

CALIFORNIA PRESCHOOL INSTRUCTIONAL NETWORK

# Engaging Children in Early Mathematical Experiences 

Mathematics Handbook

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## Priorities

Conduct regional network meetings for administrators and professional development for teachers that provide current research-based information and resources to support high quality professional development.
Provide direct preschool program support to early childhood education school readiness staff. In-depth support will include training and technical assistance, classroom observation, and on-site support to enhance the quality of preschool programs.
Inform program practitioners of resources, events, and opportunities through an established network of communication.
Network to ensure all children have access to high quality early care and education experiences, including English language learners and children with special education needs.


Engaging Children in Early Mathematical Experiences

Dedication
CPIN's Capital Service Region's Engaging Children in Early Mathematical Experiences resource is dedicated to all early childhood educators and parents who seek to create mathematically-rich environments and to develop engaging relationships with children.

## Acknowledgements

Heartfelt gratitude is extended to the CPIN Region 3 Consultants who contributed research-based information and creative learning activities for the resource. The CPIN Region 3 team appreciates the collaborative effort that these early childhood education experts provided throughout the development process.

## CPIN Consultants

> Liz Aguilar
> Marilyn Astore
> Sue Hudnall
> Kelly Twibell

CPIN sincerely appreciates and thanks the teachers, parents, and children who field-tested the Engaging Children in Early Mathematical Experiences resource by piloting it in homes and classrooms to engage children and families in early mathematics activities. The final product incorporates the valuable feedback of all who participated in the field-testing of this resource.

## Field Test Sites

A Circle of Friends Children's Center - Mary Kragel
Hand In Hand Child Development Center - Shelly Oswalt
Elk Grove Unified School District - State Preschool Program/Head Start
Sacramento Valley Christian Academy - Tracee Lewis
Stepping Stone Learning Academy - Sheila Haddox
U.C. Davis Center for Child and Family Studies


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Engaging Children in Early Mathematical Experiences

## Introduction

The CPIN Region 3 Engaging Children in Early Mathematical Experiences publication was developed to provide families, early childhood educators, and family childcare providers with research-based early learning experiences focused on the California Preschool Learning Foundations. The five key mathematics developmental strands of the foundations are: Number Sense, Algebra and Functions (Classification and Patterning), Measurement, Geometry, and Mathematical Reasoning.

The activities in this resource are closely aligned with the developmental trajectories described in the California Preschool Learning Foundations, Volume 1, Mathematics Domain. The Foundations can be used by professionals and families in conjunction with this resource as a tool for understanding young children's growth and learning. This resource can serve as a guide for selecting and implementing mathematical learning experiences that match individual developmental needs.

Research has shown when children frequently engage in everyday mathematical activities, they are more likely to flourish in mathematical development. Early concrete experiences with small numbers, simple patterns, basic shapes, and nonstandard measurement are the basic building blocks for mathematical achievement. These skills form the foundation for learning algebra, geometry, and data analysis in elementary school and are linked to later outcomes in reading and mathematics. The purposeful learning activities contained in this resource are designed to support and enhance informal and school mathematical experiences that form an essential foundation to later achievement.

# "Throughout the early years of life, children notice and explore mathematical dimensions of their world. They compare quantities, find patterns, navigate in space, and grapple with real problems, such as balancing a tall block building or sharing a bowl of crackers fairly with a playmate. Mathematics helps children make sense of their world outside of school and helps them construct a solid foundation for success in school." 

## Mathematics <br> Prekindergarten Curriculum Focal Points

## Number Sense

mportant aspects of counting, number relationships and operations

Preschool children develop an initial qualitative understanding of a quantity of small groups of objects without actually counting the objects. This understanding is referred to as visually knowing or "subitizing." It supports the ability to compare small groups of objects: to know if the groups are the same, if one group is larger (smaller), or which has more (fewer). Also developing is the ability to approach simple arithmetic-like operations on groups of objects with ideas such as "adding to," "putting together," "taking apart," "taking away," and so forth. Preschool is the time when children begin to learn to recite the numbers in order, recognize some numerals, and begin to incorporate the idea of one-to-one correspondence and true counting. This is also a time when preschool children begin to learn about cardinality, which is the concept of knowing that the last number named is the quantity of objects counted


## Geometry

Identifying shapes and describing properties of objects (shape, size, position) and the spatial relationships of objects in space

Geometry is a tool for understanding relations among shapes and spatial properties. Children develop spatial reasoning and can visualize shapes in different positions (orientation). They learn to describe the direction, distance and location of objects in space, in their own words. Children build pictures and designs by combining two- and three- dimensional shapes, and they solve such problems as deciding which piece will fit into a space in a puzzle. Opportunities to explore attributes of different shapes and to use mathematical vocabulary words help children's development of geometry and spatial thinking.

Algebra \& Functions (Classification and Patterning) Sorting and classifying objects: recognizing, extending, and creating patterns

During the preschool years, children develop beginning algebraic concepts as they sort and classify objects, observe patterns in their environment, and begin to predict what comes next, based on a recognized pattern. Sorting items, classifying them, and working with patterns help children to bring order, organization, and predictability to their world. Classification and the analysis of patterns provide a foundation for algebraic thinking as children develop the ability to recognize relationships, form generalizations, and see the connection between common underlying structures.

## Mathematical Reasoning

Using mathematical thinking to solve problems in play and everyday activities

Children in preschool encounter situations in play and everyday activities that require them to adapt and change their course of action. Although they may not realize it, some situations call for mathematical reasoning - to determine a quantity (e.g., "How many spoons?") or to reason geometrically (e.g., "What shape will fit?"). Other situations require general reasoning or problem solving. For preschoolers, when the context is familiar and comfortable enough, a simple strategy may be applied to solve an immediate problem - even something as simple as counting the number of objects held in the hand or carrying a block over to see if there are others like it. A young preschool child may begin to develop this process by trying a strategy that is not always effective. An older preschool child may try several strategies, finally finding one that works. Children learn through reasoning mathematically. Encouraging young children to engage in mathematical reasoning is not only beneficial in itself, but it also opens the door to children's exploration of the other mathematical foundations, such as geometric shapes, counting, and classification.

## Measurement

Identifying measurable attributes and comparing objects by length, weight, or capacity; precursors to measurement

Measurement is assigning a number of units to some property, such as length, height, or amount of an object. Children begin to compare, order, and measure in preschool. They identify objects as "the same" or "different", and then "more" or "less", on the basis of attributes that they can measure. They identify measurable attributes, such as length and weight, and use language such as "heavier", "taller", or "longer" to solve problems by making direct comparisons of objects on the basis of those attributes. Measurement is one of the main real-world applications of mathematics.

National Council of Teachers of Mathematics (2006) \& California Preschool Learning Foundation (2008)

## Early Childhood Mathematics: Promoting Good Beginnings

 Recommendations
## In high-quality mathematics education for 3 to 6 year-old children, teachers and other key professionals should:

- Enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social worlds.
- Build on children's experiences and knowledge, including their family, linguistic, cultural, and community backgrounds; their individual approaches to learning, and their informal knowledge.
- Base mathematics curriculum and teaching practices on the knowledge of young children's cognitive, linguistic, physical, and social-emotional development.
- Use curriculum and teaching practices that strengthen children's problem-solving and reasoning processes, as well as representing, communicating, and connecting mathematical ideas.
- Ensure that the curriculum is coherent and compatible with known relationships and sequences of important mathematical ideas.
- Provide for children's deep and sustained interaction with key mathematical ideas.
- Integrate mathematics with other activities and other activities with mathematics.
- Provide ample time, materials, and teacher support for children to engage in play, a context in which they explore and manipulate mathematical ideas with keen interest.
- Actively introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies.
- Support children's learning by thoughtfully and continually assessing all children's mathematical knowledge, skills, and strategies.

A joint position statement of the National Association for the Education of Young Children (NAEYC) and the National Council for Teachers of Mathematics (NCTM). (2002)

"High quality teaching in mathematics is about challenge and joy, not imposition and pressure. Math instruction includes providing a lot of unit blocks, along with loads of time to use them. It invites children to experience mathematics as they play in, describe, and think about their world."

## How to Use This Resource Guidance for Parents

Engaging Children in Early Mathematical Experiences contains research-based, hands-on activities and intentional practices that can be used at home to support your child's early mathematical development.

Research indicates that children enjoy and benefit from a variety of mathematical explorations. The activities in this resource are fun and offer opportunities for family members to spend meaningful time together. Interacting with your child while doing these activities, for example, in the car, on a walk, in the library, and in the grocery store can become a part of daily routines. The more opportunities your child has to experience mathematics in everyday play, the more mathematical concepts he/she will develop.

As you and your child begin to explore these activities, share information that will help him/her understand mathematical concepts. For example, "I see you counted three bears from the pile of ten bears. Do you want to do that again? I would love to do it again with you." Another example might be: "I just saw the number 25 on the speed limit sign. I wonder what other numbers we can find on the way to school." Incorporating mathematical vocabulary into your conversation will enhance the activities and support your child's vocabulary development.

One way that children learn is through repetition of meaningful activities. For this reason, some days on the learning activity pages are intentionally left blank as an opportunity to repeat and reinforce previously introduced concepts. The activities in this resource are paced in a manner that will provide your child with adequate time to understand mathematical concepts. Follow your child's lead, and repeat the experiences that she/he enjoys. Your child will want to engage in many of these fun learning activities again and again. The more you show your enjoyment of these interactive experiences, the more encouraged your child will be to grow in his/her mathematical thinking. 3

## Guidance for Parents (continued)

The activities in this resource are structured in a developmental progression within each section and can be completed in order. If an activity seems too difficult for your child, return to a previously completed task. Choosing activities that are less challenging at first makes it easier for your child to succeed. The goal is to interact with your child, and not whether he/she is doing the activity right or wrong.

All children can engage in the activities included in this resource. Some adaptations and modifications may be necessary in order to meet the learning level of your child. It is important to remember that your child is unique and will gain new skills at his or her own pace.

Engaging your child in the experiences contained in this resource will expose him/her to foundational concepts and skills that are the building blocks of success in mathematics. Providing meaningful math activities also supports the development of your child's language and vocabulary. Prior to engaging in the activities, review the Glossary of Terms on pages 54-56 in order to become familiar with the mathematical vocabulary included in this resource. Pages for taking notes are provided throughout the resource at the end of each section. These note pages can be used to record any pertinent information you may want to reflect on after engaging in the activities. You may also wish to communicate with your child's teacher regarding what he/she is learning at home. Most importantly, remember to have fun and enjoy doing the activities with your child!

## Guidance for Early Childhood Educators

Engaging Children in Early Mathematical Experiences contains research-based, hands-on activities and intentional teaching practices aligned with the California Preschool Learning Foundations, Volume I. These activities can easily supplement and be integrated into an existing curriculum to provide additional support for children's mathematical learning.

Children have a natural interest in exploration and learning. Research shows that high-quality math education can and should begin early. Early intervention in mathematics can prevent later learning problems in school for all children. The early childhood educator's role is to support children's emerging skills and interests in order to plan meaningful learning experiences. The activities included in this resource are fun and offer opportunities to engage children in playful, planned, and purposeful learning. The more exposure children have to practice math concepts throughout the day, the more they will be prepared for school and later life.

One of the ways that children learn is through repetition. Thus, they will enjoy engaging in many of the fun learning activities again and again. For this reason, some days on the learning activity pages are intentionally left blank as an opportunity to repeat and reinforce previously introduced concepts. The activities in this resource are paced in a manner that will provide children with adequate time to understand the mathematical concepts. Follow the children's lead, and repeat the math experiences they enjoy. The more you show enthusiasm during these interactive experiences, the more encouraged the children will be to learn.

## Guidance for Early Childhood Educators (continued)

The recommended activities in this resource are structured in developmental progression within each section and can be completed in order. If an activity seems too difficult, reference the California Preschool Learning Foundations for guidance. Choosing activities that are less challenging at first makes it easier for children to succeed. The goal is to interact with the children and scaffold their learning. When children are given opportunities to build on their existing knowledge, it becomes easier to master new skills as they link them to familiar concepts.

It is highly recommended that the guidance for parents on pages nine and ten be provided at the time any portion of Engaging Children in Early Mathematical Experiences is disseminated. Information on how to use this resource can be shared in a newsletter, during parent meetings, parent teacher conferences, open house, or on back-to-school night. Plan to give a brief overview of the resource, and explain how you expect parents to use the content. Encourage parents to enjoy engaging their child in the activities. You may suggest parents work on only one math concept at a time or use one page per month. Providing the entire resource at one time could overwhelm parents.

All children can engage in the experiences included in this resource. Providing meaningful math activities also supports the development of children's language and vocabulary. Prior to engaging children in the activities, review the Glossary of Terms on pages $54-56$ in order to become familiar with the mathematical vocabulary included in this resource. Some adaptations and modifications may be required in order to meet the diverse individual needs of children. Engaging children in the experiences contained in this resource will expose them to fundamental math concepts, prepare them for kindergarten, and build a foundation for many important mathematical abilities that will come later in school.

## Tools You Can Count On

Below is a list of items that you will need to engage in the fun learning activities within each strand of mathematical development. The materials list is adapted from The Intentional Teacher, a NAEYC publication. These items are readily available; you probably already have many of them in your classroom or home. The materials are also inexpensive and may be found at a dollar store. In addition, you are encouraged to visit the public library to access the wide selection of books recommended throughout this resource.

| Number Sense | Algebra \& Functions (Classification and Patterning) | Measurement | Geometry | Mathematical Reasoning |
| :---: | :---: | :---: | :---: | :---: |
| Printed items containing numbers and mathematical or scientific symbols (e.g., signs, labels, brochures, advertisements with charts and graphs) | Materials with visual patterns (e.g., toys in bright colors and black-and-white, dress-up clothes, curtains, upholstery) | Ordered sets of materials in different sizes (e.g., nesting blocks, measuring spoons, mixing bowls, measuring cups) | Materials and tools for filling and emptying (e.g., water, sand, scoops, shovels) left over containers (e.g., yogurt, salsa) | Tools for recording data (e.g., clipboards, paper, pencils, crayons, markers, chalk) |
| Items that display numbers (e.g., calculators, puzzles, simple board games with dice, or spinners) | Materials to copy and create series and patterns (e.g., beads, sticks, small blocks, pegs and pegboards, writing and collage materials) | Storage containers in graduated sizes | Everyday things to fit together and take apart (e.g., puzzles, boxes and lids, clothing with different types of fasteners) | Materials for diagramming or graphing data (e.g., newsprint pads and easels, graph paper with large grids, posterboard) |
| Numbers made of wood, plastic, or cardboard (Make sure they are sturdy, so children can hold, sort, copy, and trace them). | Shells and other patterned items from nature | Things in nature that grow, move, or change (e.g., germinating seeds) | Container and covers in different shapes and sizes | Small objects to represent counted quantities (e.g., buttons, acorns, pebbles, beans, marbles, pennies) |
| Discrete items children can easily count (e.g., blocks, shells, buttons, bottle caps, counting bears, old keys, and plant seeds) | Original artwork and reproductions featuring patterns (e.g., weavings, baskets) | Unconventional measuring tools (e.g., yarn, ribbon, blocks, cubes, paper clips, shoes) | Materials to create two-dimensional shapes (e.g., string, pipe cleaners, yarn, scissors) | Boxes and string for sorting and tying materials into groups |
| Paired items to create one-to-one correspondence (e.g., pegs and pegboards, colored markers and tops, egg cartons and eggs, socks) | Pattern blocks, keys, colored bottles, caps, cups, socks | Conventional measuring tools (e.g., tape measures, scales, grid paper, checkered table cloths, rulers, measuring spoons, graduated cylinders, string) | Moldable materials to create three-dimensional shapes (e.g., clay, dough, sand, beeswax, toothpicks) | Laundry baskets, socks |
| Materials to create numerals (e.g., string, playdough, pipe cleaners, yarn, shaving cream, flexible wire) | Plastic or magnetic letters | Sand and/or water, shovel, bucket | Things with moving parts (e.g., kitchen utensils, musical instruments, cameras) | Different size cups, turkey baster |

More Tools You Can Count On

| Number Sense | Algebra \& Functions (Classification and Patterning) | Measurement | Geometry | Mathematical Reasoning |
| :---: | :---: | :---: | :---: | :---: |
| Books that engage children in counting and exploring numerals (e.g., cookbook) | Stories, poems, and chants with repeated words and rhythms | Clear plastic recyclables of different shapes and sizes, boxes, tiles | Books that feature shapes and locations with illustrations from different perspectives | Muffin tin, plastic tub, containers of all shapes and sizes |
| Number BINGO | Songs with repetition in melody, rhythm, and words | Small bags of snack food, empty cereal boxes | Materials that change with manipulation or time (e.g., clay, play dough, computer drawing programs, sand, water, plants, animals) | Toy cars, plastic bears |
| Paper, stamps, stickers, scissors, glue, tape, chalk, and pencils | Computer programs that allow children to recognize and create series and patterns | Wrapping paper, crayons, paper | Materials to explore spatial concepts (over/under, up/down) and to view things from different heights and positions (e.g., climbing equipment, empty boxes, boards) | Different types of shoes |
| Margarine tubs | Crackers, animal crackers | Magazines, newspapers | Maps and diagrams | Playing cards |
| Dining items (e.g., plates, forks, spoons, napkins) | Ice cube trays | Stuffed and plastic animals, toy cars | Assorted pattern blocks, sets of basic two and three-dimensional shapes | Pizza, oranges |
| Socks | Colored bottle caps | Beans | Paper, drawing tools | Dessert |

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## Mathematics Vocabulary

The way adults communicate about mathematics can increase a child's vocabulary and build her/his conceptual understanding of numbers, shapes, quantities, direction, position, and properties. Incorporating the words listed below into your co-explorations with children supports integrated learning and serves as a foundation for later formal schooling.

Comparatives

| Same/different | High/low |
| :--- | :--- |
| Soft/hard | Near/far |
| Rough/smooth | More/less |
| Big/small | Empty/full |
| Short/long | Heavy/light |
| Tall/short | Many/few/fewer |
| Wide/narrow | Before/after |
| Thick/thin | Even/uneven |

Shapes/descriptions

| Curved | Triangle |
| :--- | :--- |
| Sides | Rectangle |
| Round | Hexagon |
| Sphere | Trapezoid |
| Pointed | Parallelogram |
| Cone | Corner |
| Flat | Cube |
| Square | Cylinder |
| Circle | Sphere |

Verbs

| Compare | Count |
| :--- | :--- |
| Group | Estimate |
| Sort | Subitize |
| Match | Measure |
| Repeat | Investigate |

## Miscellaneous

| Numbers | Together |
| :--- | :--- |
| Order | Same As |
| Next | Height |
| Then | Weight |
| Pattern | Angle |
| Design | Length |

## What Else?

| Average | Capacity |
| :--- | :--- |
| Altogether | Orientation |
| Direction | Transformation |
| Exactly | Symmetry |

Refer to Glossary of Terms on pages 54-56 for additional vocabulary words.



## Number Sense

Number Sense
The research says....

- "...Counting, which seems natural to young children, is essential to all other number operations..."

Seefeldt \& Galper (2008)

- "Counting involves learning the sequence of number words, identifying the quantity of items in a collection (knowing that the last counting word tells "how many"), and recognizing counting patterns (such as 21-22, 31-32, 41-42...)."
- "Researchers have determined that finger counting plays an important role in the development of mathematical understanding of young children. Using fingers to count, compare, and perform simple operations can provide children with a basis for understanding relationships between numbers up to 10. ."
- "When children understand counting, they are able to count in many different situations. We can't assume children understand what they need to know about counting if we only see them count in one kind of setting."

Clements (2004)


Number Sense
Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Count body parts - 1 nose, 2 eyes, 2 ears, 10 toes. |  | Read a counting book. Point to the written numbers as you say them. |  | Teach the rhyme "1, 2, Buckle My Shoe." |  | Count the number of people sitting at the dinner table. "How many are there altogether?" |
|  | Read the book, One, Two Skip a Few! | $3$ |  | Provide magnetic numbers for your child to count and play with. |  | Teach the rhyme, "One Potato, Two Potato," while you shop and cook. |
|  | Count the number of pockets on a piece of clothing. "How many are there altogether?" |  | Count how many cups of water it takes to fill a pitcher. |  | Have your child choose a favorite book. Ask math questions as you read. |  |
| Help your child set the table, counting the items as they are placed. "1, 2, 3, 4, 5. Five plates." |  | Teach a finger play, such as " 5 Little Monkeys" or one you know. |  | Help your child count the number of people in the family who are wearing red, blue, or yellow. |  | Use play dough to have your child make a pretend pie for each member of the family. Count how many pies are made. |
| Teach the rhyme, "5 Little Speckled Frogs." | Play a matching number game using numbers one to five. |  | Make a number book up to five with your child. Choose to stamp, draw, or attach stickers to match the written numbers. |  | Go on a number scavenger hunt with your child. Search for groups of three in your neighborhood, for example, "I see three ants." |  |

Asking "What page comes next?" provides a lesson in prediction. Copley (2000)

## Number Sense

Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Place a few small items in a sock. Have your child put his/her hand in the sock and make a guess. "How many items are inside?" Check and count together. | "You are correct! There were three buttons inside!" Gradually increase the number inside the sock as your child gains counting skills |  | Take wooden blocks, and write a numeral from 1-8 on each. Help your child put them in numerical order. | Have fun writing numbers in shaving cream. |  | Play with a number puzzle with your child. Touch, and say each numeral while playing and when the puzzle is complete. |
|  | Have your child count out loud to 10 every time she/ he washes his/her hands with soap and water. |  | Teach your child to form numerals using pipe cleaners, flexible wire, yarn or play dough. |  | Play a fun game with your child. Roll a die. Have your child jump, hop, or clap the number rolled. |  |
| Remember to count when doing ordinary things such as folding laundry, e.g., "How many towels did we fold?" |  | Go to the library, and choose a counting book with your child. Check it out, and take it home to enjoy. |  | Match objects with number cards up to 8. "We have one sock under the number 1 , and two spoons under the number 2...we have eight pennies under the number 8." |  | Point out numbers in environmental print: 4-H Club, 7Up, license plates, addresses, etc. |
| Read the book, Let's Count. | Count the number of steps you climb with your child. |  | Teach the rhyme "10 in the Bed". |  | Take a walk with your child. Count how many dogs you see. Cats? Birds? | Count how many snaps or claps your child can make in 15 seconds. |
| Teach the song "The Ants Go Marching". |  | Count the number of blocks in a tower with your child. |  | Read the book, Anno's Counting Book. | Help your child count the number of small objects in a collection - bugs, cars, rocks. |  |

Adult-guided experiences are especially important for learnings such as counting. Epstein (2007)

## Number Sense

Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Have fun writing numbers with chalk on the sidewalk. Say the numerals as you write. |  | Play "Hopscotch" outside with your child. |  | Play "Number BINGO," using numbers up to ten. |  |
| Have your child build a tower with up to ten blocks. Touch each block as you count to ten together. |  | Help your child count how many buttonholes are on a shirt. How many buttons should there be? |  | Tell a story to your child that includes numbers. | Read the book, Grandfather Counts. | Make a grocery list with your child. Draw attention to the numerals as you write. "We need eight apples. I'm going to write " 8 ," so we remember." |
|  | Count the number of windows in your classroom with the children. | Count the number of beds in each bedroom with your child. | Count the number of chairs in the kitchen with your child. | Count the number of doors in your home with your child. | Count the number of windows in your home with your child. Keep a tally as you count. At the end, help your child write how many windows are in your home altogether. |  |
| Make a paper chain to count how many days until an upcoming event (up to ten). | Have your child remove one link each day, and count how many are left until the big day. |  | Match numeral cards up to ten with the correct number of objects. |  | Plant seeds in small pots. Count out collections of ten seeds before you add them to the soil. |  |
| Play a board game. Take turns and count, using one-to-one correspondence. |  | When passing out food, ask "Does everyone have the same number of strawberries? Who has more? Who has fewer?" |  | Read a recipe with your child. Draw attention to numerical print and count the number of ingredients you will need. |  | Go to the park. Ask your child how many pushes he/she would like on the swing. Have him/her count as you push. |

Counting involves thinking, perception, and movement. National Research Council (2001)


## Notes on Number Sense

Questions to reflect on after engaging in the activities....

- What activities did the child/children enjoy most?
- Which activity needed more time?
- What activity should I repeat more often?
- What new vocabulary words can I use next time while engaged in a Number Sense Learning Activity?
- How can I extend an activity in the classroom to implement the California Preschool Learning Foundations?


# Algebra \& Functions (Classification and Patterning) 

## Algebra \& Functions <br> (Classification and Patterning) <br> The research says...

- "Perhaps more than any other content domain, the area of patterns, functions, and algebra can be emphasized throughout the young child's day."

Copley (2000)

- "The word mathematics makes many adults think of rote procedures for getting correct answers, a holdover from our own school days. But mathematics is essentially the search for sense and meaning, patterns and relationships, order and predictability."

Copley (2000)

- "Challenging number activities do not just develop children's number sense; they can also develop children's competencies in such logical competencies as sorting and ordering."

Clements (1984)

- "Thinking about patterns is another important precursor for learning mathematics in general and for learning algebra in particular."

Clements (2004)

- "Teacher facilitation and modeling are particularly important in introducing the notion of patterns, extending it to more aspects of the child's environment and daily activities, and encouraging the child's attempts to create patterns."

California Preschool Learning Foundations (2008)

Sorting (Math Counts)
Henry Arthur Pluckrose Publisher: Children's Press ISBN: 0516454587

Pattern (Math Counts) Henry Arthur Pluckrose Publisher: Children' Press ISBN: 0156454552

Pattern Fish
Trudy Harris
Publisher: Millbrook Press
ISBN: 0761317120
Beep Beep, Vroom Vroom! Stuart J. Murphy Publisher: HarperCollins
ISBN: 0739843729
The Best Bug Parade
Stuart J. Murphy
Publisher: HarperCollins
ISBN: 0064467007

## Algebra \& Functions (Classification and Patterning) Books

Patterns Everywhere
Julie Dalton
Publisher: Children's Press
ISBN: 0516253670
The Button Box
Margarette S. Reid
Publisher: Puffin
ISBN: 0140554955
Mr. Noisy's Book of Patterns
Rozanne Lanczak Williams
Publisher: Creative Teaching Press
ISBN: 1574711008
Food Patterns
Nathan Olson
Publisher: Capstone Press
ISBN: 0736878475
City Patterns
Nathan Olson
Publisher: Capstone Press
ISBN: 0736878483


Busy Bugs: A Book About Patterns Jayne Harvey \& Bernard Adnet Publisher: Penguin Group ISBN: 0448431599

## Sorting

Lynn Peppas
Publisher: Crabtree Publishing Co.
ISBN: 0778743675
Three Little Firefighters
Stuart J. Murphy
Publisher: HarperCollins
ISBN: 0060001208
Sorting By Size
Jennifer L. Marks
Publisher: Capstone Press
ISBN: 0736867406
Lots and Lots of Zebra Stripes:
Patterns in Nature
Stephen R. Swinburne
Publisher: Boyds Mills Press
ISBN: 1563979802

## Algebra \& Functions <br> (Classification and Patterning)

Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sort two colored blocks into separate piles. Add another color, and make three piles. Play again and make four piles. |  | Chant a clap/slap knees pattern, or a clap/snap fingers pattern. "Clap, snap, clap, snap;" or chant, "Hands, knees, hands, knees." |  | Talk about the attributes of the materials you plan to sort (e.g., shapes, crackers). |  |
| Collect plastic bottle caps to sort by color. |  | Sort socks into two color piles. | Sort socks into pairs. |  |  | March in a circle doing a hop/stop pattern; next, try a forward/side pattern |
|  | Read the book, Sorting (Math Counts) |  | Copy a pattern of red/blue blocks, yellow/ green, orange/brown. |  | Sort plastic letters into groups of straight lined letters, circle letters, curved letters. |  |
|  | Play a sorting game with family or friends. Make groups, such as boys/girls, shoes/ boots, walkers/riders. |  | Read the book, Food Patterns. | Copy patterns made with objects from around the house, such as forks/ spoons or paperclips/ rubber bands. | sel | Use a muffin tin to sort buttons by color, then by size, then by shape. |
|  | Extend patterns of shapes. "Circle, square, circle, square, circle, $\qquad$ ?" |  | Make a pattern with two colors or two shapes. |  | Go to the library, and ask the librarian to recommend a children's book on sorting and patterns. Check it out to read at home. |  |

Patterns help children learn to find order, cohesion, and predictability in seemingly disorganized situations.
California Preschool Learning Foundations (2008)


Algebra \& Functions
(Classification and Patterning)
Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sort crayons by color, then by size. |  | Sort keys by color, then by size, then by shape. |  | Make a place mat. Glue a pattern around the edge of a rectangle. |  | Sort plastic animals by color, then by size, and then by where they live. "Which animals live on a farm? Which live in the jungle?" |
|  | Look at some pens and pencils and ask, "How are they the same or different?" |  | Read the book, Pattern (Math Counts). | Go on a walk, and look for patterns. Look at the pattern on the sidewalk. Square, line, square, line. What other patterns can you find in the environment? | Use stamps or stickers to make patterns on paper. |  |
|  |  | Read the book, City Patterns. Help children find the pattern in brickwork, fences, and wallpaper. |  | Create a pattern based on size. Big, small, big, small. | Read the book, The Button Box. | How many ways can you sort buttons (e.g., two holes, no holes, round holes, four holes)? |
| Talk about the attributes of blocks (e.g., shape, color, size). |  | Read the book, Busy Bugs: A Book About Patterns. | Go to the park. Look for patterns in natural objects. "Does a flower have a pattern?" |  | Get up and move: clap, clap, hop, hop. Repeat, clap, clap, hop, hop. |  |
| Read the book, Sorting. |  | Use ice cube trays to sort objects (e.g., buttons) of one type in one row and a different type in the other row. |  |  | Sort plastic animals or animal crackers by animals that fly, crawl, and swim. |  |

Algebra begins with a search for patterns. Clements (2004)


Algebra \& Functions
(Classification and Patterning)
Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Make an ABAB pattern with colors, and ask your child to describe it (e.g., red, blue, red, blue, etc.). |  | Make an ABAB pattern, and ask your child to extend it (e.g., green, yellow, green, yellow, green, yellow, etc.). |  | When you get home from the grocery store, help your child sort the items into groups: fresh, boxed, canned, or frozen. |  |
| Make AB patterns with old keys up/down, up/down. |  | Read the book, Mr. Noisy's Book of Patterns. | Make ABC patterns with bottle caps by color. |  |  | Make ABC patterns with shapes, and ask your child to describe them. |
|  | Make an ABC pattern with shapes, and ask your child to extend it. |  | Provide two colors of crayons and have your child color a pattern. Try this with three colors of crayons. |  | Read the book, Sorting By Size. | Sort beads by size, then by color into margarine tubs. |
| Read the book, Patterns Everywhere. |  | Make an A-B-A-B pattern bracelet with beads. | Make an A-B-A-B pattern necklace with beads. |  | Make an A-A-B-B pattern bracelet with beads. | Make an A-A-B-B pattern necklace with beads. |
| Make an A-B-B-A-B-B pattern bracelet with beads. | Make an A-B-B-A-B-B pattern necklace with beads. |  | Stand up, sit down, stand up, sit down, repeat, and extend. Stand up, sit down, turn around, do it again. What comes next? |  | Read the book, Lots and Lots of Zebra Stripes: Patterns in Nature. | Collect objects from your neighborhood. Create a pattern: rock, leaf, rock, leaf, etc. |

Mathematics is the science and language of patterns. Copley (2003)


- What activities did the child/children enjoy most?
- Which activity needed more time?
- What activity should I repeat more often?
- What new vocabulary words can I use next time while engaged in an Algebra and Functions Learning Activity?
- How can I extend an activity in the classroom to implement the California Preschool Learning Foundations?



# Measurement 

## Measurement

## The research says....

- "Measurement...bridges two critical realms of mathematics: geometry or spatial relations and real numbers."

Clements \& Stephen (2004)

- "Although uninstructed preschool children have little understanding of the significance of unit size for measurement-related problems, their understanding grows rapidly when they have the opportunity to make relevant observations."

Sophian (2004)

- "Prekindergarten children know that properties such as mass (amount), length, and weight exist, but they do not initially know how to reason about these attributes or to measure them accurately."

Clements (2004)

- "Children's familiarity with the language required to describe measurement relationships-such as longer, taller, shorter, the same length, holds less, holds the same amount--is an important foundation for the concept of measurement."
Measurement Books


## Sea Squares

Joy N. Hulme
Publisher: Hyperion Paperbacks for Children ISBN: 0439272793

Inch by Inch
Leo Lionni
Publisher: Scholastic Inc
ISBN: 0688132839
Mr. Cookie Baker
Monica Wellington
Publisher: Dutton Juvenile
ISBN: 0525477632
How Big is a Foot?
Rolf Myller
Publisher: Yearling Books
ISBN: 0440404959
How Many Bugs in a Box?
David A. Carter
Publisher: Simon \& Schuster Children's
ISBN: 1841215120

Tracks in the Sand Loreen Leedy Publisher: Doubleday Dell ISBN: 038530658X

Now I'm Big Margaret Miller Publisher: Ashley Books ISBN: 0688140777

The Bicycle Man
Allen Say
Publisher: Sea Shell Books
ISBN: 0395506522
How Long Is It?
Donna Loughran
Publisher: Children's Press
ISBN: 0756957966
Just a Little Bit
Ann Tompert
Publisher: Houghton Mifflin
ISBN: 043920240X


Length (Math Counts)
Henry Arthur Pluckrose Publisher: Childrens Press ISBN: 0516454536

Weight (Math Counts) Henry Arthur Pluckrose Publisher: Childrens Press ISBN: 0516454609

Size (Math Counts)
Henry Arthur Pluckrose
Publisher: Childrens Press
ISBN: 0516454579
Who Sank the Boat?
Pamela Allen
Publisher: Penguin Putnam
ISBN: 0698113732
How Long or How Wide: A Measuring Guide
Brian P. Cleary
Publisher: Lerner Publishing Group
ISBN: 1580138446

Measurement
Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Build with blocks. Compare the height of two towers. "Wow! Your tower is taller than mine!" |  |  | Use recycled magazines and newspapers. Separate into two stacks, and talk about which stack is higher. |  | Act out Goldilocks and the Three Bears with your child. Let your child help select appropriately sized props, for example, "We need the smallest bowl for Baby Bear." |  |
|  | Compare and arrange stuffed or plastic animals by height. Start with two, and then add three or more items. | "Fuzzy Bear is bigger than Dino Baby, and Chickie is the smallest of them all!" |  | Help your child select his/her appropriate shoe size at a store. "This shoe is too small for your foot. Let's see if we can find a bigger one." |  | Use string to measure the length of your child's arm. Compare with other family members' arm lengths. |
| Give your child measuring cups to use in pretend play cooking. |  |  | Use a hand shovel (trowel) to measure the distance between garden rows. "We need extra space between rows. Let's measure three trowels this time." |  | Bring plastic containers of different sizes to the park to use in the sandbox. Talk about how many scoops of sand fit in the containers. |  |
| Read a book about measurement, such as Size (Math Counts). |  | Count the number of steps from the front door to another landmark, such as a mailbox. |  | Talk about weight during daily activities. "That basket is too heavy. If I put the clothes in two baskets, they will be lighter loads to lift." |  | Use paper clips laid end to end to measure a line of toy animals. |
| Read a book about measurement used in cooking, such as Mr. Cookie Baker or Cook-a-doodle-doo! | Invite your child to cook breakfast. Help him/her measure the ingredients in the recipe. |  | Build ramps for play cars to travel down. Use a checkered tablecloth to measure the length of car travel. "My car flew 20 squares." |  | Plant seeds. Observe, and measure growth. Compare the different plant heights. |  |

Measurement is one of the main real world applications of mathematics. Clements (2004)


## Measurement

Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Place two crayons side by side. Discuss which one is longer. |  | Build a tall tower of cardboard blocks or shoeboxes with your child. Measure the height with a length of string. | Invite him/her to make an even larger tower. Compare with the previous length of string. "This tower is taller than the last!" | $3$ |  | Line up paper strips (same size) or use your hands to measure the size of a household rug. Be sure not to leave any gaps between strips. |
|  | Explore the weight of different snack bags. "Which is heavier the bag of cherries or the crackers?" |  |  |  | Play "How much does it hold?" Use empty containers and turkey basters. "This cup holds three turkey basters of water." |  |
| At bath time, provide your child with plastic containers of different sizes. Offer measuring cups and spoons to explore. | Ask, "How many scoops do you think it will take to fill this container? What about the large one?" | "What happens when we use the large cup? What's different when we use the small spoon to fill a container?" |  | Read the book, Inch by Inch. |  | Provide a visual of approximate length. "An inch is about this long." Demonstrate length using your thumb and index finger. |
|  | Keep track of your child's height over time. Draw attention to how many units, such as inches, he/she has grown. |  | Play a game with beans. Talk about how many beans your child can hold in his/her hand. | Compare with the number of beans another family member can hold in his/her hand. | Discuss why the amount might be different. "Grandpa's hand is bigger than mine. He can hold more." |  |
| Make homemade playdough. Follow a recipe using measuring cups and spoons. | Count out the amount in cups needed for the recipe together - "one cup, one-half cup, one and one-half cups of flour." | Play with the playdough. Make different lengths of playdough rope. Compare which rope is longer. |  |  | Plan a water race. Mark a length of space. Add small amounts of water to a bucket, and take turns walking between the markers. | Discuss the increased challenge of added weight. "At first we moved very fast. Why do you think we are walking slower now?" |

Measurement provides an ideal bridge between geometry and number, one that comes up frequently in everyday situations.
Copley (2000)

## Measurement

Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model the use of measurement tools during home improvement projects | "The couch is 40 inches wide, and we need to figure out if it will fit through the front door. Let's use a tape measure to find out if it will fit." |  | Read the book, Just a Little Bit. |  | Go on a nature walk, and compare objects by lifting. "That's a pretty heavy rock. This one is much lighter." |  |
| Decorate trees in your neighborhood with streamers. Guess (estimate) how long a piece you will need for each different tree. |  | At a grocery store, discuss the different sizes of milk containers; and talk about which size is most appropriate for your family. | "I know we really like milk. I think a pint is too small. Let's buy a gallon of milk. That way we will have enough for everybody to enjoy." |  |  | Check your child's height. Has she/he grown? |
|  | Wrap a package together. Estimate the amount of paper needed to cover the package. "It looks like we have too much. Let's cut off the extra paper." |  | Make and fly paper airplanes. Measure the distance of the airplanes' flight with a measuring tape. Compare. "Your plane flew 56 inches. My plane flew 44 inches." |  | Play "How far can you jump?" Stack small cereal boxes (of the same size) in a line or upright. Invite your child to take a running jump. | Start with one and add more. "You jumped the distance of two boxes. Let's see if you can jump the distance of three." |
| Make a family decision, using a water graph. Have two clear cups labeled with two choices. Pour one tablespoon to indicate a vote. | Use the water levels to indicate the outcome of the vote. "It looks like 'Play a board game' has more water. Our family decision is to play a board game." |  |  | Use square tiles to cover and explore the surface area of a coffee table. How many tiles did it take to cover the table? |  |  |
|  | Use a balance scale to explore weight. Provide a variety of items, and discuss which weighs more. | Invite your child to cook dinner. Help her/him to measure the ingredients in the recipe. |  | Say to your child, "I want to know how many shoes tall you are." Trace your child's body outline with chalk outside. | With your child's help, begin at the bottom of the outline and place his/her shoe where the previous shoe unit ended. | "You are 12 shoes tall." Mark as you go with chalk. |

To estimate, children use their various senses, such as eyeballing (visual), lifting (kinesthetic), or listening (auditory).
Epstein (2007)

Engaging Children in Early Mathematical Experiences

## Notes on Measurement <br> Questions to reflect on after engaging in the activities....

- What activities did the child/children enjoy most?
- Which activity needed more time?
- What activity should I repeat more often?
- What new vocabulary words can I use next time while engaged in a Measurement Learning Activity?
- How can I extend an activity in the classroom to implement the California Preschool Learning Foundations?



## Geometry

## Geometry <br> The research says....

- "According to the National Council of Teachers of Mathematics (2000), preschool through grade 2 geometry teaching begins with naming and describing shapes."

Seefelt \& Galper (2008)

- "The study of shapes should focus on the attributes and properties of both two- and threedimensional shapes."

Seefelt \& Galper (2008)

- "Very young children can learn rich concepts about shape, if provided with varied examples and non-examples, discussions about shapes and their characteristics, and interesting tasks."

Clements \& Sarama (2004)

- "The more children work with geometric concepts, the more they can learn to explore spatial relations and experience mathematics."

Seefelt \& Galper (2008)


## Geometry Books

The Greedy Triangle
Marilyn Burns and Gordon Silveria
Publisher: Scholastic, Inc.
ISBN: 9780545042208
The Shape of Things
Dayle Ann Dodds and Julie Lacome
Publisher: Candlewick Press
ISBN: 9781564026989
The Secret Birthday Message
Eric Carle
Publisher: Harper Trophy
ISBN: 0064430995
Where's the Fly?
Caron Lee Cohen
Publisher: Greenwillow Books
ISBN: 0688140440
Outside, Inside
Carolyn Crimi
Publisher: Simon \& Schuster
ISBN: 0590868810

Architecture: Shapes
Michael Crosbie and Steve Rosenthal
Publisher: Preservation Press
ISBN: 0471143669
Color Zoo
Lois Ehlert
Publisher: Harper-Collins
ISBN: 0397322593
The Village of Round and Square Houses
Ann Frifalconi
Publisher: Little, Brown
ISBN: 0316328626
Look Inside
Lilly Ernesto and Linda Hendry
Publisher: D. C. Heath
ISBN: 0669302317
Ed Emberley's Picture Pie 2: A Drawing Book and Stencil
Ed Emberley
Publisher: Little, Brown
ISBN: 0316789801

Sea Shapes Suse MacDonald Publisher: Harcourt Brace ISBN: 0152017003

Close, Closer, Closest Shelley Rotner and Richard Olivo Publisher: Simon \& Schuster ISBN: 0689807627

Look at Annette
Marion Walter
Publisher: M. Evans
ISBN: 0871310716
Over, Under \& Through
Tana Hoban
Publisher: Simon \& Schuster
ISBN: 1416975411
Make a Bigger Puddle. Make a Smaller Worm.
Marion Walter
Publisher: M. Evans
ISBN: 1903142156

## Two-Dimensional Shape Descriptions

- What to say when talking about a TRIANGLE:
- Triangles have three sides.
- All three sides are straight (not curved).
- All three sides are connected.
- Triangles have three points or corners or angles.

- Triangles can be different sizes, or point in different directions (orientation); some sides may be the same length, but they can all be different lengths; some triangles can be long and skinny.
- What to say when talking about a RECTANGLE:
- Rectangles have four sides.
- All sides are straight (not curved).
- All sides are connected.
- The opposite sides on a rectangle are the same length
- Rectangles have four right angles.

- Rectangles can be different sizes, or point in different directions (orientation), they can be long and skinny or they can look like a square.
- What to say when talking about a SQUARE:
- All four straight sides are the same length (all connected).
- All four have right angles (squares are "regular" polygons).
- A square has two pairs of parallel sides.
- Squares can be different sizes or point in different directions (orientation).


Building Blocks-SRA Real Math/TRIAD Projects, Grade PreK. Clements \& Sarama (2007 \& 2008) Dimensional
criptions (continued)

- What to say when talking about a RHOMBUS:
- A rhombus has four straight sides.
- All four sides are straight (not curved).
- All four sides are the same length.
- All four sides are connected.
- A rhombus has four points or corners or angles.
- What to say when talking about a TRAPEZOID:
- A trapezoid has four straight sides.
- All four sides are straight (not curved).
- All four sides are connected.
- A trapezoid has four points or corners or angles.
- A trapezoid has one pair of parallel lines.
- What to say when talking about a HEXAGON:
- A hexagon has six straight sides.
- All six sides are straight (not curved).
- All six sides are angled.
- All six sides are connected.
- A hexagon has six points or corners or angles.


Building Blocks-SRA Real Math/TRIAD Projects, Grade PreK. Clements \& Sarama (2007 \& 2008)


- What to say when talking about a SPHERE:
- A sphere is a solid shape.
- Spheres are perfectly round.
- A sphere is completely symmetrical.
- All points on the surface lay the same distance from the center.
- What to say when talking about a PRISM:
- A right prism is a solid shape.
- Prism sides can be straight or curved.
- All sides are connected.
- What to say when talking about a CUBE:
- A cube is a solid object bound by six square faces, facets or sides.
- All six sides are straight.
- Cubes have eight corners.
- Cubes have 12 equal edges.
- All sides are connected.


- What to say when talking about a CYLINDER:
- A cylinder is a solid figure with straight parallel sides.
- A cylinder has a circular or oval section on the top and bottom.
- What to say when talking about a CONE:
- A cone comes to a point.
- A cone tapers smoothly from a flat, round base to a point called the apex or vertex.
- What to say when talking about a PYRAMID:
- A pyramid has a square or triangular base.
- Each base edge and apex form a triangle.



## Open-ended Questions Related to Geometry

- How is that shape like this one? How is it different?
-Why isn't this shape an oval (circle, square)? What makes this shape an oval?
- What if I turned this shape or flipped it? What would it look like if I slid it from your paper to my paper?
- Where have you seen this shape before?
- Where do you think you would find something like this shape?
- How did you decide what to copy/draw?
- How can we get to the cafeteria/office/playground from here?
- I wonder if this shape would roll or slide? What do you think? How could we stack these?

Adapted from The Young Child and Mathematics, Juanita Copley, NAEYC 2000

## Open-ended Questions Related to Geometry

- How could you cut this paper to make another shape?
-What shape could you make out of these shapes?
- Could we make the cone roll straight, or would it roll crooked? What about the cylinder?
- How do you know that you have found all the ways to put those shapes together?
- What would happen if I cut off an end of this? What would it look like?
- What do you think could be another name for this shape?
- How would you make a square/circle/triangle/ with pipe cleaners? How would you make a ball/box/cone?

Geometry

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Read the book, Color Zoo. | Help your child learn to describe twodimensional shapes as they are introduced to him/her. "This is a circle. It is round and has no corners." |  | Point out circles in the home. Draw them in the air. Draw them on paper. | $\square$ | Give your child the opportunity to hold a twodimensional shape and look at it carefully. | Then, have him/her do the same thing with his/ her eyes closed, and talk about the attributes. |
| Play with blocks with your child. Name the two-dimensional shapes of the sides of the blocks. |  | When preparing a sandwich, cut it into shapes. Help your child identify the two-dimensional shapes. |  |  | Read the book, Over, Under, Through. | Help your child experience himself/ herself in spaces around the house. Go under, over, through, into, and on top of objects. |
|  | Draw two-dimensional shapes on the sidewalk outside. Use chalk or water and a paintbrush. |  | Have children create two-dimensional shapes with their bodies on the carpet. Take pictures and display the shapes. |  | Teach your child how to sort two-dimensional shapes. "These shapes look a bit different, but they are all triangles. | They all have three sides and three corners." |
| Teach your child how to cut out twodimensional shapes that are drawn on paper. |  | Go for a walk in your neighborhood. Look for two-dimensional shapes in the environment. |  | Play "I Spy" at home, searching for twodimensional shapes. "I spy a blue triangle on the refrigerator." |  | Form three-dimensional shapes, using play dough. |
|  | Have your child create two-dimensional shapes, using yarn. Ask, "Which shape is the hardest to make?" |  | Read the book, Architecture: Shapes. |  | Have your child form two-dimensional shapes with toothpicks. Ask, "Is there a shape you can't make? Why?" |  |

Children learn through exploring their world; thus, interests and everyday activities are natural vehicles for developing mathematical
thinking. Shapes are all around us!
Seefelt \& Galper (2008)


## Geometry

## Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Play "Simon Says," using position words "Simon says: Go under the table. |  | Hide two-dimensiona shapes, and have your child find them. | Read the book, The Village of Round and Square Houses. | Have your child point out the two-dimensional shapes on houses or buildings. |  | Help your child understand position words. "You are between the slide and the swing." |
|  | Go to the library and ask the librarian to help you select a book about shapes. Take it home and enjoy! |  | Count the corners on squares, rectangles, and triangles. |  | Count the sides on squares, rectangles, and triangles. |  |
| Find examples of the same two-dimensional shape in different sizes. |  | Look for shapes in picture books. |  | Place plastic twodimensional shapes in a bucket. Have your child fish them out with a net and name each one. |  | Hold a two-dimensional shape behind your back or put it in your pocket. Describe the shape and let your child guess what shape it is. |
|  | Look for twodimensional shapes around the house. Tally how many you find of each. |  | Read the book, The Shape of Things. |  | Sort two-dimensional shapes by attributes such as the number of corners, and how many sides it has. |  |
| Look at a magazine with your child to find and identify two dimensional shapes. |  | Create and then recreate twodimensional shapes using pattern blocks. |  | Play "Hot Potato" using two-dimensional shapes. Pass the shape until the music stops, and then the child describes and identifies the shape he/she is holding. |  | Read the book, The Secret Birthday Message. |

Teachers will want to take advantage of incidental learning, but they will want to intentionally plan explorations with problems,
materials, and children's books that facilitate emergent concepts of geometry.
Seefelt \& Galper (2008)
 2p
eometry
ning Activities

## Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Give your child the opportunity to hold a three-dimensional shape and look at it carefully. Then, do the same thing with their eyes closed. |  | Help your child learn to describe threedimensional shapes as he/she is introduced to them. "This is a sphere. It is round like a ball." |  | Play "What Shape is This?" Use objects around the home or classroom. "What shape is the basketball?" Or, "What shape is the soup can?" |  |
| Read the book, Goldilocks and the Three Bears. Emphasize position words. | "Goldilocks went into the house, and sat on the biggest chair." |  | Show your child a simple map and explain how people use maps. | Help your child build a town with blocks. Draw a simple map of the town and talk about it with your child. |  | Draw attention to maps when you are away from home. Point out the map at the mall or at the zoo. "We are here now, and we want to go there." |
|  | Go to the park or playground. Use positional words to describe what your child is doing. | "You are climbing up the ladder." Or, "You are going around the equipment." |  | Read the book, Over, Under and Through. | Help your child build a fort with blankets. Go inside the fort. What is next to the fort? What is behind the fort? What is under the fort? |  |
| Use kitchen utensils to paint prints. The prints should not touch each other. | Enjoy looking at the prints while making it a game. Name the prints. Point to the print above the fork. | Point to the print below the spatula. | If the prints touch use position words. Point to the print inside, behind, against and on top of. |  | Play with threedimensional shapes. Will this shape roll? Will it roll straight or to the side? Will it slide? | Look for threedimensional shapes at the grocery store. "I see lots of boxes! And there is a cylinder!" |
| Go for a walk in your neighborhood. Look for three-dimensional shapes in the environment. |  | Sort your recyclables into three-dimensional shape groups. "Let's put all the cans in the cylinder pile." |  |  | Help your child find a hidden object using a simple map that you have created. |  |

Research strongly supports the use of a variety of manipulative's to help children understand geometric shapes and develop spatial sense.

Copley (2000)


## Notes on Geometry

## Questions to reflect on after engaging in the activities....

- What activities did the child/children enjoy most?
- Which activity needed more time?
- What activity should I repeat more often?
- What new vocabulary words can I use next time while engaged in a Geometry Learning Activity?
- How can I extend an activity in the classroom to implement the California Preschool Learning Foundations?


## Mathematical Reasoning

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## Mathematical Reasoning

The research says...

- "Data Analysis involves formulating information and answering questions by collecting, organizing, and analyzing information."

HighScope Educational Research Foundation (2009)

- "Even in the early years, children need to have experiences that help them begin to develop clarity and precision in their thinking."

Copley (2006)

- "Most preschool children by at least three years of age show that they can solve problems involving simple addition and subtraction, often by modeling with real objects or thinking about sets of objects."

California Preschool Learning Foundation (2008)

- "Four-and-five-year-olds engage in advanced mathematical explorations spontaneously in their play."

Ginsberg, Inoue, \& Seo (1999)

## Building Mathematical Reasoning

Reasoning activities are used to solve problems. Young children generally relate a person's ability to solve problems to luck, because they do not yet understand how reasoning works. Children do use reasoning skills at a young age, but their reasoning tends to be intuitive. It is unclear how they come to a conclusion or arrive at an answer. For example, a child may determine after collecting five cups to serve juice to a group of eight friends that she/he needs three more cups. When asked how she/he knew to collect three more cups, the child may simply shrug her/his shoulders. Children may initially respond to "why" questions with short, vague answers. Over time children develop, clarity and accuracy in their thinking.

Problem-solving and reasoning require children to communicate about mathematics. To share their ideas, children must articulate, clarify, organize, and consolidate their thinking. Children communicate in different ways. They share their ideas by manipulating objects, drawing pictures, using fingers, or communicating their thoughts with words. Children may write, draw diagrams and simple charts, and/or utilize mathematical symbols to explain their mathematical reasoning.

Through interactions with adults and their peers, children develop an ability to express themselves. When children are engaged in meaningful play experiences, they have a natural interest in sharing their ideas and investigations with others. Adults can encourage children's mathematical reasoning by offering tools for recording data (e.g., a clipboard and pencil for tallying the numbers of doors in a home) and diagramming or graphing data (e.g., buttons and two cups for recording a family vote). Asking questions can also support the development of mathematical reasoning. Questions such as, "How can you tell?" or, "How do you know?" encourage a child to stop and reflect. "I wonder..." questions, such as "I wonder why this group is bigger?" draws a child's attention to details in an investigation.

Some suggestions for prompting mathematical reasoning are included in the learning activities described in this resource; however, other thoughts may be generated spontaneously as you co-explore with a child. Wonder along with your child, and encourage communication about her/his observations and conclusions. A child's mathematical reasoning will develop over time, through rich experiences and with adult encouragement.


Diez Perros en la Tienda
Claire Masurel
Publisher: Ediciones Norte-Sur
ISBN: 0735813035
Pictographs
Vijaya Bodach
Publisher: Capstone Press
ISBN: 1429600411
Ten Black Dots
Donald Crews
Publisher: Harper Trophy
ISBN: 0688135749
Just Graph It
Sandi Hill
Publisher: Creative Teaching Press
ISBN: 1574713752
More, Fewer, Less
Tana Hoban
Publisher: Greenwillow Books
ISBN: 0688156932

Mathematical Reasoning Books

Ten, Nine, Eight
Molly Bang
Publisher: Harper Trophy
ISBN: 0688104800
One to Ten...and Back Again
Betty Ann Schwartz
Publisher: SmartInk Books
ISBN: 1392403116
Tally Charts
Vijaya Bodach
Publisher: Capstone Press
ISBN: 1429600438
Let's Eat Lunch
Susan Vaughn
Publisher: Rosen Publishing Group
ISBN: 0823988856
Each Orange Had 8 Slices
Paul Giganti
Publisher: Greenwillow Books
ISBN: 068813985


12 Ways to Get to 11
Eve Merriam
Publisher: Aladdin Picture Books
ISBN: 0689808925
Ten Flashing Fireflies
Philemon Sturges
Publisher: Night Sky Books
ISBN: 01558586741
The Doorbell Rang
Pat Hutchins
Publisher: HarperCollins
ISBN: 0688092349
Shoes, Shoes, Shoes
Anne Morris
Publisher: HarperCollins
ISBN: 0688161669

Who's Hiding?
Satoru Onishi
Publisher: Kane/Miller Book Pub
ISBN: 1933605243

Mathematical Reasoning
Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Invite your child to make a collection of items gathered on a nature walk. | Have your child put the collected items in a muffin tin. Encourage him/her to explain and describe the collection. |  | Read the book, Who's Hiding? | Place up to three items under a plastic tub, lift quickly and ask, "How many?" |  |
|  | Show three or four plastic bears, then hide one or two in the cave (plastic tub). Ask, "How many are hiding?" |  | Count a few objects then add one more. How many in all? Add two more. How many total? |  |  | Read the book, More, Fewer, Less. |
|  | Play "More or Less" with playing cards. Give each player one set of three to five numeral cards. Each player turns over one card at a time. | Highest card gets to take both. Keep playing until one player has all the cards. Play the opposite way, lowest card gets to take both. | Help your child compare quantities using words like "more" or "less/fewer." |  | Make two groups, each containing one to five items. Have your child guess if one group has more, fewer, or the same. | Jump two times. Jump one more time. How many times altogether? |
| Hop three times. Hop one more time. How many hops altogether? |  | Read the book, Pictographs. | Tape a small collection of buttons in organized lines on a piece of paper to create a pictograph. | Have your child explain how he/she organized the buttons. |  | Make or order a pizza. Cut it into halves, quarters, and eights. Discuss how many parts each family member will eat. |
| Read the Book, Tally Charts. |  | After dinner, ask your child to make a tally of the number of people wanting dessert. |  | Chart your family's preferences in pizza toppings. Summarize the data your child collected. | "We can look at the chart and find out that, in our home, everyone likes mushrooms, half like olives, and nobody likes peppers." | Invite your child to build a train. Help your child chart the number of train cars on the tracks. |

It is vital for young children to develop confidence in their ability to understand and use mathematics. NAEYC \& NCTM (2002)


## Mathematical Reasoning

Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hide four or five buttons in your hand; then show them quickly, and have your child guess how many you had. | Let your child take a turn, too, showing the buttons to you in his/her hand, then hiding them, so you can "guess" how many he/she had shown you. | Read the book, Shoes, Shoes, Shoes. | At a family gathering, line up shoes in different columns to demonstrate the number of sandals, tennis shoes, dress shoes, etc. worn to the event. | Discuss which type of shoe was most popular, in between, and least popular. | During bath time, fill empty cups with a turkey baster. Prompt your child, "How many basters-full of water have you added so far? Is the cup full, or is there room for more?" | Ask your child, "Help me find a cup that holds more water than this one?" |
|  | Read the book, Ten Flashing Fireflies. |  | Play "I Spy". Give descriptive clues about the mystery item. "I'm thinking of something that is green." | After each clue discuss with your child what the item might be. "It could be the couch, the couch is green. Or maybe the plant. It's green too." |  |  |
| Read the book, Tally Charts. |  | Play basketball with a laundry basket and socks. Play three rounds with 10 tosses each. Keep a tally, with pen and paper, on the number of baskets made each round. | Ask your child to indicate if he/she scored higher/ lower (or made more/fewer baskets) in the first game, second game, and so on. Point to and label corresponding columns (e.g., first, second, etc.). | Help your child make a representative number of tally marks on the chart. "You made one, two, three, four baskets. Make a line for each number." | Ask your child, "How can you tell you made three baskets? How do you know I made four?" |  |
|  | "How many hops on one foot can you make without falling?" Invite your child to predict the outcome, record his/her predictions, and then compare them with the results. |  |  | Sort laundry with your child. Ask, "Which dirty clothes do we have more of? Whites? Or bright colors?" "How many loads do we have of each?" | Make two groups, each containing, one to five items. Have your child estimate if one group has more, fewer, or the same. |  |
| Put some marbles or pennies in a baby food jar. Have each child estimate how many, then count. Play again with different quantities. |  | Read and act out stories in which size, voice, or other graduated qualities play a role, such as The Three Bears or The Three Billy Goats Gruff. | Have your child arrange plastic bears in a repeating graduated series. |  | Use a variety of containers, ask your child if he/she thinks the lid you picked will fit. | Have him/her try to put on the lid. Talk about why it may or may not fit, for example, "This one is too big!" |

Preschool children can directly model different types of problems using concrete objects, fingers, and other strategies.
Carpenter, Ansell, Franke, Fennema, \& Weisbeck (1993)


Mathematical Reasoning
Learning Activities

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Read the book, Jack and the Beanstalk. Next have each child count one to five beans and hold them in his/her hand. | Then the children can show each other and compare if they have the same or different amounts. |  |  | Have fun drawing together. Wonder aloud, "I wonder if you can make your dog twice as big as mine," or "Can you make the flower half as big as mine?" |  |  |
| Read a picture book that compares more, fewer, and less. |  |  | Count small groups of objects; then combine the groups to show the total. "We have three oranges and two apples. That's five altogether." |  |  | Pose a question to your child that requires gathering and analyzing information. "How many boxes of oatmeal do you think we need to feed our family for one month?" |
|  | Read the book, The Doorbell Rang. | Make up word stories for your child. For example, "Three children were playing then one went home. How many are left?" |  | Read the book, 12 Ways to Get to 11. | Have fun using objects to see how many ways to get to 11. Five and six, nine and two, ten and one. |  |
| Make lines of bears or toy cars. Talk about which is first, last, second, third, and so forth. |  | Read the book, Each Orange Had 8 Slices. | Cut an orange in half and then into fourths. Talk about the parts that make up the whole. Do the same with other fruits and vegetables. |  |  | Draw a chalk circle on the sidewalk. Collect and place four insects or snails in the middle. Note which moved out of the circle first, second, third, and fourth. |
| Organize your child's toys. Work together to determine how to store different types of toys. | "Where should we keep the Lego people, with the Lego blocks or with our action figures?" Ask her/him to explain her/his reasoning. |  | Read the book, Just Graph It. | Make plans for family game night. Select three games and then help your child to ask each family member, "What is your favorite game?" | Make a bar graph to illustrate family preferences. Using graph paper, make three columns, one for each game. For each family member's vote, color in one square. | Discuss the complete graph. "It looks like Chutes and Ladders is our favorite game. It has the most bars. Candyland and Go Fish both have fewer bars." |

Experiences that are appropriate to children's developmental level enable children to experience success with mathematics.
Bredekamp \& Copple (1997)


## Notes on Mathematical Reasoning

## Questions to reflect on after engaging in the activities....

- What activities did the child/children enjoy most?
- Which activity needed more time?
- What activity should I repeat more often?
- What new vocabulary words can I use next time while engaged in a Mathematical Reasoning Learning Activity?
- How can I extend an activity in the classroom to implement the California Preschool Learning Foundations?
- Algebra - Strand of mathematics that provides tools for picturing, stating, and simplifying relationships; generalization of arithmetic ideas by which unknown values and variables can be found in problem-solving. Young children build a foundation for later algebra learning through activities involving sorting and classifying, patterns, relationships, mathematical situations and structures, and change.
- Algorithm - A specific set of instructions for carrying out a procedure or solving a problem, usually with the requirement that the procedure terminates at some point.
- Attribute - A property or characteristic of an object or a person; attributes such as size, color, or shape would be apparent to a preschool child and would be used in grouping or sorting.
- Cardinality - The concept that the number name applied to the last object counted represents the total number of objects in the group (the quantity of objects counted).
- Classification - The sorting, grouping, or categorizing of objects according to established criteria.
- Cylinder - A three-dimensional figure with one curved surface and two parallel, congruent circles as bases.
- Data - Facts, such as measurements, collected about people or things; for example, how much the plants in the window grew over a one-week period or how many children have pets.
- Estimate - To use judgement to find a number close to an exact amount.
- Graph - A drawing or diagram showing a numerical relationship or displaying data.

O ${ }_{\text {Glossary of Terms (continued) }}^{1}$

- Line symmetry - Characteristic describing a figure or shape that can be divided along a line so that the sections on either side of the line match exactly.
- Manipulatives - Easily handled concrete objects, such as beads, puzzles, buttons, and peg boards, that children can work with in ways that help them understand or explore mathematic concepts.
- One-to-one correspondence - One and only one number word is used for each object in the array of objects being counted.
- Polygon - A multi-sided, two-dimensional, closed figure whose sides are line segments connected end to end, with each segment intersecting two others at its endpoints. Polygons are named for their number of sides; for example, a five-sided polygon is a pentagon.
- Predict - To tell what might happen.
- Property - Feature or characteristic common to several items, for example, both blocks are red.
- Quantification - Determination of how much or how many.
- Repeating pattern - A sequence of colors, shapes, sounds, or other attributes that occur again and again, for example, loud-soft-soft, loud-soft-soft...; circle-square, circle-square...
- Seriation - The process of ordering or creating a sequence based on a specific attribute, such as length, height, capacity, weight, or amount of time.

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- Simple repeating pattern - Has two repeating elements. Examples are as follows: A-B-A-B (e.g., red-blue-red-blue); A-A-B-B (e.g., dog-dog-cat-cat); A-B-B-A-B-B (e.g., clap-stomp-stomp-clap stomp-stomp).
- Skip-counting - Counting by 2's, 5's, 10's, or other intervals, beginning with any whole number. Examples: 2, 4, 6, 8; 5, 10, 15, 20; 7, 9, 11, 13.
- Sorting - Creating collections of objects that share a common attribute.
- Spatial orientation - Understanding and operating on relationships between different positions in space.
- Spatial relationship - The positions of objects and people in space relative to each other.
- Subitize - The ability to quickly and accurately determine the quantity of objects in a small group (of up to five objects) without actually counting the objects.
- Tally - Count data using slash marks, with every fifth slash mark crossing the prior four.
- Three-Dimensional - An object that has length, width and depth.
- Trapezoid - A quadrilateral with one pair of parallel sides.
- Two-Dimensional - An object that has length and width.


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## Web Site Resources

- Building Blocks
www.gse.buffalo.edu/org/buildingblocks/
Building blocks is funded by the National Science Foundation to create mathematics curriculum materials for young children.
- Kitchen Math for Preschoolers
www.preksmarties.com/math/kitchen.htm
Beginning math activities using "real objects in the kitchen".
- Math Resources
www.math.com
Math resources for teachers and parents.
- Mathematial Activities
www.ed.gov/pubs/EarlyMath/index.html
Mathematical activities for parents and their 2- to 5-year-old children.
- PBS Kids
www.pbskids.org
Free Pre-K-12 resources to support learning at home and at school.
- National Council of Teachers of Mathematics
www.nctm.org
NCTM serves math teachers, math educators, and administrators by providing math resources and professional development opportunities.
"Number and operations on them are essential for most everyday activities. That is, understanding their applications is a basic survival skill in our high technological and information-dependent society, and thus, a key basis of mathematical literacy, which is now as important as language literacy."

California Preschool Instructional Network

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