

# Applied Cryptology

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Keep in mind there are *two* PDFs available (of which this is the latter):

1. a PDF of examinable material used as lecture slides, and
2. a PDF of non-examinable, extra material:
  - ▶ the associated notes page may be pre-populated with extra, written explanation of material covered in lecture(s), plus
  - ▶ anything with a “grey’ed out” header/footer represents extra material which is useful and/or interesting but out of scope (and hence not covered).

Notes:

Notes:

► Agenda:

1. a 2-part unit summary:
  - recap re. motivation, i.e., why the unit exists,
  - what did and didn't we do in the unit,
2. drop-in slot re. coursework assignment.

Notes:

A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (1)

Quote

*The function `BN_nist_mod_384` (in `crypto/bn/bn_nist.c`) gives wrong results for some inputs.*

*– Reimann [5]*

Notes:

Algorithm (NIST-P-256-REDUCE, per Solinas [6, Example 3, Page 20])

**Input:** For  $w = 32$ -bit words, a 16-word integer product  $z = x \cdot y$  and the modulus  $p = 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$

**Output:** The result  $r = z \pmod{p}$

1. Form the nine, 8-word intermediate variables

$$\begin{aligned} S_0 &= \langle z_0, z_1, z_2, z_3, z_4, z_5, z_6, z_7 \rangle \\ S_1 &= \langle 0, 0, 0, z_{11}, z_{12}, z_{13}, z_{14}, z_{15} \rangle \\ S_2 &= \langle 0, 0, 0, z_{12}, z_{13}, z_{14}, z_{15}, 0 \rangle \\ S_3 &= \langle z_8, z_9, z_{10}, 0, 0, 0, z_{14}, z_{15} \rangle \\ S_4 &= \langle z_9, z_{10}, z_{11}, z_{13}, z_{14}, z_{15}, z_{13}, z_8 \rangle \\ S_5 &= \langle z_{11}, z_{12}, z_{13}, 0, 0, 0, z_8, z_{10} \rangle \\ S_6 &= \langle z_{12}, z_{13}, z_{14}, z_{15}, 0, 0, z_9, z_{11} \rangle \\ S_7 &= \langle z_{13}, z_{14}, z_{15}, z_8, z_9, z_{10}, 0, z_{12} \rangle \\ S_8 &= \langle z_{14}, z_{15}, 0, z_9, z_{10}, z_{11}, 0, z_{13} \rangle \end{aligned}$$

2. Compute

$$r = S_0 + 2S_1 + 2S_2 + S_3 + S_4 - S_5 - S_6 - S_7 - S_8 \pmod{p}.$$

3. Return  $0 \leq r < p$ .

Notes:

Algorithm (NIST-P-256-REDUCE, per OpenSSL 0.9.8g)

**Input:** For  $w = 32$ -bit words, a 16-word integer product  $z = x \cdot y$  and the modulus  $p = 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$

**Output:** The (potentially incorrect) result  $r = z \pmod{p}$

1. Form the nine, 8-word intermediate variables

$$\begin{aligned} S_0 &= \langle z_0, z_1, z_2, z_3, z_4, z_5, z_6, z_7 \rangle \\ S_1 &= \langle 0, 0, 0, z_{11}, z_{12}, z_{13}, z_{14}, z_{15} \rangle \\ S_2 &= \langle 0, 0, 0, z_{12}, z_{13}, z_{14}, z_{15}, 0 \rangle \\ S_3 &= \langle z_8, z_9, z_{10}, 0, 0, 0, z_{14}, z_{15} \rangle \\ S_4 &= \langle z_9, z_{10}, z_{11}, z_{13}, z_{14}, z_{15}, z_{13}, z_8 \rangle \\ S_5 &= \langle z_{11}, z_{12}, z_{13}, 0, 0, 0, z_8, z_{10} \rangle \\ S_6 &= \langle z_{12}, z_{13}, z_{14}, z_{15}, 0, 0, z_9, z_{11} \rangle \\ S_7 &= \langle z_{13}, z_{14}, z_{15}, z_8, z_9, z_{10}, 0, z_{12} \rangle \\ S_8 &= \langle z_{14}, z_{15}, 0, z_9, z_{10}, z_{11}, 0, z_{13} \rangle \end{aligned}$$

2. Compute

$$\begin{aligned} S &= S_0 + 2S_1 + 2S_2 + S_3 + S_4 - S_5 - S_6 - S_7 - S_8 \\ &= t + c \cdot 2^{256} \end{aligned}$$

3. Compute

$$\begin{aligned} r &= t - c \cdot p \pmod{2^{256}} \\ &= t - \text{sign}(c) \cdot T[[c]] \pmod{2^{256}} \end{aligned}$$

for pre-computed  $T[i] = i \cdot p$ .

4. If  $r \geq p$  (resp.  $r < 0$ ) then update  $r \leftarrow r - p$  (resp.  $r \leftarrow r + p$ ), return  $r$ .

Notes:

## A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (3)

Issue 1: arithmetic on NIST-P-[256,384]

### ► Observation(s):

- **good:** BN\_nist\_mod\_256 (resp. BN\_nist\_mod\_384) is more efficient.
- **bad:** BN\_nist\_mod\_256 (resp. BN\_nist\_mod\_384) can produce an incorrect result, e.g.,
  1. triggered deliberately with special-form operands

$$\begin{aligned} x &= (2^{32} - 1) \cdot 2^{224} + 3 \cdot 2^{128} + x_0 \\ y &= (2^{32} - 1) \cdot 2^{224} + 1 \cdot 2^{96} + y_0 \end{aligned}$$

for random  $0 \leq x_0, y_0 < 2^{32}$ , or

2. triggered randomly with probability  $\sim 10 \cdot 2^{-29}$ .

Notes:

## A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (4)

Issue 2: (opt-out) ephemeral-static EC-DHE

### Algorithm (EC-DH(E) key agreement [7, Section 8.1][8, Section 2.1])

$\mathcal{A}$

$\mathcal{B}$

Knows  $\mathbf{G} = E(\mathbb{F}_q) = \langle G \rangle$  of order  $n$ ,  
 $pk_{\mathcal{B}}, (pk_{\mathcal{A}})^{\dagger}, (sk_{\mathcal{A}})^{\dagger}$

Knows  $\mathbf{G} = E(\mathbb{F}_q) = \langle G \rangle$  of order  $n$ ,  
 $(pk_{\mathcal{A}})^{\dagger}, pk_{\mathcal{B}}, sk_{\mathcal{B}}$

$$k_{\mathcal{A}}^{(i)} \stackrel{\$}{\leftarrow} \{1, 2, \dots, n-1\}$$

$$Q_{\mathcal{A}}^{(i)} \leftarrow [k_{\mathcal{A}}^{(i)}]G$$

$Q_{\mathcal{A}}^{(i)}$



$Q_{\mathcal{B}}^{(i)}$



$$R_{\mathcal{A}}^{(i)} \leftarrow [k_{\mathcal{A}}^{(i)}]Q_{\mathcal{B}}^{(i)} = [k_{\mathcal{A}}^{(i)} \cdot k_{\mathcal{B}}^{(i)}]G$$

Use  $R_{\mathcal{A}}^{(i)}$

$$k_{\mathcal{B}}^{(i)} \stackrel{\$}{\leftarrow} \{1, 2, \dots, n-1\}$$

$$Q_{\mathcal{B}}^{(i)} \leftarrow [k_{\mathcal{B}}^{(i)}]G$$

$$R_{\mathcal{B}}^{(i)} \leftarrow [k_{\mathcal{B}}^{(i)}]Q_{\mathcal{A}}^{(i)} = [k_{\mathcal{B}}^{(i)} \cdot k_{\mathcal{A}}^{(i)}]G$$

Use  $R_{\mathcal{B}}^{(i)}$

Notes:

- A high-level overview of how the above relates to OpenSSL can be found at

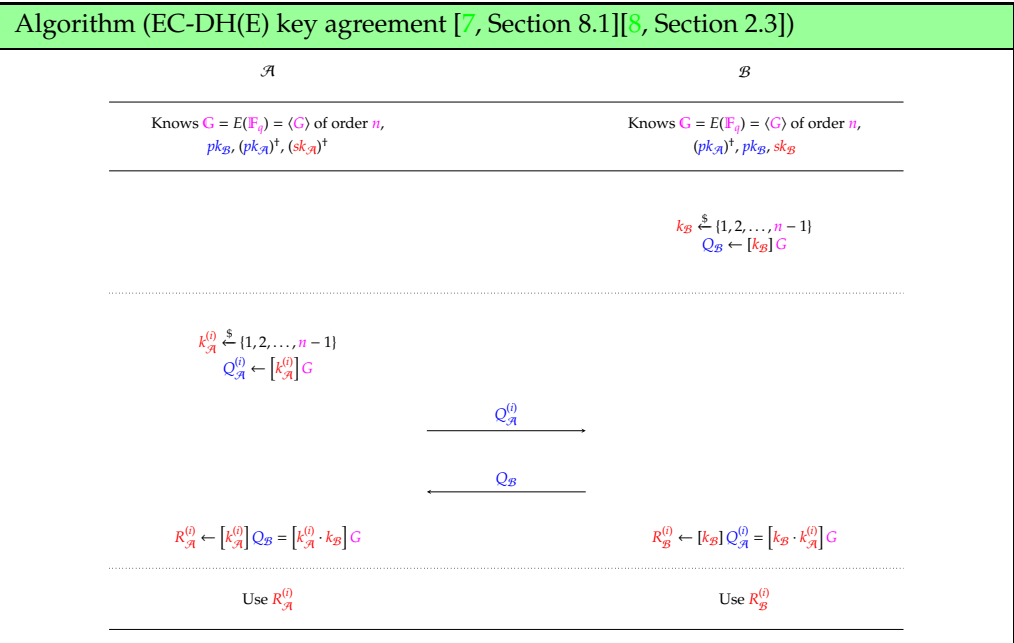
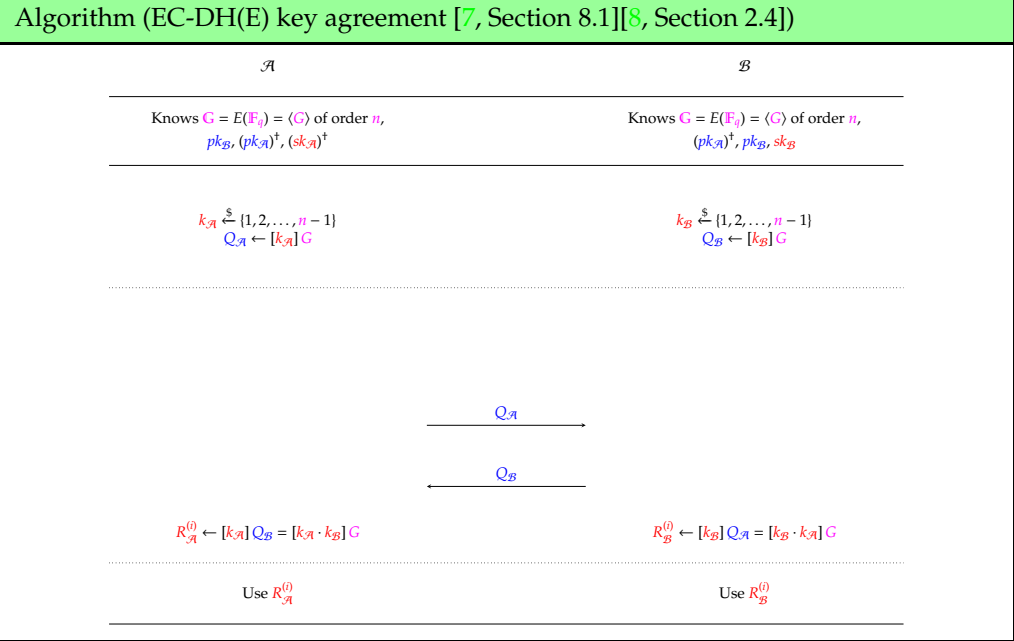
[http://wiki.openssl.org/index.php/Diffie\\_Hellman](http://wiki.openssl.org/index.php/Diffie_Hellman)

and

[http://wiki.openssl.org/index.php/Elliptic\\_Curve\\_Diffie\\_Hellman](http://wiki.openssl.org/index.php/Elliptic_Curve_Diffie_Hellman)

Note that the former explicitly warns against use of anonymous variants, offering a way to exclude them from the cipher suite list.

- It seems reasonable to say that the static-static and ephemeral-static options are confusion with respect to, e.g., the ECDHE cipher suite identifier (which implies ephemeral, but not which, if any party respects this).



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- It seems reasonable to say that the static-static and ephemeral-static options are confusion with respect to, e.g., the ECDHE cipher suite identifier (which implies ephemeral, but not which, if any party respects this).

► **Observation(s):**

- **good:** the key agreement is more efficient (for the server).
- **good:** input points are validated by testing whether

$$P_y^2 \stackrel{?}{=} P_x^3 + a_4 P_x + a_6$$

given  $P = (P_x, P_y)$ .

- **bad:** ephemeral-static EC-DHE is the default i.e.,
  - uses a per-invocation (of the library) rather than a per-session key, *unless*
  - one explicitly uses `SSL_CTX_set_options` using `SSL_OP_SINGLE_ECDH_USE` which means  $k_B$  is a static, fixed target for any attack.

- **bad:** if we select  $P = (P_x, P_y)$  as follows

1. Select  $P_x$  such that during the computation of the RHS  $t' = (P_x^2 + a_4) \cdot P_x + a_6 \pmod{p}$

- the step  $t'_0 = P_x^2 \pmod{p}$  does not trigger the bug, and
- the step  $t'_1 = (t'_0 + a_4) \cdot P_x \pmod{p}$  does trigger the bug, and
- $t'$  is a quadratic residue modulo  $p$ .

2. Compute  $P_y = \sqrt{t'} \pmod{p}$ .

then  $P$  passes validation, but is on some curve  $E'$  rather than  $E$ .

Notes:

A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (6)  
An attack!

Quote

*Decrypting ciphertexts on any computer which multiplies even one pair of numbers incorrectly can lead to full leakage of the secret key, sometimes with a single well-chosen ciphertext.*

– Biham et. al. [1], Page 1]

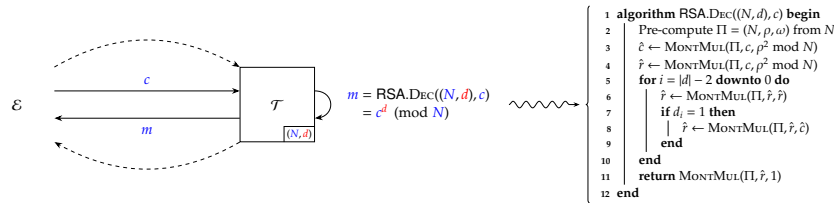
Notes:

## A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (7)

An attack!

### Scenario:

- given the following interaction between an **attacker**  $\mathcal{E}$  and a **target**  $\mathcal{T}$



### and noting that

- there are no countermeasures implemented,
- the Montgomery multiplication implementation is FIOS-based [3],
- the  $(w \times w)$ -bit integer multiplier hardware has a bug: when computing  $r = x \times y$  if

$$\begin{aligned} x \neq \alpha \quad \vee \quad y \neq \beta &\Rightarrow r \text{ is correct} \\ x = \alpha \quad \wedge \quad y = \beta &\Rightarrow r \text{ is incorrect} \end{aligned}$$

for some known (but arbitrary)  $\alpha$  and  $\beta$ .

- how can  $\mathcal{E}$  mount a successful attack, i.e., recover  $d$ ?

Notes:

## A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (8)

An attack!

### Attack [1, Section 4.2]:

- in some  $t$ -th step,  $\mathcal{E}$ 
  - knows some more-significant portion of the binary expansion of  $d$ , and
  - aims to recover  $d_t$ , the next less-significant unknown bit,
- select a  $c$  so during decryption when  $i = t$  and just after line #6

$$\begin{aligned} \exists j \text{ such that } \hat{r}_j &= \alpha \\ \exists j \text{ such that } \hat{c}_j &= \beta \end{aligned}$$

i.e.,  $\alpha$  and  $\beta$  occur in the representations of  $\hat{r}$  and  $\hat{c}$ ,

- this selection means

$$\begin{aligned} d_t = 0 &\Rightarrow \hat{r} \text{ is not multiplied by } \hat{c} \Rightarrow \text{the bug is not triggered} \\ d_t = 1 &\Rightarrow \hat{r} \text{ is multiplied by } \hat{c} \Rightarrow \text{the bug is triggered} \end{aligned}$$

- test whether

$$m^e \pmod{N} \stackrel{?}{=} c$$

and infer

$$\begin{aligned} m \text{ is correct} &\Rightarrow \text{the bug was not triggered} \Rightarrow d_t = 0 \\ m \text{ is incorrect} &\Rightarrow \text{the bug was triggered} \Rightarrow d_t = 1 \end{aligned}$$

Notes:

## A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (9)

An attack!

Feature	Biham et. al. [1, Section 4.2]	Brumley et. al. [2, Section 3]
Target	Fixed $d$	Fixed $k_{\mathcal{T}}$
Input	Arbitrary poisoned integer $c \in \mathbb{Z}_N^*$	Controlled distinguisher point $Q_{\mathcal{E}} = [k_{\mathcal{E}}]G \in E(\mathbb{F}_p)$
Computation	Left-to-right binary exponentiation	Left-to-right (modified) wNAF scalar multiplication
Leakage	Re-encrypt $m$ using $e$ , check against $c$	Handshake success/failure

Notes:

## A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (10)

A patch?

### ► Epilogue:

#### ► good(ish):

#### Quote

*We appreciate you reporting this issue to us but, unfortunately, we aren't inclined to handle this vulnerability because it is already patched and only affects obsolete Linux distributions.*

– CERT

Notes:

- The analysis paper by Martin et al. [4] was published in 2013: the attack paper by Brumley et al. [2] was published in 2012, but OpenSSL 0.9.8g was released in 2007 (i.e., much earlier).



# A real-world story: an attack [2] on TLS 1.2 + OpenSSL 0.9.8g (10) A patch?

## ► Epilogue:

► **bad**: even circa 2013, the reality [4] seemed to differ somewhat:

Version	Percentage	Distribution	OSSL Version	CVEs
0.9.8e-fips-rhel5	37.25	Debian Squeeze (6.0)	0.9.8o	11
0.9.8g	14.50	Debian Lenny (5.0)	0.9.8g	24
0.9.7a	7.02	Debian Etch (4.0)	0.9.8c	26
0.9.8o	4.76	RHEL 6	0.9.8e/1.0.0-fips	0/14
1.0.0-fips	4.36	RHEL 5	0.9.7a/0.9.8e-fips	14/0
0.9.7d	2.91	RHEL 4	0.9.6b/0.9.7a	9/14
0.9.8n	2.75	Fedora 18	1.0.1c	3
0.9.7e	1.94	Fedora 17	1.0.0i	3
0.9.8c	1.80	Fedora 16	1.0.0e	9
0.9.8m	1.74			
0.9.8e	1.72			
0.9.8r	1.71			

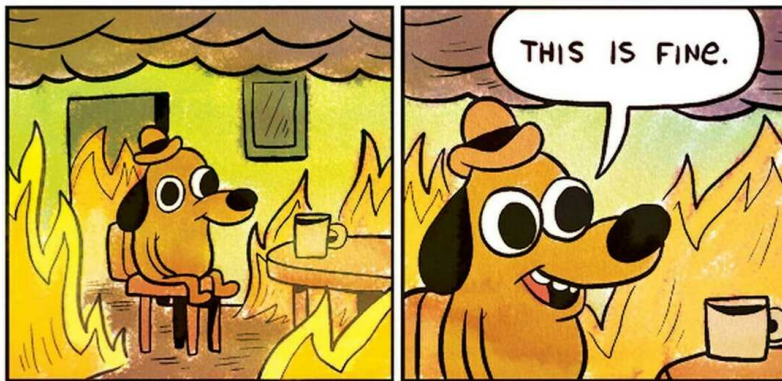
Table 2: Most popular OpenSSL versions on the Internet.

Table 3: Default OpenSSL versions shipping with popular Linux distributions.

<https://jscholarship.library.jhu.edu/items/00b58834-a88c-449e-ab23-db2f44207383>

## Unit summary (1)

## ► Summary:



<http://memegenerator.net>

Notes:

- The analysis paper by Martin et al. [4] was published in 2013: the attack paper by Brumley et al. [2] was published in 2012, but OpenSSL 0.9.8g was released in 2007 (i.e., much earlier).

Notes:

## Unit summary (2)

### ► Summary: what *have* we done includes

#### 1. exposed some low-level detail:

- concrete versus abstract (e.g., AES versus generic block cipher),
- written standards (e.g., FIPS-197 versus lecture slides),
- ...

#### 2. highlighted some high-level principles:

- most effective implementation will be domain-specific,
- apply adversarial thinking to *everything*,
- need for and value in well-considered trade-offs,
- don't *over*-optimise to the point efficiency > security,
- apply "*inverse* Postel's Law", i.e., be *very* strict re. what you accept as input,
- ...

#### 3. focused on some high-level outcomes:

- improved



- general concepts (versus specific examples) ⇒ long-term (versus short-term) value.

Notes:

## Unit summary (3)

### ► Summary: what *haven't* we done includes

#### 1. greater *depth*, i.e., more $X$ for $X \in \text{COMS30048}$ :

- more implementation
  - platforms (e.g., FPGAs, ASICs, GPUs, ..., JavaScript versus C)
  - constraints (e.g., from use-case, platform, tooling, ...)
  - co-design (e.g., hardware/software, specification/implementation, ...)
  - ...
- more attacks
- more countermeasures
- more primitives (e.g., PQC, LWC, hash functions, ..., FHE, MPC, ...)
- more protocols (e.g., DNSSEC, IPSec, ...)

#### 2. greater *breadth*, i.e., more $X$ for $X \notin \text{COMS30048}$ :

- hardware security (e.g., TEEs, HSMs, secure boot and update, FDE, ...)
- formal verification
- key management (e.g., secure generation, storage, and erasure, ...)
- social-technical (e.g., usability, politics, risk analysis, supply chain, disclosure, ...)
- certification and standardisation processes
- ...

Notes:

## References

- [1] E. Biham, Y. Carmeli, and A. Shamir. “Bug Attacks”. In: *Advances in Cryptology (CRYPTO)*. Vol. 5157. LNCS. Springer-Verlag, 2008, pp. 221–240 (see pp. 23, 27, 29).
- [2] B. Brumley et al. “Practical realisation and elimination of an ECC-related software bug attack”. In: *Topics in Cryptology (CT-RSA)*. LNCS 7178. Springer-Verlag, 2012, pp. 171–186 (see pp. 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31–34).
- [3] Ç.K. Koç, T. Acar, and B.S. Kaliski. “Analyzing and comparing Montgomery multiplication algorithms”. In: *IEEE Micro* 16.3 (1996), pp. 26–33 (see p. 25).
- [4] P.D. Martin et al. *Classifying Network Protocol Implementation Versions: An OpenSSL Case Study*. Tech. rep. 13-01. Johns Hopkins University, 2013. URL: <http://www.michaelrushman.org/pdf/martin.pdf> (see pp. 31–34).
- [5] H. Reimann. *BN\_nist\_mod\_384 gives wrong answers*. openssl-dev mailing list #1593. 2007. URL: <http://marc.info/?t=119271238800004> (see p. 7).
- [6] J.A. Solinas. *Generalized Mersenne Numbers*. Tech. rep. CORR 99-39. Centre for Applied Cryptographic Research (CACR), University of Waterloo, 1999 (see p. 9).
- [7] T. Dierks and E. Rescorla. *The Transport Layer Security (TLS) Protocol version 1.2*. Internet Engineering Task Force (IETF) Request for Comments (RFC) 5246. 2008. URL: <http://tools.ietf.org/html/rfc5246> (see pp. 15, 17, 19).
- [8] E. Rescorla. *Diffie-Hellman Key Agreement Method*. Internet Engineering Task Force (IETF) Request for Comments (RFC) 2631. 1999. URL: <http://tools.ietf.org/html/rfc2631> (see pp. 15, 17, 19).

Notes: