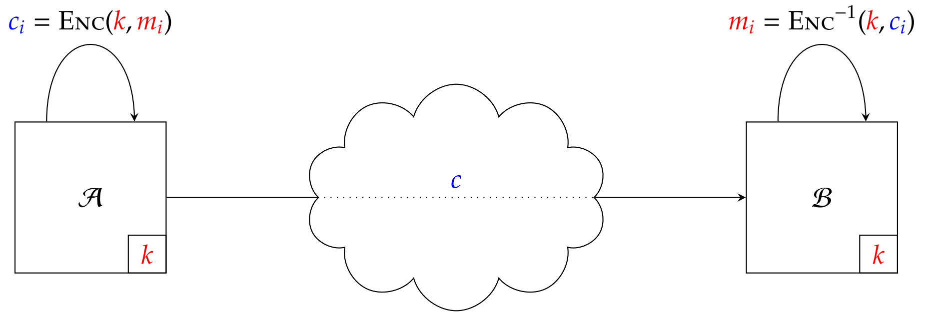
COMS30048 hand-out: exam-style revision questions

Q1. Imagine some party $\mathcal{A}$ wants to send an n-bit plaintext message m (e.g., an email) to party $\mathcal{B}$ via a public network (e.g., the Internet). Since M contains sensitive content, the parties intend to secure the communication using cryptography. First, they agree a shared n_k-bit key k. Then, m is split into l blocks with the i-th such n_b-bit block denoted m_i. Next, each block of m is encrypted using a block cipher Enc to produce a corresponding ciphertext c. Finally, c can be communicated using the existing network stack. The setting can be roughly illustrated as follows:



a This setting describes a solution based on symmetric cryptography.

i Name a concrete choice for the block cipher Enc.

ii Briefly explain how an alternative solution using asymmetric cryptography would work.

b The parties could encrypt m themselves as above, *or* the network stack could be tasked with doing it for them: if the latter were true, where, within the OSI model for example, might you expect the process to happen?

c Currently, authenticity of the parties is not ensured. Informally outline a potential problem with this based on the existence of an attacker $\mathcal{E}$.

d Use of the block cipher ensures the confidentiality but *not* the integrity of m: informally describe what these terms mean, and why they are ensured (or not).

e A Message Authentication Code (MAC) can be used to detect manipulation of the message, and hence ensure integrity. Given a second shared key k', at least two options are possible:

MAC-then-encrypt: compute a tag $\tau = \textsc{Tag}(k', m)$, then communicate $c = \textsc{Enc}(k, m \parallel \tau)$.

encrypt-then-MAC: compute $c = \textsc{Enc}(k, m)$ and a tag $\tau = \textsc{Tag}(k', c)$, then communicate $C' = C \parallel \tau$.

Outline a possible advantage of **each** option versus the other.

f Currently, the relationship

$$c_i = \textsc{Enc}(k, m_i)$$

details how blocks of c are produced from blocks of m.

i Name this mode of operation, and informally explain why using it might not be ideal.

ii Name and describe (using a relationship between blocks of m and c as above) a more preferable mode of operation. Informally explain why this alternative solves the problem you outlined above.

g Imagine n_b does not divide n exactly, meaning the final block m_{l-1} has fewer than n_b bits in it.

i Outline a problem faced by $\mathcal{A}$ and $\mathcal{B}$ when this situation occurs.

ii Stating any assumptions you make, outline a scheme the parties could use to solve this problem.

Q2. The following questions concern a block cipher defined by the following two functions

$$\begin{array}{lcl} \textsc{Enc} & : & \{0,1\}^{n_k} \times \{0,1\}^{n_b} \rightarrow \{0,1\}^{n_b} \\ \textsc{Dec} & : & \{0,1\}^{n_k} \times \{0,1\}^{n_b} \rightarrow \{0,1\}^{n_b} \end{array}$$

and that uses a n_k-bit key to encrypt an n_b-bit plaintext into an n_b-bit ciphertext (and visa versa).

a A cryptographic hash function

$$\textsc{Hash} : \{0,1\}^* \rightarrow \{0,1\}^{n_d}$$

maps an arbitrary (but finite) length input message m into an n_d-bit digest d.

i Explain how to construct Hash using the block cipher.

ii Describe the purpose of an MDC versus that of a MAC. Given Hash is unkeyed, is it an MDC or a MAC?

b It is suggested the block cipher could be used as a Pseudo-Random Number Generator (PRNG).

i Explain what a PRNG is, and describe **one** application for which such a primitive might be used.

ii Design a PRNG construction based on Enc, making sure to include

- an algorithm to update the state and produce output,
- the size of the PRNG state and output,
- the maximum period of the PRNG, and
- any advantages and disadvantages of this approach versus (named) alternatives.

Q3. Consider a block cipher with a n_k-bit key size and n_b-bit block size.

a Imagine someone selects a value for n_k. Being careful to state any assumptions, describe how one might reason whether a choice of n_k is too small for

i encryption of government documents,

ii use on an RFID tag attached to boxes of chocolate.

b Imagine there are two choices, i.e., either

i $n_k = 16$ and $n_b = 128$, or

ii $n_k = 128$ and $n_b = 8$

Explain why **both** choices are inadvisable (from a security perspective), and how one might break the resulting block ciphers.

Q4. Given the goal of key recovery, informally differentiate between

a a known ciphertext attack,

b a ciphertext only attack,

c a chosen ciphertext attack, and

d an adaptive chosen ciphertext attack.

Q5. a For some multiplicative group $\mathbb{G}$ of order q generated by g, define the following:

i Discrete Logarithm Problem (DLP),

ii Diffie-Hellman Problem (DHP),

iii Decisional Diffie-Hellman Problem (DDH).

b For some problems X and Y, explain what it means to write $X \leq_P Y$.

c Explain why (and how) $DDH \leq_P DLP$.

Q6. a Consider the elliptic curve

$$E : y^2 = x^3 + a_4 x + a_6$$

defined over the field $\mathbb{F}_p$ where $p = 7$ and $a_4 = a_6 = 1 \in \mathbb{F}_p$.

i List all the rational points on this curve.

ii Explain how a point $P = (P_x, P_y) \in E(\mathbb{F}_p)$ can be compressed and then decompressed to reduce the cost of communicating it, detailing what saving (in bits) is made.

b Consider the elliptic curve

$$E : y^2 = x^3 + a_4 x + a_6$$

of order n defined over the field $\mathbb{F}_p$ where p is a 256-bit prime and $a_4, a_6 \in \mathbb{F}_p$, and two public-key encryption schemes: EC-IES using $E(\mathbb{F}_p)$, and RSA using a 1024-bit modulus.

i Compare the key length and key generation algorithms for the two schemes, taking care to note advantages and disadvantages.

ii The public key in RSA is a pair (N, e) where N is the modulus and e is the encryption exponent; "small" values of e are permitted, without negative impact on security, to accelerate encryption operations. Explain whether or not, and why, there is an analogous concept in EC-IES.

iii Describe how standard binary exponentiation can be improved in order to compute $Q = [k]P$ more efficiently (given both $P \in E(\mathbb{F}_p)$ and $0 \le k < n$), and why your approach is more efficient.

Q7. a Define the Elliptic Curve Discrete Logarithm Problem (EC-DLP), and estimate the size of group required to provide a sufficient level of security.

b EC-DSA is an elliptic curve version of the DSA digital signature scheme whose security is based on the EC-DLP.

i Define *how* EC-DSA works, including

- domain parameters,
- key generation,
- signing, and
- verification.

ii Explain *why* EC-DSA works, i.e., why one can successfully verify a valid signature on some message.

iii Highlight **three** reasons why EC-DSA is usually more efficient than DSA.

iv Some elliptic curve based signature schemes need to hash an arbitrary length message m into a valid elliptic curve point P. Explain how this is possible.

Q8. Imagine you need to implement the RSA public key encryption scheme, and must therefore generate key material including

- a modulus $N = p \cdot q$, and
- a public and private exponent, e and d, such that $e \cdot d \equiv 1 \pmod{\Phi(N)}$.

Briefly outline

a **two** properties that the generated p and q must satisfy to ensure security, and

b **two** guidelines that might be followed to ensure the key material, once generated, *remains* secure and hence can be used for as long as possible.

Q9. The following text represents the output of OpenSSL when used to dump an X.509 certificate:

```
Certificate:
  Data:
    Version: 3 (0x2)
    Serial Number:
        4f:9d:96:d9:66:b0:99:2b:54:c2:95:7c:b4:15:7d:4d
    Signature Algorithm: sha1WithRSAEncryption
    Issuer: C=ZA, O=Thawte Consulting (Pty) Ltd., CN=Thawte SGC CA
    Validity
        Not Before: Oct 26 00:00:00 2011 GMT
        Not After : Sep 30 23:59:59 2013 GMT
```

```
    Subject: C=US, ST=California, L=Mountain View, O=Google Inc, CN=www.google.
        com
    Subject Public Key Info:
        Public Key Algorithm: rsaEncryption
            Public-Key: (1024 bit)
            Modulus:
                00:de:b7:26:43:a6:99:85:cd:38:a7:15:09:b9:cf:
                0f:c9:c3:55:8c:88:ee:8c:8d:28:27:24:4b:2a:5e:
                a0:d8:16:fa:61:18:4b:cf:6d:60:80:d3:35:40:32:
                72:c0:8f:12:d8:e5:4e:8f:b9:b2:f6:d9:15:5e:5a:
                86:31:a3:ba:86:aa:6b:c8:d9:71:8c:cc:cd:27:13:
                1e:9d:42:5d:38:f6:a7:ac:ef:fa:62:f3:18:81:d4:
                24:46:7f:01:77:7c:c6:2a:89:14:99:bb:98:39:1d:
                a8:19:fb:39:00:44:7d:1b:94:6a:78:2d:69:ad:c0:
                7a:2c:fa:d0:da:20:12:98:d3
            Exponent: 65537 (0x10001)
    X509v3 extensions:
        X509v3 Basic Constraints: critical
            CA:FALSE
        Authority Information Access:
            OCSP - URI:http://ocsp.thawte.com
            CA Issuers - URI:http://www.thawte.com/repository/Thawte_SGC_CA.crt
Signature Algorithm: sha1WithRSAEncryption
    21:ac:d5:ae:ca:34:89:5a:c2:ab:52:d2:b2:34:66:9d:7a:ab:
    ee:e6:7c:d5:7e:c2:5c:28:bb:74:00:c9:10:1f:42:13:fc:69:
    8a:1e:24:a0:02:00:e9:ba:5b:ca:19:04:b2:d3:af:01:b2:7e:
    5f:14:db:a6:db:52:b9:9a:f3:12:7f:7c:a2:9c:3b:6f:99:7d:
    ea:50:0d:76:23:12:ff:f7:66:73:29:b7:95:0a:ad:d8:8b:b2:
    de:20:e9:0a:70:64:11:08:c8:5a:f1:7d:9e:ec:69:a5:a5:d5:
    82:d7:27:1e:9e:56:cd:d2:76:d5:79:2b:f7:25:43:1c:69:f0:
    b8:f9
```

Note that some of the output has been highlighted. Imagine you are a client web-browser that, in the process of engaging in an SSL handshake with the server www.google.com, downloads this certificate: explain the purpose of **each** highlighted fragment within the context of said handshake.

Q10. Imagine you are involved in the design and configuration of an SSL server for an e-commerce web-site which will have a high volume of traffic.

a Various products exist that accelerate cryptographic operations via dedicated hardware housed on a plug-in card. This is an interesting option for the server: ignoring budget, what sort of accelerator (i.e., for what operation) would you select and why?

b Your employers are concerned about the threat of Denial of Service (DoS) attacks on the server. Although confident the computational and network capacity mean the server will cope with a huge number of rogue connections, they have read about something called a "resource depletion" attack. Explain what resources you think the attack might refer to, and why their depletion might represent a threat.

Q11. In late 2011 the servers of DigiNotar, a Dutch Certificate Authority (CA) which issued SSL certificates, were attacked; the gained access to the servers, and (presumably) copied data include private key material. Explain what the implications for this are within the context of SSL.

Q12. Consider `TLS_DHE_RSA_WITH_AES_128_CBC_SHA`, a common TLS cipher suite identifier. State what algorithm is used for each of

a end point authentication,

b application data authenticity,

c key exchange, and

d application data encryption,

and, in detail, how each algorithm is used for the associated role (based on communication between two end points, a client and server).

Q13. A new company aims to produce a product based on secure video downloads: a given video stream is split into frames and then encrypted, on-demand, by a server. Users pay for a key that allows films to be downloaded, decrypted and then viewed using client software. Licensing issues mean the server and client systems must be implemented from scratch: you must provide advice during the development process.

a The company is trying to select between DES, AES and RSA; explain which encryption scheme would you recommend and why.

b Neither DES, AES nor RSA should be used as a raw (or "textbook") encryption primitive; explain why this is and what you recommended as an alternative.

c In an effort to reduce their bandwidth requirements, the company decide to compress the video; explain why they should compress before encrypting rather than the other way round.

d The users can either

i each be given the same key, or

ii each be given a different key.

Explain the advantages **and** disadvantages of **each** choice.

e To improve performance on the server, the company want to utilise the large amount of memory available; using your choice from the first part of the question, explain if this is possible or not.

Q14. Recall that a Linear Congruential Generator (LCG) parameterised by constants a, c and p starts with a seed x_0, and generates successive pseudo-random numbers via the equation

$$x_i = a \cdot x_{i-1} + c \pmod{p}.$$

Imagine the parameters

$$\begin{array}{rcr} a & = & 9821 \\ c & = & 6925 \\ m & = & 65535 \end{array}$$

are selected for a hardware implementation within a new smart-card whose clock frequency is 8kHz; the LCG produces a 16-bit pseudo-random number every clock cycle. The smart-card is used in a cryptographic protocol in which LCG outputs are used as nonces.

a Imagine the randomness of nonces generated by the LCG is crucial: if they are not random, the protocol can be attacked. What issue can you identify with the design as outlined?

b Given the implementation is in hardware, explain whether or not, and why, you think an LCG is a suitable approach.

c The smart-card has a hardware-based block cipher implementation which is used within the same protocol: explain an alternative approach, using this component, which would be preferential for generating the nonces.

Q15. In a stack smashing attack against some target device $\mathcal{D}$, an attacker $\mathcal{E}$ sends a malign string `x`, which is written into a buffer `t` within $\mathcal{D}$. Since `x` is too large to fit into `t`, it overwrites an return address and the control-flow is redirected into `t`: $\mathcal{D}$ subsequently executes some shellcode chosen by $\mathcal{E}$.

a Consider two arrays

```
uint8_t shellcode1[] = { 0x8D, 0x4C, 0x24, 0x04, 0x83, 0xE4, 0xF0, 0xFF,
                         0x71, 0xFC, 0x55, 0x89, 0xE5, 0x53, 0xE8, 0x00,
                         0x00, 0x00, 0x00, 0x5B, 0x83, 0xC3, 0xED, 0x8D,
                         0x83, 0x42, 0x00, 0x00, 0x00, 0x89, 0x45, 0xF0,
                         0xC7, 0x45, 0xF4, 0x00, 0x00, 0x00, 0x00, 0x8D,
                         0x4D, 0xF0, 0xB8, 0x0B, 0x00, 0x00, 0x00, 0x31,
                         0xD2, 0x53, 0x8B, 0x5D, 0xF0, 0xCD, 0x80, 0x5B,
                         0x83, 0xC4, 0x10, 0x59, 0x5B, 0xC9, 0x8D, 0x61,
                         0xFC, 0xC3, 0x2F, 0x62, 0x69, 0x6E, 0x2F, 0x73,
                         0x68, 0x00 };

uint8_t shellcode2[] = { 0xEB, 0x0D, 0x5E, 0x31, 0xC9, 0xB1, 0x4A, 0x80,
                         0x36, 0x01, 0x46, 0xE2, 0xFA, 0xEB, 0x05, 0xE8,
                         0xEE, 0xFF, 0xFF, 0xFF, 0x8C, 0x4D, 0x25, 0x05,
                         0x82, 0xE5, 0xF1, 0xFE, 0x70, 0xFD, 0x54, 0x88,
                         0xE4, 0x52, 0xE9, 0x01, 0x01, 0x01, 0x01, 0x5A,
                         0x82, 0xC2, 0xEC, 0x8C, 0x82, 0x43, 0x01, 0x01,
                         0x01, 0x88, 0x44, 0xF1, 0xC6, 0x44, 0xF5, 0x01,
                         0x01, 0x01, 0x01, 0x8C, 0x4C, 0xF1, 0xB9, 0x0A,
                         0x01, 0x01, 0x01, 0x30, 0xD3, 0x52, 0x8A, 0x5C,
                         0xF1, 0xCC, 0x81, 0x5A, 0x82, 0xC5, 0x11, 0x58,
                         0x5A, 0xC8, 0x8C, 0x60, 0xFD, 0xC2, 0x2E, 0x63,
                         0x68, 0x6F, 0x2E, 0x72, 0x69, 0x01 };
```

that **both** represent x86 machine code programs that execute `/bin/sh`, a command shell. $\mathcal{E}$ aims to use one of the programs in the attack described above: explain **two** features that could guide a choice between them.

b Outline **two** software-only countermeasures that could be used to protect $\mathcal{D}$ from this attack.

Q16. The following questions concern security of the ElGamal signature scheme against fault attacks; in each case, provide a concise answer (i.e., use only a few sentences).

a The ElGamal signature scheme consists of three algorithms, namely key generation, signature generation and signature verification. Given the first two are defined (on the left- and right-hand side respectively) as

Input: Security parameters λ_p and λ_q
Output: A public/private key pair

i Select a suitable λ_p-bit prime p, and g, an λ_q-bit generator of $\mathbb{Z}_p^*$.

ii Select a random x such that $1 < x < p-1$.

iii Compute $y = g^x \pmod{p}$.

iv Return the public key (p, g, y) and private key (p, g, x).

Input: A private key (p, g, x), a message m
Output: A signature $\sigma = (r, s)$ on m

i Select a random k st. $0 < k < p-1$ and $\gcd(k, p-1) = 1$.

ii Compute $r = g^k \pmod{p}$.

iii Given a suitable hash function μ, compute $s = (\mu(m) - x \cdot r)/k \pmod{p-1}$.

iv The pair (r, s) is a signature on the message m.

describe the third algorithm for signature verification.

b Imagine this scheme is implemented on a basic smart-card (i.e., no cryptographic accelerator, or counter-measures against attack); what types of fault attack can you image being applied to such a target?

c Explain how a fault attack might influence the ephemeral value r, and the effect this might have on signatures generated. Carefully including any assumptions, outline such an attack that can recover x, the private key.

Q17. Consider a new, light-weight block cipher design which is based loosely on AES. To encrypt an 8-bit plaintext `m` with the cipher key `k`, the following C function is used:

```
uint8_t encrypt( uint8_t* k, uint8_t m ) {
  uint8_t t = m;

  for( int i = 0; i < r; i++ ) {
    t = sbox_encrypt[ t ^ k[ i ] ];

    if( t & 0x80 ) {
      t = 0x1B ^ ( t << 1 );
    }
    else {
```

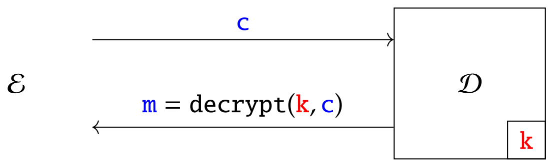
```
      t =         ( t << 1 );
    }
  }

  return t;
}
```

That is, the function processes the state `t` in `r` rounds, each of which contains three steps:

- XOR the state with `k`,
- pass the state through the AES S-box for encryption, then
- update the state via the AES `xtime` function.

The corresponding decryption function is used in a set-top-box device which will decrypt ciphertexts with an embedded (and unknown) key, then return the corresponding plaintext:



The designers are confident that brute-force attacks are unfeasible within applications the device will be used for; they are, however, worried about the threat of attacks relating to physical security.

a Assuming the AES decryption S-box is available (i.e., the inverse of `sbox_encrypt` is held in `sbox_decrypt`), write the decryption function used by the device.

b Comment on any sources of information leakage from the device; describe how you could solve **each** problem identified (e.g., a potential countermeasure).

c Imagine you are able to mount a specific fault attack against the device: before any one round, you can corrupt `sbox_decrypt` by setting any number of elements to any value you choose. Given you aim to recover the embedded key, explain

- which round you target,
- which elements you corrupt with which values, and
- why this approach is advantageous to you, the attacker.

Q18. Consider a client tasked with recording then communicating sensor data to a server. Authenticity of messages is not important, but they are encrypted using a standard block cipher under a shared 128-bit key k_{root} (which is refreshed periodically in a secure manner).

Both client and server have access to a Key Derivation Function (KDF) called KeyTree which can generate message keys from k_{root}:

Input: The 128-bit root key k_{root}, an n-bit unsigned integer p
Output: A 128-bit message key $k_{message}$

$k_{message} \leftarrow k_{root}$
for $i = n - 1$ **downto** 0 **do**
  $k_{message} \leftarrow f(k_{message}, p_i)$
end
return $k_{message}$

The algorithm uses two functions

$$\begin{array}{rcl} H_0 & : & \{0,1\}^{128} \rightarrow \{0,1\}^{128} \\ H_1 & : & \{0,1\}^{128} \rightarrow \{0,1\}^{128} \end{array}$$

to define $f(x, i) = H_i(x)$, meaning f applies the i-th function to input x. p_i denotes the i-th bit of p, which is represented in binary.

a Suppose that $p = 6$. Write an equation for $k_{message}$, as returned by KeyTree, in terms of H_0, H_1 and k_{root}; using a diagram and assuming $n = 3$, illustrate how the space of all message keys is generated by KeyTree.

b For a given k_{root}, how many possible message keys exist?

c There are two versions of the client. Both use the same software implementation of a block cipher denoted Enc, but different approaches for encryption of messages:

- The first version uses the root key k_{root} to encrypt each message block using Enc.
- The second version first calls
$$k_{message} = \textsc{KeyTree}(k_{root}, id)$$
with a random integer id, then uses the message key $k_{message}$ to encrypt each message block using Enc.

The second version of the client was produced because of an known SPA attack on the implementation of Enc: the attack can recover the cipher key used during encryption or decryption of a given message block.

i Informally explain what an SPA attack is, including any assumptions that must be satisfied to consider such an attack.

ii The server must be able to decrypt messages sent by the client: define a message format that allows version two of the client to satisfy this requirement.

iii The designers believe the second version of the client is protected from the SPA attack: explain whether **and** why they are correct (or not).

iv The designers want to instantiate H_0 and H_1 with Enc, and define
$$\begin{array}{rcl} H_0(x) & = & \textsc{Enc}(x, 0) \\ H_1(x) & = & \textsc{Enc}(x, 1) \end{array}$$
so the i-th function encrypts a fixed message, i.e., 0 or 1, under the key x. Discuss the impact this might have on security for the second version of the client.

v Briefly explain **two** ways to implement a hiding countermeasure against the SPA attack described.

d Irrespective of the above, assume the second version of the client is immune to SPA attacks and the designers did not opt to instantiate H_0 and H_1 with Enc after all. However, they are now worried about DPA attacks in the same setting (i.e., which recover the cipher key).

i Informally explain why a DPA attack might be feasible if the message is 1000 blocks long, but not 10 blocks.

ii How could KeyTree be used to improve resilience against the DPA attack? In your answer, carefully describe the new message encryption approach **and** discuss why it improves security.