Applied Cryptology

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Keep in mind there are *two* PDFs available (of which this is the latter):

1. a PDF of examinable material used as lecture slides, and
2. a PDF of non-examinable, extra material:
   - the associated notes page may be pre-populated with extra, written explaination of material covered in lecture(s), plus
   - anything with a "grey'ed out" header/footer represents extra material which is useful and/or interesting but out of scope (and hence not covered).

Notes:

Notes:

- Agenda: explore **(pseudo-)random bit generation**, via
  1. an "in theory", i.e., design-oriented perspective, and
  2. an "in practice", i.e., implementation-oriented perspective.
- Caveat!

$$\sim 2 \text{ hours} \quad \Rightarrow \quad \text{introductory, and (very) selective (versus definitive) coverage.}$$

Notes:

COMS30048 lecture: week #20

- Bad news: in *theory*, we need to consider each of
  1. random *bit*, i.e., an
     $$x \in \{0, 1\}$$
     which is random,
  2. random *bit sequence*, i.e., an
     $$x \in \{0, 1\}^n$$
     which is random (e.g., for an AES cipher key k),
  3. random *number*, i.e., an
     $$x \in \{0, 1, \ldots, n-1\}$$
     which is random (e.g., for an RSA modulus $N = p \cdot q$).

Notes:

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- Good news: in *practice*, we don't because
  - 1. $\Rightarrow$ 2.
    - concatenate n random bits together, i.e.,

$$x = x_0 \parallel x_1 \parallel \cdots \parallel x_{n-1},$$

    - produce x as output.
  - 2. $\Rightarrow$ 3.
    - if $n = 2^{n'}$ for some integer n', then
      - generate an n'-bit sequence x' per the above,
      - interpret x' as the integer

$$x = \sum_{i=0}^{i<n'} x'_i,$$

      - produce x as output.
    - if $n \neq 2^{n'}$ for any integer n', then
      - let n' be the smallest integer such that $2^{n'} > n$,
      - generate an n'-bit sequence x' per the above,
      - interpret x' as the integer

$$x = \sum_{i=0}^{i<n'} x'_i,$$

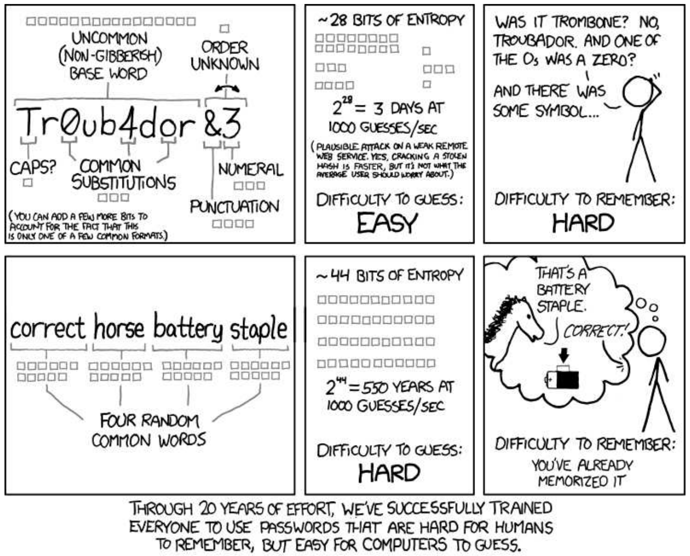
      - if $x \geq n$, reject (or discard) it and try again; otherwise, if $x < n$, produce x as output.

$\therefore$ we can focus on random bits (and ignore numbers).

Notes:

Part 1: in theory (1)

Entropy



http://xkcd.com/936

Notes:

Definition

The concept of **entropy** is a measure of uncertainty with respect to a random variable. Less formally, the entropy of some x relates to how much you know (resp. do not know) about x: if some x could be one of 2^n possible values, it is said to have n bits of entropy. In addition, we say

1. an x with $n > 0$ bits of entropy is termed **entropic**, and
2. if an entropic x has negligible probability of having been generated before, it is deemed **fresh entropy**.

Notes:

Part 1: in theory (2)
Entropy

Definition

The concept of **entropy** is a measure of uncertainty with respect to a random variable. Less formally, the entropy of some x relates to how much you know (resp. do not know) about x: if some x could be one of 2^n possible values, it is said to have n bits of entropy. In addition, we say

1. an x with $n > 0$ bits of entropy is termed **entropic**, and
2. if an entropic x has negligible probability of having been generated before, it is deemed **fresh entropy**.

- Example: given a 32-bit sequence x,
  - if x is random, then it has 32 bits of entropy,
  - if $x_0 = 0$ and $x_1 = 1$ (i.e., the two LSBs of x are known), then it has 30 bits of entropy,
  - if $\mathrm{HW}(x) = 14$ (i.e., x has Hamming weight 14), then it has ~ 29 bits of entropy.

Notes:

Part 1: in theory (3)

Entropy

> **Definition**
>
> A **noise source** is a non-deterministic, physical process which provides a means of generating an *unconditioned* (or raw) entropic output.

Notes:

Part 1: in theory (3)

Entropy

> **Definition**
>
> A **noise source** is a non-deterministic, physical process which provides a means of generating an *unconditioned* (or raw) entropic output.

- Example (see [8, Section 5.2], or [19, Section 3]):
  1. hardware-based:
     - time between emission of (e.g., α or β) particles during radioactive decay,
     - thermal (or Johnson-Nyquist) noise stemming from a resistor or capacitor,
     - frequency instability (or "jitter") of a ring oscillator,
     - fluctuation of hard disk seek-time and access latency,
     - noise resulting from a disconnected audio input (or ADC),
     - ...
  2. software-based:
     - a high resolution system clock or cycle counter,
     - elapsed time between user input (e.g., key-presses or mouse movement),
     - content of input/output buffers (e.g., disk caches),
     - operating system state (e.g., load) or events (e.g., network activity),
     - ...

Notes:

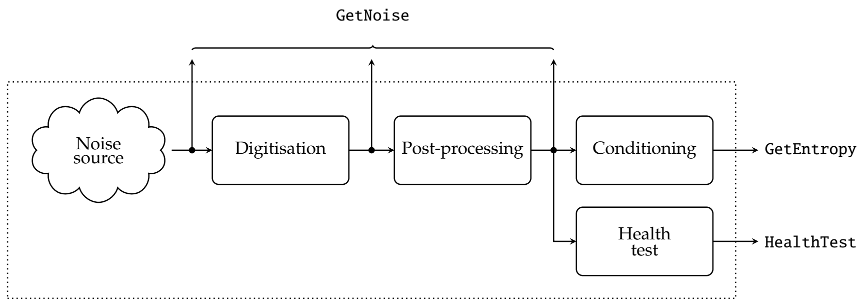
Part 1: in theory (4)

Entropy

Definition

An **entropy source** is a construction, based on a noise source, which provides a means of generating a *conditioned* entropic output.

Model [21, Section 2.2]



Notes:

Part 1: in theory (5)

Randomness

Definition

Per [20, Section 4], an **ideal random bit-sequence**

$$x = \langle x_0, x_1, \ldots x_{n-1} \rangle$$

will exhibit the following properties

1. **unpredictable** $\Rightarrow$ the probability of guessing x_i is close to $\frac{1}{2}$
2. **unbiased** $\Rightarrow$ $x_i = 0$ and $x_i = 1$ occur with equal probability
3. **uncorrelated** $\Rightarrow$ x_i and x_j are statistically independent

and contain n bits of entropy.

Notes:

Part 1: in theory (5)

Randomness

Definition

Per [20, Section 4], a **pseudo-random bit-sequence**

$$x = \langle x_0, x_1, \ldots x_{n-1} \rangle$$

"looks random", i.e., exhibits the same properties as an ideal random sequence, *but* is generated algorithmically and thus likely contains less than n bits of entropy.

Notes:

Part 1: in theory (6)

(Pseudo-)random bit generators

Definition

A **Random Bit Generator (RBG)** can be used to generates a sequence of random bits. There are two more specific cases, namely

True Random Bit Generator (TRBG)	≡	**Non-deterministic Random Bit Generator (NRBG)**
Pseudo-Random Bit Generator (PRBG)	≡	**Deterministic Random Bit Generator (DRBG)**

with the right-hand terms preferred by [20]. Based on this, it is reasonable to say that

$$\text{TRBG} \equiv \text{NRBG} \simeq \text{entropy source.}$$

Notes:

Part 1: in theory (6)
(Pseudo-)random bit generators

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True Random Bit Generator (TRBG)	$\equiv$	**Non-deterministic Random Bit Generator (NRBG)**
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$$\text{TRBG} \equiv \text{NRBG} \simeq \text{entropy source.}$$

- Idea: informally at least,

TRBG		PRBG
unpredictable,	←+→	predictable,
hardware-based,		software-based,
may have high latency,		can have low latency,
can be high quality.		may be low quality.

$\therefore$ we'll consider a *hybrid* construction.

Notes:

Part 1: in theory (7)
(Pseudo-)random bit generators

Definition

Consider a deterministic, polynomial-time algorithm G. Given a **seed** $\varsigma \in \{0,1\}^{n_\varsigma}$ as input, it produces $G(\varsigma) \in \{0,1\}^{n_r}$ as output where $n_r = f(n_\varsigma)$ for some polynomial function f. As such, we call G a **Pseudo-Random Generator (PRG)** if

1. for every n_ς it holds that $n_r > n_\varsigma$, and
2. for all polynomial-time destinguishers D, there exists a negligible function negl such that

$$|\Pr[D(G(\varsigma)) = 1] - \Pr[D(r) = 1]| \leq \mathsf{negl}(n_\varsigma)$$

where ς and r are chosen uniformly at random from $\{0,1\}^{n_\varsigma}$ and $\{0,1\}^{n_r}$ respectively.

Notes:

Syntax

Having fixed the (finite) space $\mathcal{S}$ of states, a concrete **Pseudo-Random Generator (PRG)** is defined by

1. an algorithm $\text{Seed} : \mathbb{Z} \times \{0,1\}^{n_\varsigma} \rightarrow \mathcal{S}$ that
   - accepts a security parameter and an n_ς-bit seed as input, and
   - produces an initial state as output
2. an algorithm $\text{Update} : \mathcal{S} \rightarrow \mathcal{S} \times \{0,1\}^{n_b}$ that
   - accepts a current state as input, and
   - produces a next state and an n_b-bit block of pseudo-random bits as output.

Notes:

- Translation: assuming $n_r = l \cdot n_b$ for some l, then

1. use TRBG $\rightsquigarrow$ { generate a sufficiently large, high-entropy seed ς

2. use PRBG $\rightsquigarrow$

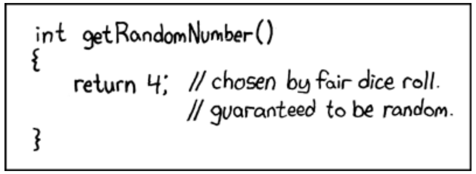
$$\left\{ \begin{array}{lll} \theta[0] & & \leftarrow \text{Seed}(\lambda, \varsigma) \\ \theta[1] & , b[0] & \leftarrow \text{Update}(\theta[0]) \\ \theta[2] & , b[1] & \leftarrow \text{Update}(\theta[1]) \\ & \vdots & \\ \theta[i+1] & , b[i] & \leftarrow \text{Update}(\theta[i]) \\ & \vdots & \end{array} \right.$$

meaning that

$$b = \underbrace{\underbrace{b[0]}_{n_b\text{-bits}} \parallel \underbrace{b[1]}_{n_b\text{-bits}} \parallel \cdots \parallel \underbrace{b[l-1]}_{n_b\text{-bits}}}_{l \cdot n_b = n_r\text{-bits}} \equiv G(\varsigma)$$

provides the output required per the PRG definition.

Notes:



http://xkcd.com/221

Notes:

- Problem: we need to assess the quality of our construction (and output from it).
- Solution:
  1. for *some* instanciations, we can develop a proof,
  2. for *some* instanciations, we must apply
     - online (e.g., continuously or periodically *during* use), and/or
     - offline (i.e., once *before* use)

     statistical tests (see, e.g., [8, Section 5.4]) to sample outputs; note that
     - the intention is to detect weakness (meaning a PRBG can only be rejected by a test),
     - the conclusion is itself probabilistic, meaning use of multiple tests amplifies confidence.

Notes:

Definition

A PRBG is said to pass all **statistical tests** iff. no polynomial-time algorithm can, with probability greater than $\frac{1}{2}$, distinguish the output from a ideal random bit-sequence of the same length.

Definition

A PRBG is said to pass the **next-bit test** iff. no polynomial-time algorithm can, with probability greater than $\frac{1}{2}$, predict the $(n+1)$-th bit of output given the previous n bits.

Theorem (Yao [14])

If a PRBG passes the next-bit test, it will pass all statistical tests.

Notes:

Definition

Per [20, Section 4], imagine an attacker compromises the PRBG state at time t: we term a PRBG **back-tracking resistant** (resp. **prediction resistant**) if said attacker cannot distinguish between an (unseen) PRBG output at time $t' < t$ (resp. $t' > t$) and an ideal random bit-sequence of the same length.

Definition

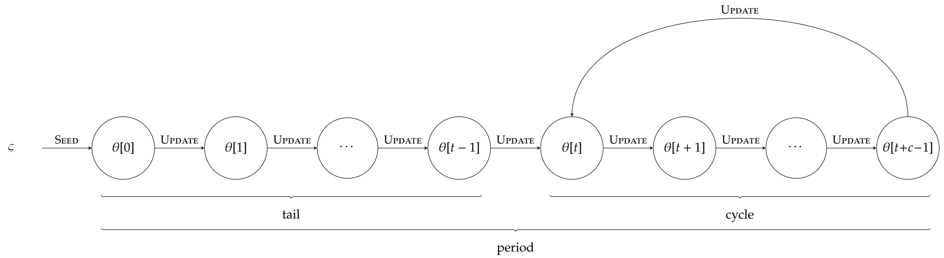
A **Cryptographically Secure Pseudo-Random Bit Generator (CS-PRBG)** is simple a PRBG whose properties make it suitable for use within a cryptographic use-case. A CS-PRBG should (at least)

1. be a PRBG of sufficient quality, i.e., pass the next-bit test, and
2. resist state compromise attacks, i.e., be back-tracking and prediction resistant.

Notes:

Part 1: in theory (13)
(Pseudo-)random bit generators

- Problem: our construction is deterministic, so
  - the same ς will yield the same $\theta[0]$ and hence any $\theta[j]$ for $j > 0$,
  - recovery of ς allows computation of any $\theta[j]$ for $j \geq 0$,
  - recovery of $\theta[i]$ allows computation of any $\theta[j]$ for $j > i$,
  - the set $\mathcal{S}$ is finite, so per



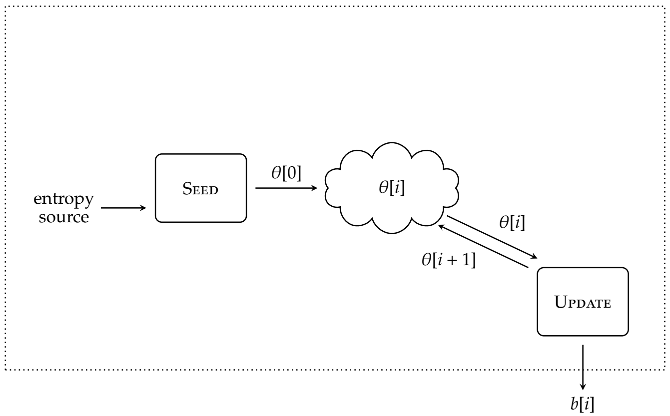
  the state, and thus also the output, will eventually cycle.

- Solution:
  1. select parameters that mitigate such issues, and
  2. introduce selected *non*-determinism.

Notes:

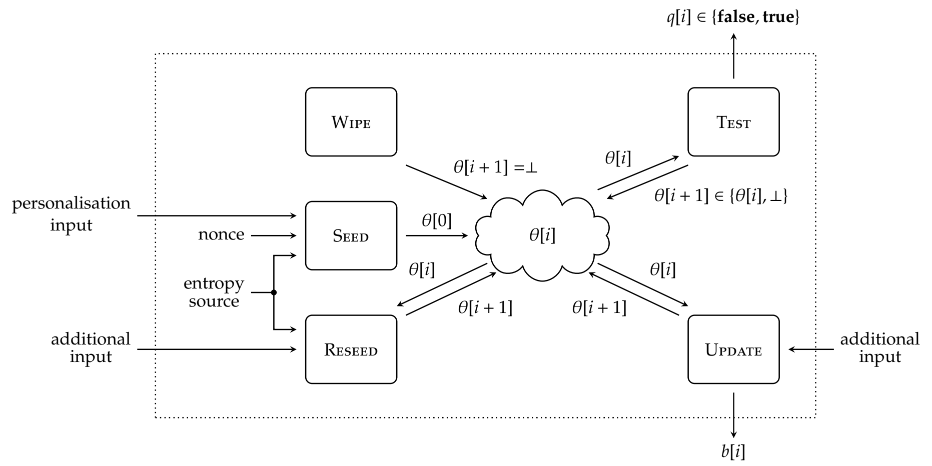
Part 1: in theory (14)
(Pseudo-)random bit generators

Definition ([20, Figure 1])



Notes:

Definition ([20, Figure 1])



Notes:

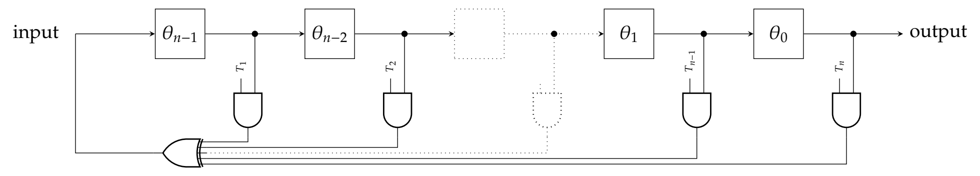
- (Sub-)agenda: explain selected, example designs, organised into 4 classes, i.e.,
  1. "classic",
  2. software-oriented,
  3. hardware-oriented,
  4. system-oriented,

  with a focus on design properties and trade-offs between them, e.g.,
  - efficiency,
  - security, i.e., quality of (pseudo-)random output,
  - interface,
  - assumptions,
  - ...

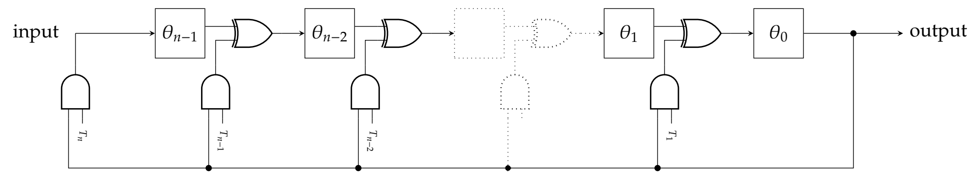
Notes:

- **Design**: **Linear-Feedback Shift Registers (LFSR)** [5, 6].

Algorithm (type-1, or "external" or Fibonacci, LFSR)



Algorithm (type-2, or "internal" or Galois, LFSR)



Notes:

- **Design**: **Blum-Blum-Shub (BBS)** [10].

Algorithm (BBS.Seed)

Input: A security parameter λ, and a seed ς
Output: An initial state $\theta[0]$

Use entropy provided by ς to perform the following steps:

1. Select two random $(\lambda/2)$-bit primes p and q such that $p \equiv q \equiv 3 \pmod 4$, and compute $N = p \cdot q$.
2. Select a random $s \in \{0, 1, \ldots, N-1\}$ such that $\gcd(s, N) = 1$.
3. Compute $s[0] = s^2 \pmod N$.
4. Return $\theta[0] = (N, s[0])$.

Notes:

- Design: **Blum-Blum-Shub (BBS)** [10].

Algorithm (BBS.Update)

Input: A current state $\theta[i] = (N, s[i])$
Output: A next state $\theta[i+1]$, and $n_b = 1$ bit pseudo-random output $b[i]$

1. Compute $s[i+1] = s[i]^2 \pmod{N}$.
2. Let $b[i] = s[i+1] \pmod{2}$, i.e., $b[i] = \text{LSB}(s[i+1])$.
3. Return $\theta[i+1] = (N, s[i+1])$ and $b[i]$.

Notes:

- Design: **ANSI X9.31** [18, Appendix A.2.4].

Algorithm (X9.31.Seed)

Input: A security parameter λ, and a seed ς
Output: An initial state $\theta[0]$

1. Use λ to select a block cipher with an n_k-bit key size and n_b-bit block size, e.g.,

$$\begin{array}{lcll} \text{3DES} & \rightsquigarrow & n_b = 64, & n_k = 192 \\ \text{AES-128} & \rightsquigarrow & n_b = 128, & n_k = 128 \\ \text{AES-192} & \rightsquigarrow & n_b = 128, & n_k = 192 \\ \text{AES-256} & \rightsquigarrow & n_b = 128, & n_k = 256 \end{array}$$

2. Use entropy provided by ς to derive an n_k-bit cipher key k (or pre-select a k for the PRBG).
3. Use entropy provided by ς to derive an n_b-bit block $s[0]$.
4. Return $\theta[0] = (k, s[0])$.

Notes:

- Kelsey et al. [13, Section 3.1] critique this design, or at least a less-general precursor named X9.17. The crux of their argument is that an attacker which recovers the block cipher key can recover the PRNG state given sample output (by guessing the timestamp, then decrypting). You *could* argue recovering the key is analogous to recovering the state, so maybe this is catastrophic by definition, but, either way, it should be clear that protecting *both* is vital.

Part 2: in practice (4)
Class #2: software-oriented

- Design: **ANSI X9.31** [18, Appendix A.2.4].

Algorithm (X9.31.Update)

Input: A current state $\theta[i] = (k, s[i])$
Output: A next state $\theta[i+1]$, and n_b-bit pseudo-random output $b[i]$

1. Compute $t' = \textsc{Enc}(k, t)$, where t is a n_b-bit representation of the current time.
2. Compute $b[i] = \textsc{Enc}(k, t' \oplus s[i])$.
3. Compute $s[i+1] = \textsc{Enc}(k, t' \oplus b[i])$.
4. Return $\theta[i+1] = (k, s[i+1])$ and $b[i]$.

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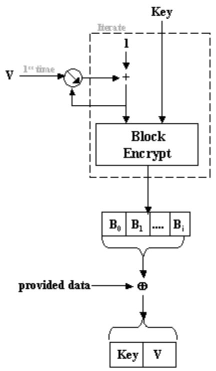
Part 2: in practice (5)
Class #2: software-oriented

- Design: **NIST CTR_DRBG** [20, Section 10.2.1].

Algorithm (CTR_DRBG.Update)

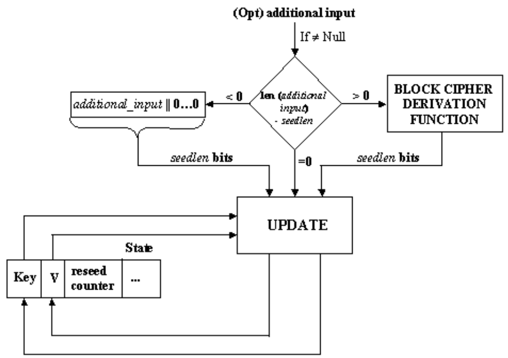


http://csrc.nist.gov/publications/nistpubs/800-90A/SP800-90A.pdf

Notes:

- Design: **NIST CTR_DRBG** [20, Section 10.2.1].

Algorithm (CTR_DRBG.Instantiate)



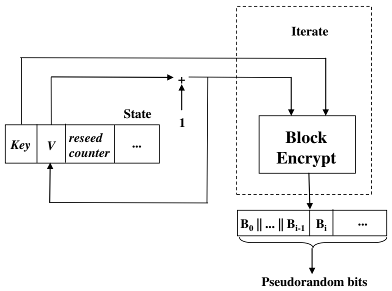
Notes:

http://csrc.nist.gov/publications/nistpubs/800-90A/SP800-90A.pdf

Part 2: in practice (5)
Class #2: software-oriented

- Design: **NIST CTR_DRBG** [20, Section 10.2.1].

Algorithm (CTR_DRBG.Generate)

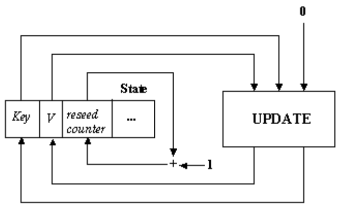


Notes:

http://csrc.nist.gov/publications/nistpubs/800-90A/SP800-90A.pdf

- **Design**: **NIST CTR_DRBG** [20, Section 10.2.1].

Algorithm (CTR_DRBG.Reseed)

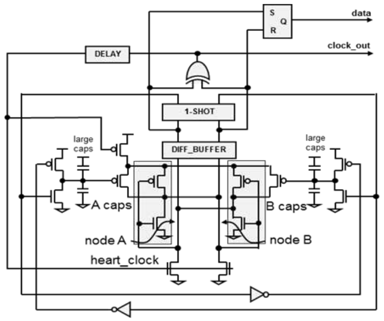


http://csrc.nist.gov/publications/nistpubs/800-90A/SP800-90A.pdf

Notes:

- **Design**: **Intel Secure Key** [16].

Algorithm (RdRand entropy source)



http://www.cryptography.com/public/pdf/Intel_TRNG_Report_20120312.pdf

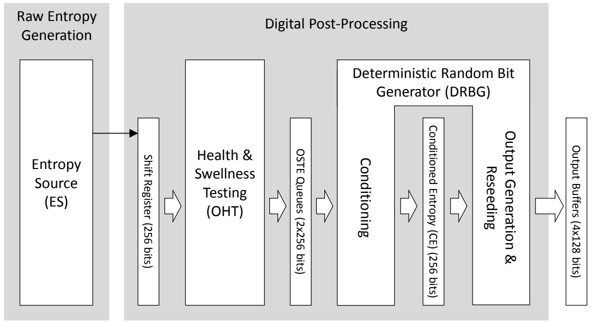
Notes:

- The entropy source diagram is for one cell, many of which are distributed in and around the associated micro-processor core. The TRNG diagram uses somewhat generic blocks: in more detail
  - the conditioning step uses a MAC, namely AES-CBC-MAC [16, Section 3.2.2],
  - the output generation and reseeding step uses a block cipher, namely AES-CTR [16, Section 3.2.3],
  - the entropy source outputs bits at a rate of $\sim 3\text{Gbit}\,\text{s}^{-1}$ [16, Section 3.2.1],
  - the TRNG outputs bits at a rate of $\sim 6\text{Gbit}\,\text{s}^{-1}$, and
  - the FIFO-based output buffer is read from by using the `rdrand` instruction.
- The 'D' in DRNG here isn't for deterministic: it's for *digital* (versus analogue, which earlier Intel designs were).
- The DRNG, and hence associated instructions, are only available on modern Intel processors; on Linux, checking `/proc/cpuinfo` highlights whether or not the features is available.
- In additional to official Intel documentation [16] (which acts as a user manual), a more technical overview is given by a technical report by Hamburg et al. [15] (with a similar analysis of an earlier Intel design [17] also interesting).

Part 2: in practice (6)
Class #3: hardware-oriented

- **Design: Intel Secure Key [16].**

Algorithm (RdRand TRBG)



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Part 2: in practice (7)
Class #3: hardware-oriented

- **Design: Intel Secure Key [16].**

Listing (RdRand interface)

```
bool rdrand64( uint64_t* r ) {
  bool success;

  asm( "rdrand %0 ; setc %1"
       : "=r" (*r), "=qm" (success) );

  return success;
}
```

Listing (RdRand interface)

```
bool rdrand64_retry( uint64_t* r, int l ) {
  int i = 0;

  do {
    if( rdrand64( r ) ) {
      return true;
    }
  } while( i++ < l );

  return false;
}
```


Notes:

Part 2: in practice (8)
Class #4: system-oriented

- **Design: Linux.**
  - circa 1994(ish):
    - maintain entropy pool $\theta[i]$, injecting entropy, e.g., from system-related events,
    - define a predicate

$$P(\theta[i]) = \begin{cases} \textbf{false} & \text{if estimated entropy in } \theta[i] \text{ is deemed insufficient} \\ \textbf{true} & \text{if estimated entropy in } \theta[i] \text{ is deemed sufficient} \end{cases}$$

      based on the concept of entropy estimation,
    - expose $\theta[i]$ to user-space via the (pseudo) files

$$\begin{array}{lcl} \text{write to } \texttt{/dev/random} & \simeq & \text{inject entropy into } \theta[i] \\ \text{read from } \texttt{/dev/random} & \simeq & \begin{cases} \text{if } P(\theta[i]) = \textbf{false}, \text{ block then sample from PRNG (re)seeded from } \theta[i] \\ \text{if } P(\theta[i]) = \textbf{true}, \text{ then sample from PRNG (re)seeded from } \theta[i] \end{cases} \\ \text{read from } \texttt{/dev/urandom} & \simeq & \text{sample from PRNG (re)seeded from } \theta[i] \end{array}$$

Notes:

- A major challenge in describing "the" Linux implementation is that it is continually evolving! On one hand, this is positive in the sense it continues to improve to 1) address new threats, and 2) serve the demands of new use-cases. On the other hand, however, this moving target makes it harder to offer an accurate, general overview. Modulo their publication date, various resources offer descriptions worth reading: see, for example, Gutterman et. al [12] and Dodis et al. [11].
- There are several notable controversies associated with the approach and implementation used by Linux. These include 1) use or non-use of RdRand (based on arguments around trust), and 2) the semantics of whatever mechanism is exposed to user-space (e.g., the concept of entropy estimation, blocking versus non-blocking).
- The associated `man` page, i.e.,

  `man -s 4 random`

  gives some operational details with respect to use of `/dev/random` and/or `/dev/urandom`. It's also possible to inspect various information about of them via `/proc/sys/kernel/random`. For example, the file `/proc/sys/kernel/random/entropy_avail` exposes the estimate of available entropy in the entropy pool(s).
- An important quote from the `man` page is:

  "If you are unsure about whether you should use `/dev/random` or `/dev/urandom`, then probably you want to use the latter. As a general rule, `/dev/urandom` should be used for everything except long-lived GPG/SSL/SSH keys."

  This seems a slightly dangerous statement: *ephemeral* keys are often just as security-critical as long term (cf. forward secrecy), so arguably a better rule would be to use `/dev/random` for *anything* cryptographically related (and `/dev/urandom` otherwise). But even this is somewhat contentious, reflecting the general difficulty of this topic. To this end,

  `http://www.2uo.de/myths-about-urandom`

  offers some perspective: some points should be obvious already (e.g., the difference between *true*- and *pseudo*-random), but offers some interesting points about use of `/dev/random` versus `/dev/urandom`.
- There are various associated or alternative user- and kernel-space systems, such as the Entropy Gathering Daemon (EGD)

  `http://egd.sourceforge.net/`

Part 2: in practice (8)
Class #4: system-oriented

- **Design: Linux.**
  - circa 2014(ish):
    - update re. additional system call

```
ssize_t getrandom( void* x, size_t n, unsigned int flags )
```

      where

$$\texttt{getrandom} \simeq \begin{cases} \text{if PRNG has not been initialised, then do block} \\ \text{if PRNG has been initialised, then do not block} \end{cases}$$

    - this yields clear(er) semantics, and avoids need for file handle.

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Part 2: in practice (8)
Class #4: system-oriented

- Design: **Linux**.
  - circa 2016(ish):
    - update re. PRNG, which is changed from being based on SHA-1 to ChaCha20,
    - this yields, e.g., lower latency with respect to sampling output.

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  ```
  man -s 4 random
  ```

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Part 2: in practice (8)
Class #4: system-oriented

- Design: **Linux**.
  - circa 2020(ish):
    - update re. file-based semantics

$$\texttt{/dev/urandom} \simeq \text{do not block}$$

$$\texttt{/dev/random} \simeq \begin{cases} \text{if PRNG has not been initialised, then do block} \\ \text{if PRNG has been initialised, then do not block} \end{cases}$$

Notes:

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  "If you are unsure about whether you should use `/dev/random` or `/dev/urandom`, then probably you want to use the latter. As a general rule, `/dev/urandom` should be used for everything except long-lived GPG/SSL/SSH keys."

  This seems a slightly dangerous statement: *ephemeral* keys are often just as security-critical as long term (cf. forward secrecy), so arguably a better rule would be to use `/dev/random` for *anything* cryptographically related (and `/dev/urandom` otherwise). But even this is somewhat contentious, reflecting the general difficulty of this topic. To this end,

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Conclusions

Quote

Any one who considers arithmetical methods of producing random digits is, of course, in a state of sin.

– von Neumann (http://en.wikiquote.org/wiki/Randomness)

Quote

The generation of random numbers is too important to be left to chance.

– Coveyou (http://en.wikiquote.org/wiki/Randomness)

Quote

The design of such pseudo-random number generation algorithms, like the design of symmetric encryption algorithms, is not a task for amateurs.

– Eastlake, Schiller, and Crocker [19]

Notes:

Conclusions

- Take away points:
  1. A high-quality source of randomness is fundamental to more or less *every* security proof: it might be an assumption in in theory, but in practice this issue requires care.
  2. Iff. you need to develop your own PRBG implementation, use a standard (e.g., NIST SP800-90A [20]) design or framework ...
  3. ... often such a design can leverage a primitive (e.g., a block cipher) you need anyway, thus reducing effort, attack surface, etc.
  4. Some golden rules:
     - use a large, high-entropy seed,
     - avoid reliance on a single entropy source where possible,
     - opt for a cryptographically secure design and ensure it is parameterised correctly,
     - hedge against failure via robust pre- and post-processing where need be,
     - include quality tests on pseudo-randomness generation (e.g., alongside functional unit testing),
     - don't compromise security for efficiency,
     - ...

Notes:

Additional Reading

- *Wikipedia: Randomness*. URL: `https://en.wikipedia.org/wiki/Randomness`.
- *Wikipedia: Pseudorandomness*. URL: `https://en.wikipedia.org/wiki/Pseudorandomness`.
- *Wikipedia:* `/dev/random`. URL: `https://en.wikipedia.org/wiki/dev/random`.
- *Wikipedia: RDRAND*. URL: `https://en.wikipedia.org/wiki/RDRAND`.
- K.H. Rosen. "Chapter 7: Discrete probability". In: *Discrete Mathematics and Its Applications*. 7th ed. McGraw Hill, 2013.
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- D. Johnston. *Random Number Generators – Principles and Practices: A Guide for Engineers and Programmers*. 1st ed. De|G Press, 2018.
- D. Eastlake, J. Schiller, and S. Crocker. *Randomness Requirements for Security*. Internet Engineering Task Force (IETF) Request for Comments (RFC) 4086. 2005. URL: `http://tools.ietf.org/html/rfc4086`.

Notes:

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[2] *Wikipedia: Pseudorandomness*. URL: `https://en.wikipedia.org/wiki/Pseudorandomness` (see p. 89).

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Notes: