120 | 120 | 120 |
| 15 | 15 | 15 |

Landsat images are in the form of arrays of numbers; one array for each band. These numbers give the reflected energy from the surface in the different bands.

The arrays of numbers on the left are the pixel values from a small area in the city of Oakland, California.

**Problem 1** – If the resolution is 30 meters/pixel, what are the dimensions of this area of the city in A) meters? B) feet? (if 1 meter = 3 feet).

**Problem 2** – Graph the spectra of each of the 9 pixels in this image.

**Problem 3** – From the calibration spectra, draw a similar-sized grid and label each pixel with its thematic content (i.e. R=rock; w = water; b = buildings; P=plants/forest)

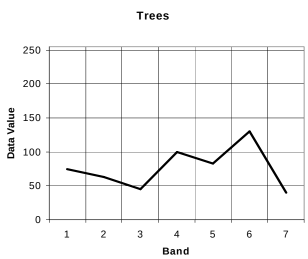
**Problem 4** – Select colors for each of the thematic types and 'colorize' your image to make a false-color picture of this area.

**Problem 5** – What do you think this area would look like if you could view it at ground level?

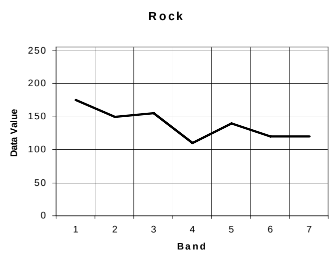
**Problem 1** – What are the dimensions of this area of the city in A) meters? B) feet? Answer: A) 3x3 pixels @ 30-meters/pixel is **90 meters x 90 meters**. B) **270 feet x 270 feet.**

**Problem 2** – Graph the spectra of each of the 9 pixels in this image. Answer: The spectra form three distinct groups:

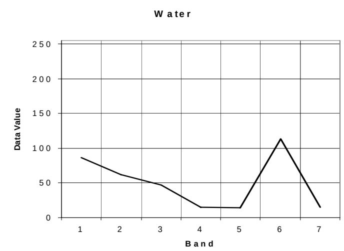
Pixel1-3




Pixel 4-6



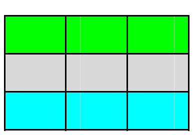

Pixel 7-9




**Problem 3** – From the calibration spectra, draw a similar-sized grid and label each pixel with its thematic content (i.e. R=rock; w = water; b = buildings; P=plants/forest) Answer: **Pixels 1-3 in top row are trees or plants; Pixels 4-6 in middle row are rocky or exposed ground; Pixel 7-9 is water.**

| | | |
|---|---|---|
| T | T | T |
| R | R | R |
| W | W | W |

**Problem 4** – Select colors for each of the thematic types and 'colorize' your image to make a false-color picture of this area. Answer:



**Problem 5** – What do you think this area would look like if you could views it at ground level?
Answer: **This 90m x 90m area borders an area containing water such as a lake, river or creek. The green area is the trees/grass or plant matter bordering a narrow beach-like area.**

# Creating an Image from Numbers

Scientific data is often represented by assigning ranges of numbers to specific colors, then representing the data by these 'false colors' rather than the actual numbers. This allows the eye to see patterns in the data that can be hidden by the numbers themselves.

**Materials:**

Colored pencils or crayons: White, Orange, Red, Yellow, Green, Blue, Black
Piece of 8.5 x 11-inch paper
Metric ruler

**Procedure:**

Step 1) From your color 'pallet', select each color to represent numbers in the indicated ranges:

| Number Range | Color |
|---|---|
| 0 - 5 | Black |
| 6-10 | |
| 11-15 | |
| 16-20 | |
| 21-25 | |
| 26-30 | |
| 31-35 | White |

Step 2) Draw a 100-cell, square grid with 10 cells (pixels) on a side, and each cell 1-centimeter on a side.

Step 3) Using the number grid below, color-in the cells on your grid with the colors you selected in from your color pallet in Step 1.

**Problem 1 -** If the numbers represent temperatures in degrees centigrade, where are the hottest and coldest areas in the false-color image?

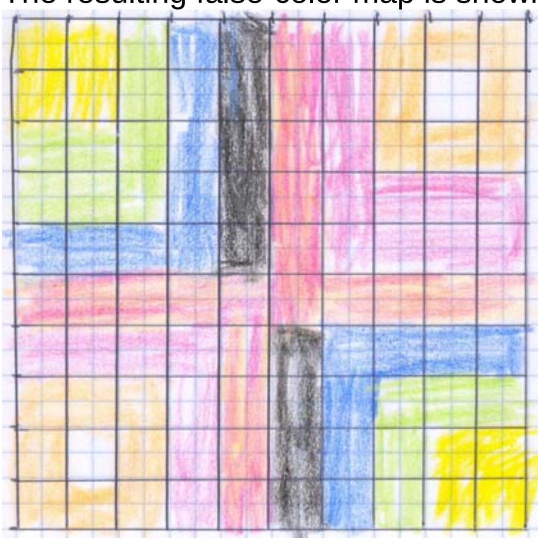
**Problem 2** - If the numbers represent the speed of a gas in kilometers per hour, where is the gas moving between 6 - 10 km/h?

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 20 | 16 | 11 | 6 | 2 | 21 | 23 | 27 | 28 | 29 |
| 17 | 18 | 12 | 8 | 1 | 22 | 22 | 26 | 33 | 28 |
| 14 | 13 | 15 | 7 | 3 | 23 | 23 | 28 | 28 | 27 |
| 14 | 12 | 13 | 9 | 4 | 24 | 24 | 23 | 24 | 25 |
| 6 | 7 | 8 | 10 | 5 | 25 | 25 | 24 | 24 | 24 |
| 21 | 22 | 21 | 22 | 22 | 25 | 24 | 25 | 24 | 23 |
| 23 | 24 | 25 | 24 | 21 | 4 | 10 | 8 | 6 | 8 |
| 26 | 27 | 26 | 24 | 21 | 2 | 8 | 15 | 14 | 12 |
| 27 | 33 | 26 | 23 | 22 | 3 | 7 | 13 | 20 | 18 |
| 26 | 27 | 27 | 24 | 23 | 1 | 6 | 13 | 22 | 17 |

Here is one combination:

| Number Range | Color |
|---|---|
| 0 - 5 | Black |
| 6-10 | Blue |
| 11-15 | Green |
| 16-20 | Yellow |
| 21-25 | Red |
| 26-30 | Orange |
| 31-35 | White |
| | |

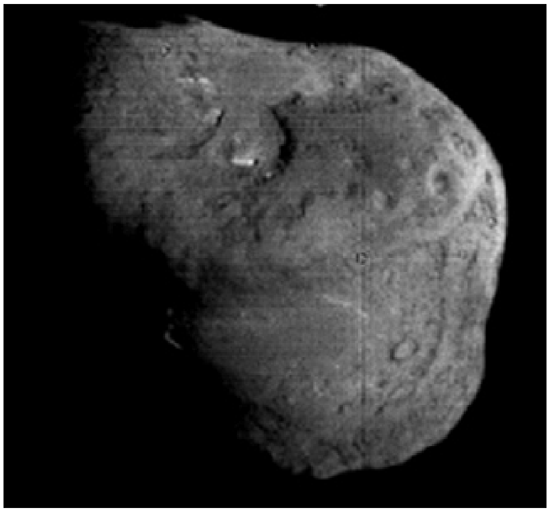
The resulting false-color map is shown below:

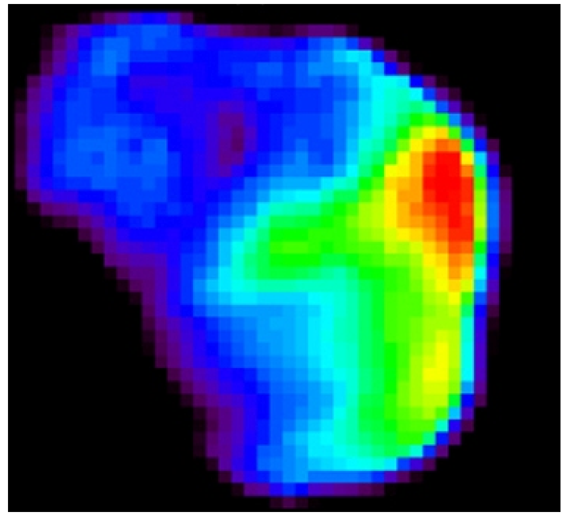


**Problem 1 -** If the numbers represent temperatures in degrees centigrade, where are the hottest and coldest areas in the false-color image?
Answer: **In the two white pixels in the upper-right and lower-left corners.**

**Problem 2** - If the numbers represent the speed of a gas in kilometers per hour, where is the gas moving between 6-10 km/h?
Answer: **In the pixels shaded blue in the above image.**

***Note:*** *This process is identical to old-fashioned 'paint by numbers' games, except that the colors do not have to represent the actual colors of a physical object. There are no 'natural' colors for temperature, speed, chemical composition or dozens of other scientific quantities that are often rendered as an image. That is why scientists call them 'false-colors'.*





The above image (right) was taken by the Deep Impact spacecraft as it passed-by the nucleus of Comet Tempel-1. The false-color image (right) shows the temperature map for this comet. The resolution of the images is 160 meters per pixel. The following color pallet was used to map the temperatures, which are given in Kelvins (K). For a comparison, on the Kelvin scale, Absolute Zero is 0 K, and typical room temperature is 293 K. A very hot day in the desert can reach 320 K.

| Temperature (K) | Color |
|---|---|
| 320-330 | Red |
| 314-319 | Yellow |
| 300-313 | Green |
| 290-299 | Light-Blue |
| 275-289 | Blue |
| 265-274 | Indigo |
| < 264 | Black |

**Problem 1** - In what regions on the surface of the comet nucleus would an astronaut feel most comfortable under typical room-temperature conditions?

**Problem 2** - Over how many square meters would the temperature conditions exceed the hottest desert conditions on Earth?

**Problem 3** - The sun is located to the right of the image so sunlight is traveling from right to left. What do you think might be causing the cold temperature region near the middle of the comet nucleus in the image to the right?

**Problem 1** - In what regions on the surface of the comet nucleus would an astronaut feel most comfortable under typical room-temperature conditions?

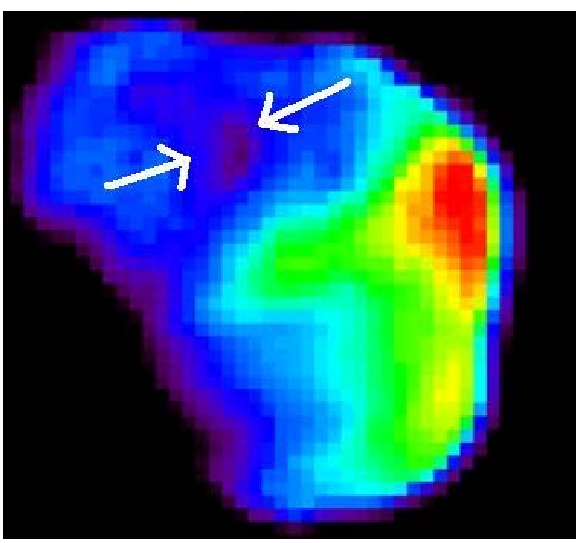
Answer: **Room temperature is 293 K, and this corresponds to a color of light-blue in the false-color temperature map.**

**Problem 2** - Over how many square meters would the temperature conditions exceed the hottest desert conditions on Earth?

Answer: These temperatures would be red-colored in the map. By counting pixels there are about 60 pixels that are reddish or red. Each pixel represents an area of 160 meters x 160 meters or 26,000 meters$^2$, so the total area is **about 1.6 million meters$^2$. This equals 1.6 square kilometers.**

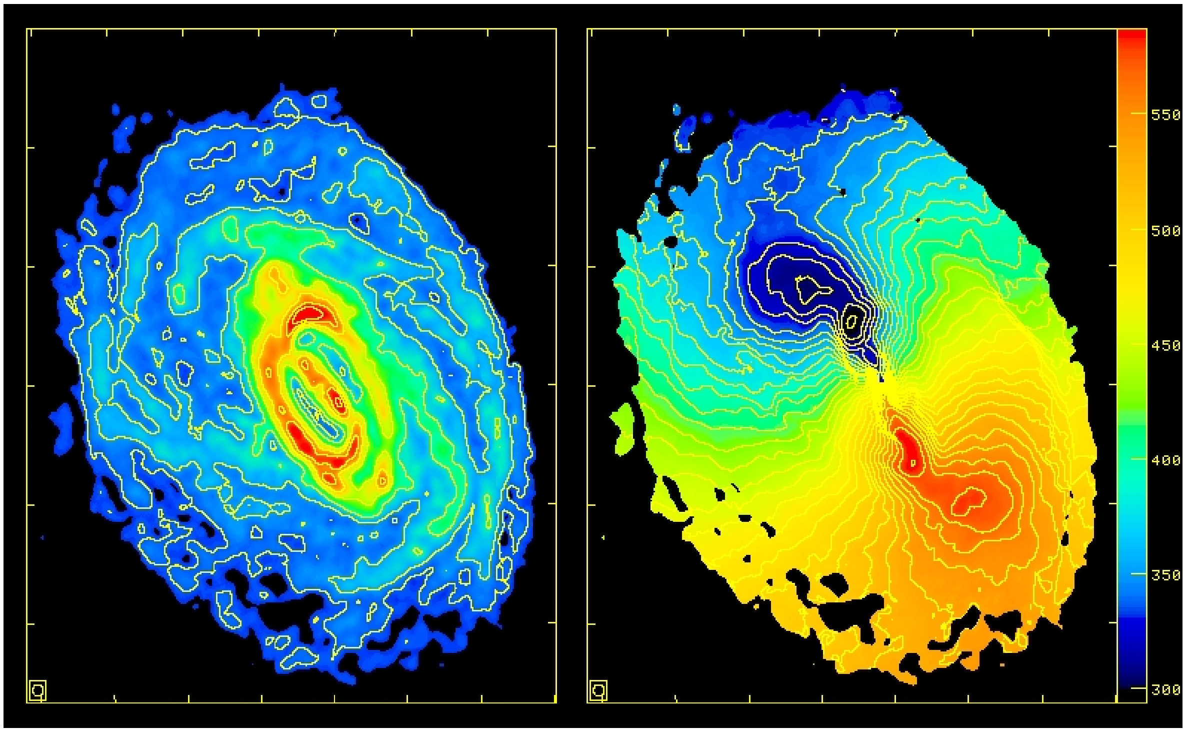
**Problem 3** - The sun is located to the right of the image so sunlight is traveling from right to left. What do you think might be causing the cold temperature region near the middle of the comet nucleus in the image to the right?

**Answer:** Cold temperatures indicate that the sun is not heating the surface, so this region, indicated by the arrow below, must represent a shadowed area on the surface of the comet.



*For more information about Tempel-1 see:*

*http://www.nasa.gov/mission_pages/deepimpact/multimedia/temperature_map-20061002.html*




The Circinus Galaxy, located 15 million light years from Earth, is a spiral galaxy that cannot be seen from Earth because of the enormous amount of dust in our Milky Way which hides it from view. However, astronomers can detect the radio-wavelength light emitted by hydrogen gas in this galaxy (left figure). They can also measure the speed of the gas in this galaxy (right). The color bar shows the pallet used to represent hydrogen gas speeds from 300 to 600 kilometers/sec.

The Doppler Effect in this moving galaxy is visually represented by the red color that indicates gas moving away from Earth, while the blue color indicates gas moving towards Earth. (Courtesy: B. Koribalski (ATNF, CSIRO), K. Jones, M. Elmouttie (University of Queensland) and R. Haynes (ATNF, CSIRO).

**Problem 1** - If the center of the galaxy is moving at 440 km/sec from Earth, what is the maximum speed of the gas moving: A) away from Earth? B) towards Earth?

**Problem 2** - From your answer to Problem 1, what is the average speed of this galaxy as it moves away from Earth?

**Problem 3** - As this galaxy is moving away from Earth: A) explain how the data also shows that the galaxy is rotating around its center; B) what is the absolute magnitude of the speed of rotation around the center of the galaxy?

**Problem 1** - If the center of the galaxy is moving at 440 km/sec from Earth, what is the maximum speed of the gas moving: A) away from Earth? B) towards Earth?

Answer: A) The red color indicates motion away from Earth. The maximum red color equals a speed of 600 km/s.
B) The blue color indicates motion away from Earth. The minimum red color equals a speed of 300 km/s.

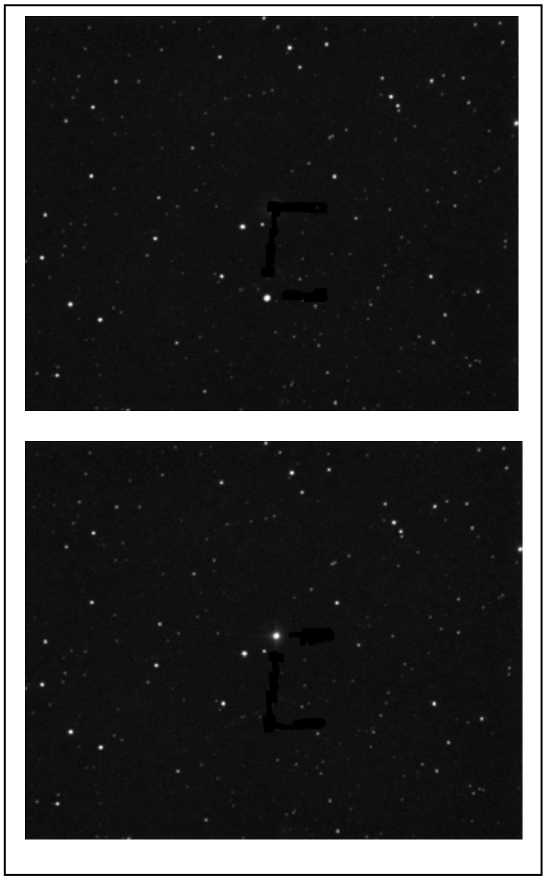
**Problem 2** - From your answer to Problem 1, what is the average speed of this galaxy as it moves away from Earth?

Answer: The average speed is $V = \frac{300 + 600}{2}$ = 450 kilometers/sec.

**Problem 3** - As this galaxy is moving away from Earth: A) explain how the data also shows that the galaxy is rotating around its center; B) what is the absolute magnitude of the speed of rotation around the center of the galaxy?

**Answer:**
A) The top-left part is moving towards the Earth while the bottom-right part is moving away. This is the speed pattern that you would see if you were looking at a rotating body; half of it would be moving towards you and half away ,with the dividing line (axis of symmetry) being the axis of rotation.

B) Relative to the center of the galaxy, the approaching half is moving at ( 300 - 450) = -150 kilometers/sec and the receding half is moving at (600-450) = +150 kilometers/sec, so **the galaxy is spinning at about 150 kilometers/sec**.



By comparing two images of the same region of the sky, astronomers can spot moving objects. Although very distant stars do not move very much even when photographs are spread out over many decades, nearby stars can be seen to move an appreciable distance. The 'remote sensing' of the motion of nearby stars is one way to identify which stars are nearest the sun, and which are much farther away.

The figure to the left shows the change in position of Barnard's Star located 6 light years from Earth. The star is called a red dwarf star, and produces a dull red light that is 280 times less luminous than our own 'yellow' sun.

The photos were taken by UK amateur astronomer Robert Johnson in 1991 (top) and 2007 (bottom). The width of the images is about 1/4 of a degree.

**Problem 1** - Can you find Barnard's Star in the two photographs?

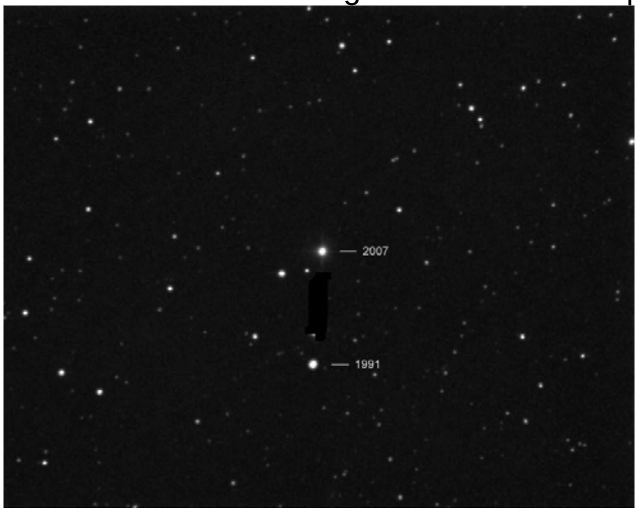
**Problem 2** - Using a millimeter ruler, what is the scale of this image in degrees per millimeter?

**Problem 3** - How many degrees did Barnard's Star travel between 1991 and 2007?

**Problem 4** - What is the angular speed of Barnard's Star in degrees per century?

**Problem 5** - The full moon is 1/2 degree in diameter. How many years will it take for Barnard's Star to travel the diameter of the full moon from its current position in the sky?

**Problem 1** - Can you find Barnard's Star in the two photographs?
Answer: See the markings below in the composite image.




**Problem 2** - Using a millimeter ruler, what is the scale of this image in degrees per millimeter?

Answer: The width of the images, in millimeters, is about 63 millimeters. This represents 0.25 degrees, so the scale is 0.25 degrees/63 mm = **0.004 degrees/millimeter.**

**Problem 3** - How many degrees did Barnard's Star travel between 1991 and 2007?
Answer: The distance between the two star images is about 14 millimeters, so the distance traveled is about 14 mm x (0.004 degrees/1mm) = **0.056 degrees**.

**Problem 4** - What is the angular speed of Barnard's Star in degrees per century?
Answer: The time between the photographs is 2007 - 1991 = 16 years during which time it traveled 0.056 degrees, so in 100 years the star will travel (100/16)x0.056 = **0.35 degrees**.

**Problem 5** - The full moon is 1/2 degree in diameter. How many years will it take for Barnard's Star to travel the diameter of the full moon from its current position in the sky?

Answer: To travel 0.5 degrees, it will take about (0.5/0.35)x100 years = **140 years**.

# Additional Resources

**The Remote Sensing Tutorial -** This NASA resource, written by Dr. Nicholas Short, is a detailed and comprehensive introduction to the history and practices of remote sensing. Dr. Short, a former NASA Goddard employee and author/editor of four NASA-sponsored books (Mission to Earth: Landsat Views the World; The Landsat Tutorial Workbook; The HCMM Anthology; and Geomorphology from Space) put his significant experience and talents to work to present an updated and expanded version of his past efforts. The result is this Internet website and a CD-ROM (also tied to the Internet) entitled "The Remote Sensing Tutorial".

**http://rst.gsfc.nasa.gov/**

**An Online Guide to Remote Sensing** - The Online Remote Sensing Guide consists of two web-based instructional modules that use multimedia technology and the dynamic capabilities of the web. These resources incorporate text, colorful diagrams, and animations to introduce selected topics in the field of remote sensing. Selected pages link to (or will soon link to) relevant current weather products, allowing the user to apply what has been learned in the instructional modules to real-time weather data. The target audience for the Online Remote Sensing Guide is high school and undergraduate level students. However, these resources have been used by instructors throughout K-12, undergraduate and graduate level education. Contents of the Online Remote Sensing Guide were developed by graduate students and faculty through our efforts in the Collaborative Visualization Project(CoVis), which was funded by the National Science Foundation. These resources have been reviewed by faculty and scientists at the University of Illinois and the Illinois State Water Survey. Many of these resources were tested in a classroom environment and have been modified based upon teacher and student feedback

**http://ww2010.atmos.uiuc.edu/(Gh)/guides/rs/home.rxml**

**Earth Observatory** - This is an extensive NASA resource covering all aspects of remote sensing as practiced by NASA earth observation satellites.

**http://earthobservatory.nasa.gov/Features/RemoteSensing/**

**Remote Sensing Core Curriculum** - Developed by the University of Minnesota and supported by grants from NASA, this extensive resource is directed at students entering the field of remote sensing, and includes content for K12 students.

**http://rscc.umn.edu/**

# A Note from the Author

*Hi Again!*

*Remote sensing sounds like an intimidating concept, but in fact we do it every day, whether watching our televisions to get the latest images from distant lands, or talking on our cell phones. It's all about gaining information about distant places using technologies that extend our senses.*

*This book is designed to let you experience many of the mathematical themes that run through remote sensing, but you will not need advanced math to appreciate the details! Many of the problems require little more than simple arithmetic, or working with proportions and unit conversions.*

*I think the most incredible thing about remote sensing is that it almost magically lets us visit places, and see things, that humans could never directly experience about the physical world. Thanks to remote sensing, we can explore the surface of a distant planet, probe the interior of the sun, or visit the nucleus of an atom.*

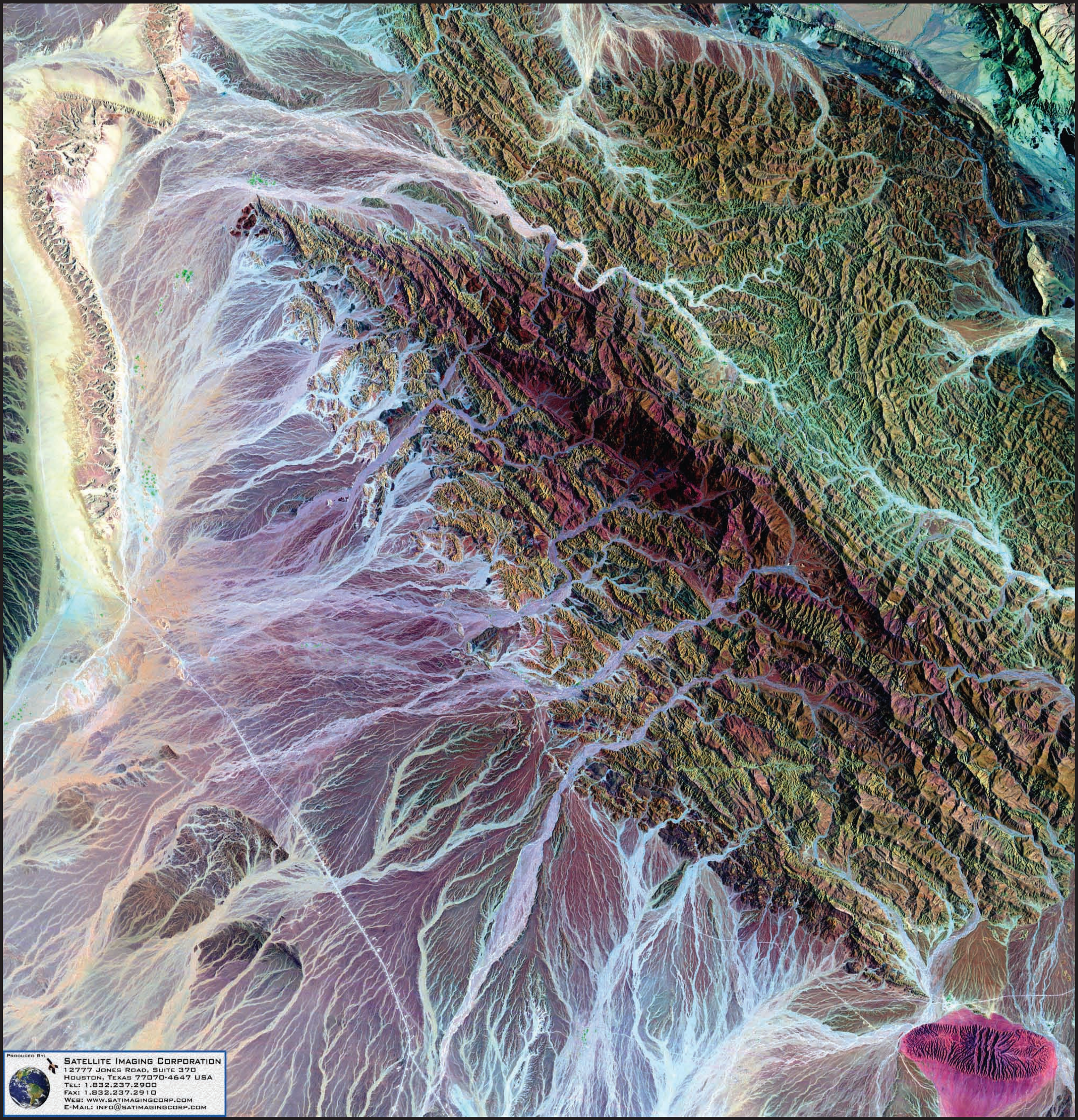
*When I was 10 years old and my Father showed me the stars in Orion's belt for the first time, I recall wondering what these stars looked like up close. For the next few years I read all that I could abut space and astronomy at the same time that NASA was working its way through the Gemini and Apollo programs and launching the Pioneer and Mariner spacecraft to visit Venus and Mars. Those were exciting times for a child, and the fantastic pictures of star clusters, nebula and galaxies were thrilling and wondrous to study. It was my introduction to the idea that the 'world' is far bigger than the small slice of it that you see on the Evening News. There is also more going on in the universe than the mere squabbling of humans on a remote planet.*

*Today, in our relentlessly commercialized and self-absorbed world, I wish that our children would be encouraged to step outside themselves and their electronic networks to recapture a sense of perspective that even many adults have lost.*

*Mathematics and remote sensing will not be the answer to all these questions and issues, but it will surely help to show their logical interconnections, and give us once again a sense of proportion. Whether the oil leaking into the Gulf represents 5,000 barrels per day or 50,000 barrels per day is a bigger question than merely a factor of ten error. It represents a complete change of livelihood for millions of people.*

*Sincerely,*

*Dr. Sten Odenwald*
*Space Math @ NASA*

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